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Proceeding Paper

Dynamic Vibration Analysis of Two Timoshenko Beams in the Presence of Friction †

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Abstract: This paper presents the effect of friction in the vibration behavior between two beams in relative motion according to Timoshenko's beam theory. The current system is composed of two cantilever beams screwed together. The nonlinear behavior can be divided into two phases: stick and slip. The differential equations of motion in the two phases are presented. In one hand, the viscous damping is added in the stick phase. In the other hand, the viscous damping and the frictional force applied are added in the slipping phase. The Galerkin method is used for the solution of differential equations to transfer the system of partial differential equations to the system of ordinary differential equations.

Keywords: Timoshenko beam; stick slip phenomenon; Galerkin method

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1. Introduction

In dynamical systems and in the presence of nonlinearity, analytical solution is limited to solve the equation of motion. However, approximate methods of solutions are used to simulate the dynamic behavior of beams. Among these methods we can cite: The Rayleigh–Ritz method, the finite-element method, and the weighted-residual methods. The Rayleigh–Ritz method is used to obtain the natural frequencies of beams. However, it is not suitable for non-conservative systems [1]. It is limited to systems of equations that can be expressed in energy or in weak forms [2]. The finite element method has not such limitations; but, it can be very expensive in terms of computation. In contrast, the Galerkin method, a weighted-residual method, is computationally efficient, capable of handling non conservative and nonlinear systems, and suits dynamic problems.

In the present work, we take the differential equations for two cantilever beams presented in the reference [3], the two beams are connected together by bolts which allow the apparition of friction at the contact interface, the effect of rotatory inertia and shear deformation are taken into account in the medialization of the two beams. However, no damping term appears in the differential equation. The main contribution of the present study is to add the viscous damping in stick and slip motions and the structural damping in slip motion. Finally, The Galerkin method is used to transfer the partial differential equation to an ordinary differential equation.

2. Methods

Considering two prismatic beams assembled with bolts. The boundary conditions of beams are clamped-free. Assuming that the section of the beams is constant, the two beams are compatible, i.e., they have the same transverse displacement and bending slope. Supposing each beam here is Timoshenko beam.

The equation of motion can be given as it is presented in [3].

In stick phase:

$$\left.\begin{aligned} &2\kappa GS(\frac{\partial^2 w}{\partial x^2}-\frac{\partial \alpha}{\partial x}) = 2\rho S\frac{\partial^2 w}{\partial t^2} \\ &2\kappa GS(\frac{\partial^2 w}{\partial x^2}-\frac{\partial \alpha}{\partial x}) - 2\rho I\frac{\partial^3 \alpha}{\partial^2 t\partial x} = -(\frac{T_h^2 ES + 4EI}{2})\frac{\partial^3 \alpha}{\partial x^3}\end{aligned}\right\} \quad (1)$$

In slip phase:

$$\left.\begin{aligned} &2\kappa GS(\frac{\partial^2 w}{\partial x^2}-\frac{\partial \alpha}{\partial x}) = 2\rho S\frac{\partial^2 w}{\partial t^2} \\ &2\kappa GS(\frac{\partial^2 w}{\partial x^2}-\frac{\partial \alpha}{\partial x}) - 2\rho I\frac{\partial^3 \alpha}{\partial^2 t\partial x} = -2EI\frac{\partial^3 \alpha}{\partial x^3}\end{aligned}\right\} \quad (2)$$

where: κ: Shear correction factor which depends on the shape of the cross section, G: Shear modulus, S: Cross section, $w(x,t)$: Transverse displacement, $\alpha(x,t)$: Bending slope, ρ: Density of material, I: Second moment of area of the cross section with respecting to the bending axis, E: Young's modulus.

The systems of Equations (1) and (2) do not contain any form of damping. To take into account the viscous damping in the system of differential equations for stick, the system of Equation (1) takes the form:

$$\left.\begin{aligned} &\kappa GS(\frac{\partial \alpha}{\partial x}-\frac{\partial^2 w}{\partial x^2}) + c_1\frac{\partial w}{\partial t} + \rho S\frac{\partial^2 w}{\partial t^2} = 0 \\ &\kappa GS(\alpha-\frac{\partial w}{\partial x}) + \rho I\frac{\partial^2 \alpha}{\partial t^2} = (\frac{EST_h^2 + 4EI}{4})\frac{\partial^2 \alpha}{\partial x^2}\end{aligned}\right\} \quad (3)$$

where c_1: viscous damping coefficient in the stick phase.

