computer sciences and mathematics forum

MDPI

Proceeding Paper

Generalized Integral Transforms and Fractional Calculus Operators Involving a Generalized Mittag-Leffler Type Function †

Ankit Pal

Department of Mathematics, School of Advanced Sciences & Languages, VIT Bhopal University, Sehore 466114 , Madhya Pradesh, India; ankit.pal@vitbhopal.ac.in

† Presented at the 1st International Online Conference on Mathematics and Applications; Available online: https://iocma2023.sciforum.net/.

Abstract: In this work, we consider a generalized Mittag-Leffler type function and establish several integral formulas involving Jacobi and related transforms. We also establish some composition of generalized fractional derivative formulas associated with the generalized Mittag-Leffler type function. Additionally, certain special cases of generalized fractional derivative formulas involving Mittag-Leffler type function have been corollarily presented.

Keywords: Mittag-Leffler functions; Mittag-Leffler type confluent hypergeometric function; generalized wright hypergeometric function; Jacobi transform; generalized fractional derivative

MSC: 33B15; 33C05; 33C60; 33E12; 44A99

check for updates

Citation: Pal, A. Generalized Integral Transforms and Fractional Calculus Operators Involving a Generalized Mittag-Leffler Type Function. *Comput. Sci. Math. Forum* **2023**, *1*, 0. https://doi.org/

Academic Editor: Firstname Lastname

Published: 28 April 2023

1. Introduction and Preliminaries

The Mittag-Leffler function was introduced by the Swedish mathematician Gösta Mittag-Leffler [1] in 1903 and can be defined by

$$E_\nu(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\nu n + 1)}, \ \Re(\nu) > 0 \tag{1}$$

In 1905, Wiman [2] generalized the Mittag-Leffler function $E_\nu(z)$ and came up with the following definition:

$$E_{\nu,\eta}(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\nu n + \eta)}, \tag{2}$$

where $\nu, \eta \in \mathbf{C}, \Re(\nu) > 0, \Re(\eta) > 0$ and is known as the Wiman's function.

Later, Prabhakar [3] gave three parameter Mittag-Leffler function $E^{\gamma}_{\nu,\eta}(z)$ in 1971 and can be represented as,

$$E^{\gamma}_{\nu,\eta}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_n}{\Gamma(\nu n + \eta)} \frac{z^n}{n!}, \tag{3}$$

where $\nu, \eta, \gamma \in \mathbf{C}$ and $\Re(\nu) > 0, \Re(\eta) > 0, \Re(\gamma) > 0$.

Wright [4] introduced the Fox-Wright function, an extension of the generalized hypergeometric function, which can be represented in the following form:

$${}_p\Psi_q(z) = {}_p\Psi_q\left[\begin{matrix}(u_i, U_i)_{1,p} \\ (v_j, V_j)_{1,q}\end{matrix}\middle| z\right] = \sum_{n=0}^{\infty} \frac{\Gamma(u_1 + U_1 n)\ldots\Gamma(u_p + U_p n)}{\Gamma(v_1 + V_1 n)\ldots\Gamma(v_q + V_q n)} \frac{z^n}{n!}, \tag{4}$$

where $u_i, v_j \in \mathbf{C}, i = 1,2,\ldots,p; j = 1,2,\ldots,q$, and the coefficients $U_1,\ldots,U_p \in \mathbf{R}^+$ and $V_1,\ldots,V_q \in \mathbf{R}^+$ satisfying the condition

$$\sum_{j=1}^{q} V_j - \sum_{i=1}^{p} U_i > -1. \tag{5}$$

In particular, when $U_i = V_j = 1$ $(i = 1,2,\ldots,p;\ j = 1,2,\ldots,q)$, (4) reduces to

$${}_p\Psi_q\left[\begin{matrix}(u_1,1),\ldots,(u_p,1)\\(v_1,1),\ldots,(v_q,1)\end{matrix}\middle|z\right] = \frac{\prod_{i=1}^{p}\Gamma(u_i)}{\prod_{j=1}^{q}\Gamma(v_j)}\,{}_pF_q\left[\begin{matrix}u_1,\ldots,u_p\\v_1,\ldots,v_q\end{matrix}\middle|z\right], \tag{6}$$

where ${}_pF_q(\cdot)$ is the generalized hypergeometric function [5].

