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A New Efficient Numerical Approach for Delay Differential Equations †

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Abstract: In this paper, different types of nonlinear delay differential equations are solved using the hybrid technique of the differential transform and the Bell polynomial. The obtained results are compared to those obtained previously. The nonlinear terms involved in delay differential equations are handled efficiently with the help of Bell polynomials. The presented technique has been tested on three concrete problems. Different types of errors, such as absolute and maximal, are computed to show the effectiveness and reliability of the method.

Keywords: delay differential equations; differential transform; bell polynomial; convergence; errors

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1. Introduction

Integer and non-integer nonlinear delay differential equations (NDDEs) find applications in mathematical modeling and processes. Many researchers have attempted different analytical and numerical methods to obtain their solution. Each method has its own strengths and limitations; the details can be seen in [1–4,6,8,10,11]. While solving NDDEs, nonlinear terms are to be handled efficiently. Hence, we require a technique that can deal with different kinds of nonlinear terms such as algebraic, logarithmic, trigonometrical, etc. Differential transformation combined with Bell polynomials is one of the suitable techniques to solve NDDEs. The details about differential transforms and Bell polynomials can be found in [5,7,9] and the references therein. The main aim of the paper is to solve NDDEs using a hybrid technique of differential transforms and Bell polynomials.

The paper is organized as follows: In Section 2, we discuss the algorithm of the method, and in Section 3, we discuss the results and discussion. The conclusion is in Section 4.

2. Description of the Method

In this paper we consider following system of p proportional delay differential equations

$$w^{(n)}(v)=\sum_{j=0}^{J}a_j(v)\,w\left(\frac{v}{b_j}\right)+h(w)+m(v), \tag{1}$$

where $w^{(n)}(v)=\left(\mathbf{w}_1^{(\mathbf{n})}(\mathbf{v}),\ldots,\mathbf{w}_\mathbf{p}^{(\mathbf{n})}(\mathbf{v})\right)^T$, $w(v)=\left(\mathbf{w}_1(\mathbf{v}),\ldots,\mathbf{w}_\mathbf{p}(\mathbf{v})\right)^T$, $m(v)=\left(\mathbf{m}_1(\mathbf{v}),\ldots,\mathbf{m}_\mathbf{p}(\mathbf{v})\right)^T$, and $h(w)=\left(\mathbf{h}_1(\mathbf{w}),\ldots,\mathbf{h}_\mathbf{p}(\mathbf{w})\right)^T$ are p-dimensional vector functions and h(w) is nonlinear function represented as

$$h(w)=\sum_{i=1}^{l}\beta_i f_i\left(g_i\left(\frac{v}{q_i}\right)\right)$$

with the initial conditions

$$\sum_{r=0}^{n-1} w^{(r)}(v_0)=w_\alpha,\alpha=0,1,\ldots,n-1, \quad (2)$$

$\beta_i\in\mathbf{R}, l, J\in\mathbf{N}, b_j\geq 1, q_i\geq 1, a_j(.)$ and $m(.)$ are some functions of v, f_i are continuous known functions, $g_i=w$ are continuous unknown functions and $(.)^{(n)}$ is the nth derivative of the function w.

Steps of the method can be described as

Step 1: Apply differential transform to initial conditions (2)

Step 2: Apply differential transform to Equation (1) and bell polynomials to nonlinear terms available in Equation (1).

Step 3: Obtain a recursion system using Step 1 and Step 2.

Step 4: Solve the recursive system obtain in Step1 and Step 2 and determine unknowns W(0), W(1).

Step 5: Using inverse differential transform obtain series solution for problem (1) and (2).

3. Results and Discussion

We present three numerical examples of NDDEs to show efficiency and reliability of the presented method. All computations have been carried out in MATHEMATICA software.

Example 1. *Consider the following nonlinear delay differential equation.*

$$w'''(v)=-2w'(v)+8w\left(\frac{v}{2}\right)\sqrt{1-w^2\left(\frac{v}{2}\right)} \quad (3)$$

with initial conditions

$$w(0)=1, w'(0)=0, w''(0)=-4 \quad (4)$$

The exact solution is given by

$$w(v)=\cos 2v \quad (5)$$

Table 1. Comparison of numerical solution w_N (v) with exact solution when N = 15 for example 1.

v	w(v)	w_N(v)	R_N(v)
0.1	0.98006658	0.98006658	0
0.2	0.92106099	0.92106099	1.2×10^{-16}
0.3	0.82533561	0.82533561	1.3×10^{-16}
0.4	0.69670671	0.69670671	1.9×10^{-15}
0.5	0.54030231	0.54030231	8.8×10^{-14}

Example 2. *Consider the following nonlinear delay differential equation*

$$w''(v) = 2\ln\left(w\left(\frac{v}{3}\right)\right) + 9w^3\left(\frac{v}{3}\right) - 2v \tag{6}$$

with initial conditions

$$w(0) = 1, w'(0) = 3 \tag{7}$$

The exact solution is given as

$$w(v) = e^{3v} \tag{8}$$

Table 2. Comparison of numerical solution w_N (v) with exact solution when N = 10 for Example 2.

v	**w(v)**	**w_N(v)**	**R_N(v)**
0.1	1.3498588	1.3498588	3.3×10^{-14}
0.2	1.8221188	1.8221188	5.2×10^{-11}
0.3	2.4596031	2.4596031	3.4×10^{-9}
0.4	3.3201169	3.3201167	6.2×10^{-8}
0.5	4.4816891	4.4816866	5.5×10^{-7}
0.6	6.0496475	6.0496286	3.1×10^{-6}
0.7	8.1661699	8.1660639	1.2×10^{-5}
0.8	11.023176	11.022702	4.3×10^{-5}
0.9	14.879732	14.877945	1.2×10^{-4}
1.0	20.085537	20.079665	2.9×10^{-4}

Example 3. *Consider following nonlinear delay differential equation*

$$(2v+1)w'(v) = 2ve^{w\left(\frac{v}{2}\right)} + e^{w\left(\frac{v}{2}\right)} - 2v^2 - 3v + 1 \tag{9}$$

with initial conditions

$$w(0) = 0 \tag{10}$$

Exact solution is given as

$$w(v) = \log(2v+1) \tag{11}$$

Table 3. Comparison of numerical solution w_N (v) with exact solution when N = 15 for Example 3.

v	**w(v)**	**w_N(v)**	**R_N(v)**
0.1	0.18232156	0.18232156	1.8×10^{-12}
0.2	0.33647224	0.33647226	5.7×10^{-8}
0.3	0.47000363	0.4700149	2.3×10^{-5}
0.4	0.58778666	0.58879092	1.7×10^{-3}
0.5	0.69314718	0.72537185	4.6×10^{-2}

4. Conclusions

We propose a new technique to handle nonlinear delay differential equations. The main advantage of this approach is application of Bell polynomials in order to resolve nonlinearity. The technique is straight forward and can be applied to other higher order nonlinear DDEs.

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