In the slipping phase in addition to the viscous damping, we take into account the structural damping in the system of Equation (2) to have the system of Equation (4).

$$\left.\begin{aligned} &\kappa GS(\frac{\partial \alpha}{\partial x}-\frac{\partial^2 w}{\partial x^2}) + c_2\frac{\partial w}{\partial t} + \rho S\frac{\partial^2 w}{\partial t^2} + F_{total}\,sign(\frac{\partial \alpha}{\partial t}) = 0 \\ &\kappa GS(\alpha-\frac{\partial w}{\partial x}) + \rho I\frac{\partial^2 \alpha}{\partial t^2} = EI\frac{\partial^2 \alpha}{\partial x^2}\end{aligned}\right\} \quad (4)$$

where c_2: viscous damping coefficient in the slip phase, F_{total}: friction force, it is given by $F_{total}(x) = \int_0^l \mu\gamma(x)dx$, where: μ: friction coefficient, $\gamma(x)$: linear lateral force due to bolt tension.

Solving the Bolted Beam Problem by the Galerkin Method

Looking for the solutions of the system of Equation (3) by the Galerkin method [4]. In dimensionless quantities, this system takes the following form:

$$\left.\begin{aligned} &\kappa GS\left(\frac{\partial\alpha}{l\partial\Delta}-\frac{\partial^2 w}{l^2\partial\Delta^2}\right)+c_1\frac{\partial w}{\partial t}+\rho S\frac{\partial^2 w}{\partial t^2}=0 \\ &\kappa GS\left(\alpha-\frac{\partial w}{l\partial\Delta}\right)+\rho I\frac{\partial^2\alpha}{\partial t^2}=\left(\frac{EST_h^2+4EI}{4l^2}\right)\frac{\partial^2\alpha}{\partial\Delta^2} \end{aligned}\right\} \tag{5}$$

where: $\Delta=\dfrac{x}{l}$.

Looking for the solutions in the form:

$$w(\Delta,t)=\sum_{i=1}^{\infty}W_i q_i(t) \tag{6}$$

$$\alpha(\Delta,t)=\sum_{i=1}^{\infty}A_i q_i(t) \tag{7}$$

W_i et A_i are calculated according to the method used in the reference [5]. $q_i(t)$: time function of the section rotation and transverse displacement.

Introducing the solutions (6) and (7) in the system of Equations (5):

$$\left.\begin{aligned} &\kappa GS\int_0^1\sum_{i=1}^{\infty}\left(\frac{dA_i}{ld\Delta}-\frac{d^2W_i}{l^2d\Delta^2}\right)W_j q_i(t)d\Delta+c_1\int_0^1\sum_{i=1}^{\infty}W_iW_j\dot{q}_i(t)d\Delta\ldots \\ &+\rho S\int_0^1\sum_{i=1}^{\infty}W_iW_j\ddot{q}_i(t)d\Delta=0 \qquad (a) \\ &\kappa GS\int_0^1\sum_{i=1}^{\infty}\left(A_i-\frac{dW_i}{ld\Delta}\right)A_j q_i(t)d\Delta+\rho I\int_0^1\sum_{i=1}^{\infty}A_iA_j\ddot{q}_i(t)d\Delta\ldots \\ &=\frac{EST_h^2+4EI}{4l^2}\int_0^1\sum_{i=1}^{\infty}\frac{d^2A_i}{d\Delta^2}A_j q_i(t)d\Delta \qquad (b) \end{aligned}\right\} \tag{8}$$

Applying in the system of Equation (8) the orthogonality property of the eigenmodes for the Timoshenko beam as formulated in the reference [6].