In the present work, we introduce a Mittag-Leffler type confluent hypergeometric function due to Ghanim and Al-Janaby [6]:

$$M_{\nu,\mu}^{\eta,\gamma}(z) = \sum_{m=0}^{\infty}\frac{\Gamma(\mu)\Gamma(\eta m+\gamma)}{\Gamma(\gamma)\Gamma(\nu m+\mu)}\frac{z^m}{m!}, \tag{7}$$

where $\nu,\eta,\gamma,\mu \in \mathbf{C}$ and $\Re(\nu) > 0$. It is worth pointing out that series representation of (7) yields a variety of connections with special functions, including confluent hypergeometric function and generalized Mittag-Leffler functions (1)–(3).

2. Jacobi and Related Integral Transform

The Jacobi integral transform [7] p. 501 of a function $f(z)$ is defined as follows:

$$\mathrm{J}^{(\alpha,\beta)}[f(z);n] = \int_{-1}^{1}(1-z)^{\alpha}(1+z)^{\beta}P_n^{(\alpha,\beta)}(z)\,f(z)dz, \tag{8}$$

where $\min\{\Re(\alpha),\Re(\beta)\} > -1; n \in \mathbb{N}_0$ and provided that the integral on the right hand side in (8) exists. Here, $P_n^{(\alpha,\beta)}(z)$ is the classical orthogonal Jacobi polynomial [8] (Chapter 10) defined by

$$\begin{aligned}P_n^{(\alpha,\beta)}(z) &= (-1)^n P_n^{(\beta,\alpha)}(-z)\\ &= \binom{\alpha+n}{n}\,{}_2F_1\left[\begin{matrix}-n,\alpha+\beta+n+1\\\alpha+1\end{matrix}\middle|\frac{1-z}{2}\right].\end{aligned} \tag{9}$$

The Jacobi polynomials $P_n^{(\alpha,\beta)}(z)$ contain, as their special cases, such other classical orthogonal polynomials as (for example) the Gegenbauer polynomials $C_n^c(z)$, the Legendre (or spherical) polynomials $P_n(z)$, and the Tchebycheff polynomials $T_n(z)$ and $U_n(z)$ of the first and second kind (see, for details, [9]). In fact, we have the following relationships with the Gegenbauer polynomials $C_n^c(z)$ and the Legendre polynomials $P_n(z)$:

$$C_n^c(z) = \binom{c+n-\frac{1}{2}}{n}^{-1}\binom{2c+n-1}{n}\mathrm{J}^{(c-\frac{1}{2},c-\frac{1}{2})}[f(z);n] \tag{10}$$

and

$$P_n(z) = C_n^{\frac{1}{2}}(z) = P_n^{(0,0)}(z), \tag{11}$$

respectively. Thus, by applying the relationships in (10) and (11) and ignoring altogether the constant binomial coefficients occurring in (10), the parameters α and β in (8) earlier can be suitably specialized to define the corresponding Gegenbauer transform $\mathrm{G}^{(c)}[f(z);n]$ and the corresponding Legendre transform $\mathrm{L}[f(z);n]$ as follows:

$$\mathrm{G}^{(c)}[f(z);n] = \binom{c+n-\frac{1}{2}}{n}^{-1}\binom{2c+n-1}{n}\mathrm{J}^{\left(c-\frac{1}{2},c-\frac{1}{2}\right)}[f(z);n] \tag{12}$$

$$= \int_{-1}^{1}(1-z^2)^{c-\frac{1}{2}}\, C_n^c(z)f(z)dz\, \left(\Re(c) > -\frac{1}{2}; n \in \mathbf{N}_0\right), \tag{13}$$

and

$$\mathrm{L}[f(z);n] = \mathrm{G}^{\left(\frac{1}{2}\right)}[f(z);n] = \int_{-1}^{1} P_n(z)f(z)dz,\ (n \in \mathbf{N}_0). \tag{14}$$

Lemma 1 ([10]). *The Jacobi transform of the power function $z^{\omega-1}$ is given by*

$$\int_{-1}^{1}(1-z)^{\delta_1-1}(1+z)^{\delta_2-1}P_n^{(\alpha,\beta)}(z)\, z^{\omega-1}dz = 2^{\delta_1+\delta_2-1}\binom{\alpha+n}{n}B(\delta_1,\delta_2) \times F^{1:2,1}_{1:1,0}\left[\begin{matrix}\delta_1 : -n, \alpha+\beta+n+1; 1-\omega \\ \delta_1+\delta_2 : \quad \alpha+1; \quad - \end{matrix}\middle| 1,2\right], \tag{15}$$

where $\min\{\Re(\delta_1),\Re(\delta_2)\} > 0; \omega \in \mathbf{C}; n \in \mathbb{N}_0$ *and* $F^{p:q,r}_{l:m,n}(\cdot)$ *is the familiar Kampé de Fériet function [11].*

Theorem 1. *The following Jacobi transform formula holds true:*

$$\mathrm{J}^{(\alpha,\beta)}\left[z^{\omega-1}M^{\eta,\gamma}_{\nu,\mu}(uz);n\right] = 2^{\alpha+\beta+1}\binom{\alpha+n}{n}B(\alpha+1,\beta+1)\sum_{k=0}^{\infty}\frac{\Gamma(\mu)\Gamma(\eta k+\gamma)}{\Gamma(\gamma)\Gamma(\nu k+\mu)} \times F^{1:2,1}_{1:1,0}\left[\begin{matrix}\alpha+1 : -n, \alpha+\beta+n+1; 1-\omega-k \\ \alpha+\beta+2 : \quad \alpha+1; \quad - \end{matrix}\middle| 1,2\right]\frac{u^k}{k!}, \tag{16}$$

where $\omega \in \mathbf{C}; n \in \mathbb{N}_0$ *and* $\min\{\Re(\alpha),\Re(\beta)\} > -1, |u| < 1$.

Proof. To prove Theorem 1, we first apply Jacobi transform (8) in conjunction with (7). Then, upon reversing the order of integration and summation and make use of Lemma 1, this leads to the right hand side of Theorem 1. □

Corollary 1. *Under the conditions stated in Theorem 1 and setting* $\alpha = \beta = c - \frac{1}{2}$*, the following Gegenbauer transform formula holds true:*

$$\mathrm{G}^{(c)}\left[z^{\omega-1}M^{\eta,\gamma}_{\nu,\mu}(uz);n\right] = 2^{2c}\binom{2c+n-1}{n}B\left(c+\frac{1}{2},c+\frac{1}{2}\right)\sum_{k=0}^{\infty}\frac{\Gamma(\mu)\Gamma(\eta k+\gamma)}{\Gamma(\gamma)\Gamma(\nu k+\mu)} \times F^{1:2,1}_{1:1,0}\left[\begin{matrix}c+\frac{1}{2} : -n, 2c+n; 1-\omega-k \\ 2c+1 : \quad c+\frac{1}{2}; \quad - \end{matrix}\middle| 1,2\right]\frac{u^k}{k!}, \tag{17}$$

where $\omega \in \mathbf{C}; n \in \mathbb{N}_0, \Re(c) > -\frac{1}{2}, |u| < 1$.

Corollary 2. *Under the conditions stated in Theorem 1 and setting* $\alpha = \beta = 0$ *or* $c = \frac{1}{2}$*, the following Legendre transform formula holds true:*

$$\mathrm{L}\left[z^{\omega-1}M^{\eta,\gamma}_{\nu,\mu}(uz);n\right] = 2\sum_{k=0}^{\infty}\frac{\Gamma(\mu)\Gamma(\eta k+\gamma)}{\Gamma(\gamma)\Gamma(\nu k+\mu)} \times F^{1:2,1}_{1:1,0}\left[\begin{matrix}1 : -n, n+1; 1-\omega-k \\ 2 : \quad 1; \quad - \end{matrix}\middle| 1,2\right]\frac{u^k}{k!},$$

where $\omega \in \mathbf{C}; n \in \mathbb{N}_0, |u| < 1$.

3. Fractional Derivative Formulas

In this section, we establish several fractional derivative formulas for the Mittag-Leffler type function. With this purpose, we recall the following pairs of left-sided and right-sided hypergeometric fractional derivative operator $\mathcal{D}_{0+}^{\lambda,\sigma,\kappa}$ and $\mathcal{D}_{\infty-}^{\lambda,\sigma,\kappa}$:

Definition 1 ([12]). *The left-sided hypergeometric fractional integral operator $\mathcal{I}_{0+}^{\lambda,\sigma,\kappa}$ and corresponding left-sided hypergeometric fractional integral operator $\mathcal{D}_{0+}^{\lambda,\sigma,\kappa}$ are defined, for $x > 0, \lambda, \sigma, \kappa \in \mathbf{C}$, by*

$$\left(\mathcal{I}_{0+}^{\lambda,\sigma,\kappa} f\right)(x) = \frac{x^{-\lambda-\sigma}}{\Gamma(\lambda)} \int_0^x (x-t)^{\lambda-1} {}_2F_1\left(\lambda+\sigma, -\kappa; \lambda; 1-\frac{t}{x}\right) f(t)\, dt, \tag{18}$$

where $\Re(\lambda) > 0$ and

$$\left(\mathcal{D}_{0+}^{\lambda,\sigma,\kappa} f\right)(x) = \left(\mathcal{I}_{0+}^{-\lambda,-\sigma,\lambda+\kappa} f\right)(x) = \left(\frac{d}{dx}\right)^n \left\{\left(\mathcal{I}_{0+}^{-\lambda+\kappa,-\sigma-\kappa,\lambda+\kappa-n} f\right)(x)\right\},$$

where $\Re(\lambda) \geq 0; n = [\Re(\lambda)] + 1$ and $[x]$ denotes the largest integer in the real number x.