Summing the Equations (8a) and (8b), and using the orthogonality property of the eigenmodes:

$$\begin{aligned} &\left(\rho S\int_0^1 W_i^2 d\Delta+\rho I\int_0^1 A_i^2 d\Delta\right)\ddot{q}_i(t)+c_1\int_0^1 W_i^2 d\Delta\dot{q}_i(t)+ \\ &\left(\kappa GS\int_0^1\left(A_i-\frac{dW_i}{ld\Delta}\right)^2 d\Delta+\frac{EST_h^2+4EI}{4l^2}\int_0^1\left(\frac{dA_i}{d\Delta}\right)^2 d\Delta\right)q_i(t)=0 \end{aligned} \tag{9}$$

In the stick phase, the differential equation of motion is given by:

$$m_{sti}\ddot{q}_i(t)+c_{sti}\dot{q}_i(t)+k_{sti}q_i(t)=0 \tag{10}$$

where: m_{sti} : generalized mass of mode i in the stick phase, it is given by $m_{sti} = (\rho S\int_0^1 W_i^2 d\Delta + \rho I\int_0^1 A_i^2 d\Delta)$, k_{sti} : generalized stiffness of mode i in the stick phase, it is given by $k_{sti} = (\kappa GS\int_0^1 (A_i - \frac{dW_i}{ld\Delta})^2 d\Delta + (\frac{EST_h^2 + 4EI}{4l^2})\int_0^1 (\frac{dA_i}{d\Delta})^2 d\Delta)$, c_{sti} : generalized viscous damping of mode i in the stick phase, it is given by $c_{sti} = c_1\int_0^1 W_i^2 d\Delta$ Taking the same procedure for the system of Equation (4) to get:

$$m_{sli}\ddot{q}_i(t) + c_{sli}\dot{q}_i(t) + k_{sli}q_i(t) + F_{total}\,sign(\dot{q}_i(t))\int_0^1 W_i d\Delta = 0 \quad (11)$$

where: m_{sli} : generalized mass in the slip phase, it is given by $m_{sli} = m_{sti}$, k_{sli} : generalized stiffness in the slip phase, it is given by $k_{sli} = \kappa GS\int_0^1 (A_i - \frac{dW_i}{ld\Delta})^2 d\Delta + \frac{EI}{l^2}\int_0^1 (\frac{dA_i}{d\Delta})^2 d\Delta$, c_{sli} : generalized damping in the slip phase, it is given by $c_{sli} = c_2\int_0^1 W_i^2 d\Delta$.

The Equations (10) and (11) are ordinary differential equations and their solution is known in literature.

3. Results and Discussion

The Figure 1a,b show that the decay in the case of pure stick motion and pure slip motion is exponential since viscous damping is dominant. The pure stick motion appears for maximum clamping force and the pure slip motion appears for zero clamping force. For intermediate clamping forces (Figure 2), the motion is stick-slip followed by stick motion. The decay is frictional for the stick-slip phase and exponential for the stick phase, this can be justified by the fact that the damping by dry friction is dominant.