Remark 1. *For $\sigma = -\lambda, \sigma = 0$, the left-sided hypergeometric fractional integral operator $\mathcal{D}_{0+}^{\lambda,\sigma,\kappa}$ coincides with the familiar Riemann-Liouville fractional derivative operator ${}_{RL}\mathcal{D}_{0+}^{\lambda}$ and the left-sided Erdélyi-Kobar fractional derivative operator ${}_{EK}\mathcal{D}_{0+}^{\lambda,\kappa}$ as given below (see [12]):*

$$\left(\mathcal{D}_{0+}^{\lambda,-\lambda,\kappa} f\right)(x) = \left({}_{RL}\mathcal{D}_{0+}^{\lambda} f\right)(x) = \left(\frac{d}{dx}\right)^n \left\{\frac{1}{\Gamma(n-\lambda)} \int_0^x \frac{f(t)}{(x-t)^{\lambda-n+1}}\, dt\right\}, \tag{19}$$

and

$$\left(\mathcal{D}_{0+}^{\lambda,0,\kappa} f\right)(x) = \left({}_{EK}\mathcal{D}_{0+}^{\lambda,\kappa} f\right)(x) = x^{\kappa}\left(-\frac{d}{dx}\right)^n \left\{\frac{1}{\Gamma(n-\lambda)} \int_0^x \frac{t^{\lambda+\kappa} f(t)}{(x-t)^{\lambda-n+1}}\, dt\right\}, \tag{20}$$

where $x > 0; \Re(\lambda) \geq 0; n = [\Re(\lambda)] + 1$.

Definition 2 ([12]). *The right-sided hypergeometric fractional integral operator $\mathcal{I}_{\infty-}^{\lambda,\sigma,\kappa}$ and corresponding right-sided hypergeometric fractional integral operator $\mathcal{D}_{\infty-}^{\lambda,\sigma,\kappa}$ are defined, for $x > 0, \lambda, \sigma, \kappa \in \mathbf{C}$, by*

$$\left(\mathcal{I}_{\infty-}^{\lambda,\sigma,\kappa} f\right)(x) = \frac{1}{\Gamma(\lambda)} \int_x^\infty (t-x)^{\lambda-1}\, t^{-\lambda-\sigma} {}_2F_1\left(\lambda+\sigma, -\kappa; \lambda; 1-\frac{x}{t}\right) f(t)\, dt, \tag{21}$$

where $\Re(\lambda) > 0$ and

$$\left(\mathcal{D}_{\infty-}^{\lambda,\sigma,\kappa} f\right)(x) = \left(\mathcal{I}_{\infty-}^{-\lambda,-\sigma,\lambda+\kappa} f\right)(x) = \left(-\frac{d}{dx}\right)^n \left\{\left(\mathcal{I}_{0+}^{-\lambda+\kappa,-\sigma-\kappa,\lambda+\kappa-n} f\right)(x)\right\},$$

where $\Re(\lambda) \geq 0; n = [\Re(\lambda) + 1]$ and $[x]$ denotes the largest integer in the real number x.

Remark 2. For $\sigma = -\lambda, \sigma = 0$, the right-sided hypergeometric fractional integral operator $\mathcal{D}_{\infty-}^{\lambda,\sigma,\kappa}$ coincides with the Weyl fractional derivative operator ${}_{W}\mathcal{D}_{\infty-}^{\lambda}$ and the right-sided Erdélyi-Kobar fractional derivative operator ${}_{EK}\mathcal{D}_{\infty-}^{\lambda,\kappa}$ as given below (see [12]):

$$\left(\mathcal{D}_{\infty-}^{\lambda,-\lambda,\kappa} f\right)(x) = \left({}_{W}\mathcal{D}_{\infty-}^{\lambda} f\right)(x) = \left(-\frac{d}{dx}\right)^n \left\{\frac{1}{\Gamma(n-\lambda)} \int_x^\infty \frac{f(t)}{(t-x)^{\lambda-n+1}}\, dt\right\}, \tag{22}$$