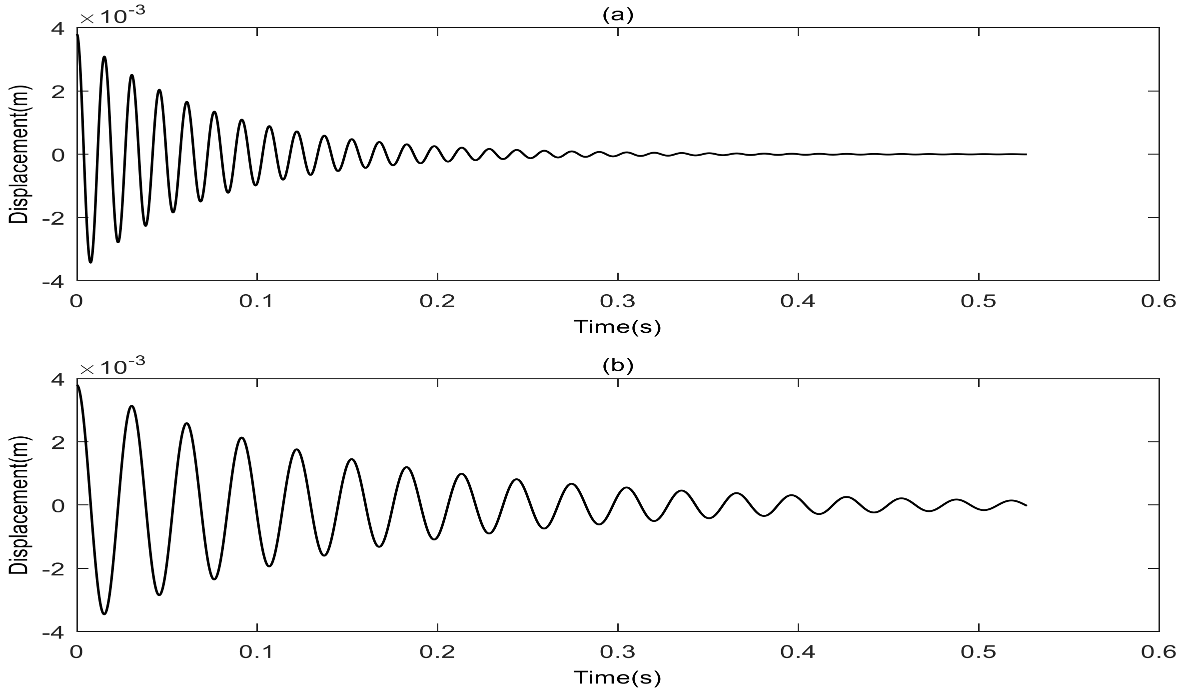

Figure 1. Free response of the bolted beam: (**a**) Pure stick motion, (**b**) Pure slip motion.

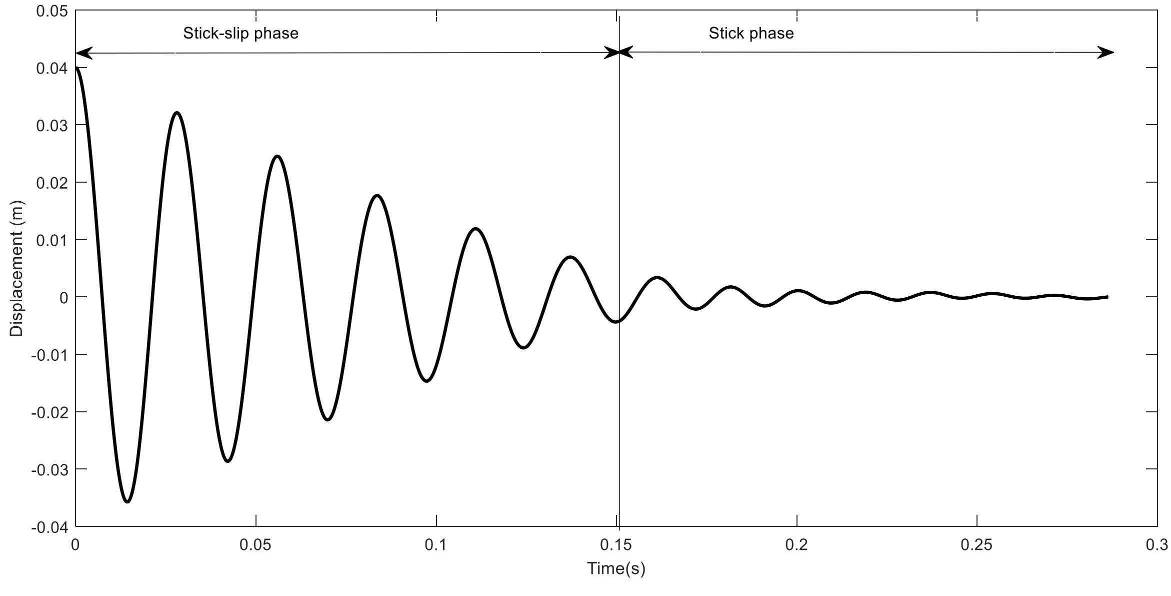

Figure 2. Free response of the bolted beam in intermediate clamping forces.

4. Conclusions

This work focuses on the solution of differential equation of motion of two clamped Timoshenko beams in the presence of friction. The viscous damping is added in stick and slip phases, and the structural damping is added in the slip phase. The application of Galerkin's method allows transferring the partial differential equations to an ordinary differential equation. The decay is exponential for zero clamping force and maximum clamping force. In case of application of clamping force, the decay is frictional in stick-slip phase and exponential in stick phase until the movement stops.

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