and

$$\left(\mathcal{D}_{\infty-}^{\lambda,0,\kappa} f\right)(x) = \left({}_{EK}\mathcal{D}_{\infty-}^{\lambda,\kappa} f\right)(x) = x^{\lambda+\kappa}\left(\frac{d}{dx}\right)^n \left\{\frac{1}{\Gamma(n-\lambda)} \int_x^\infty \frac{t^{-\kappa} f(t)}{(t-x)^{\lambda-n+1}}\, dt\right\}, \tag{23}$$

where $x > 0; \Re(\lambda) \geq 0; n = [\Re(\lambda)] + 1$.

We need to recall the following Lemma 2 [12] to prove the Theorems 2 and 3.

Lemma 2. *Let $\lambda, \sigma, \kappa, \mu \in \mathbf{C}$ and $x > 0, \Re(\lambda) \geq 0$. Then each of the following hypergeometric fractional derivative formulas holds true:*

$$\left(\mathcal{D}_{0+}^{\lambda,\sigma,\kappa}\, t^{\mu-1}\right)(x) = \frac{\Gamma(\mu)\Gamma(\mu+\lambda+\sigma+\kappa)}{\Gamma(\mu+\sigma)\Gamma(\mu+\kappa)}\, x^{\mu+\sigma-1}, \tag{24}$$

where $\Re(\mu) > -\min\{0, \Re(\lambda+\sigma+\kappa)\}$ and

$$\left(\mathcal{D}_{\infty-}^{\lambda,\sigma,\kappa}\, t^{\mu-1}\right)(x) = \frac{\Gamma(1-\sigma-\mu)\Gamma(1-\mu+\lambda+\kappa)}{\Gamma(1-\mu)\Gamma(1-\mu+\kappa-\sigma)}\, x^{\mu+\sigma-1}, \tag{25}$$

where $\Re(\mu) < 1 + \min\{-\Re(\sigma+\kappa), \Re(\lambda+\kappa)\}$.

Theorem 2. *Let $\lambda, \sigma, \kappa, \mu, \nu, \eta, \gamma \in \mathbf{C}$ and $x > 0, \Re(\lambda) \geq 0, \Re(\nu) > 0$. Then the following left-sided hypergeometric fractional derivative formula holds true:*

$$\left[\mathcal{D}_{0+}^{\lambda,\sigma,\kappa}\, t^{\mu-1} M_{\nu,\mu}^{\eta,\gamma}(ut^{\nu})\right](x) = x^{\mu+\sigma-1}\frac{\Gamma(\mu)}{\Gamma(\gamma)}\, {}_2\Psi_2\left[\begin{matrix}(\gamma,\eta),(\mu+\lambda+\sigma+\kappa,\nu)\\ (\mu+\sigma,\nu),(\mu+\kappa,\nu)\end{matrix}\middle| ux^{\nu}\right], \tag{26}$$

where $\Re(\mu) > -\min\{0, \Re(\lambda+\sigma+\kappa)\}$

Proof. Using (7), we have

$$\left[\mathcal{D}_{0+}^{\lambda,\sigma,\kappa}\, t^{\mu-1} M_{\nu,\mu}^{\eta,\gamma}(ut^{\nu})\right](x) = \sum_{n=0}^{\infty}\frac{\Gamma(\mu)\Gamma(\eta n+\gamma)}{\Gamma(\gamma)\Gamma(\nu n+\mu)}\frac{u^n}{n!} \times \left(\mathcal{D}_{0+}^{\lambda,\sigma,\kappa}\, t^{\nu n+\mu-1}\right)(x). \tag{27}$$

Using (24), this leads to the right hand side of (26). □

Corollary 3. *Under the conditions already stated in Theorem 2 and setting $\sigma = -\lambda$ and $\sigma = 0$, the following Riemann-Liouville fractional derivative formula and the left-sided Erdélyi-Kobar fractional derivative formulas holds true:*

$$\left[{}_{RL}\mathcal{D}_{0+}^{\lambda}\, t^{\mu-1} M_{\nu,\mu}^{\eta,\gamma}(ut^{\nu})\right](x) = x^{\mu-\lambda-1}\frac{\Gamma(\mu)}{\Gamma(\mu-\lambda)}\, M_{\nu,\mu-\lambda}^{\eta,\gamma}(ux^{\nu}). \tag{28}$$

$$\left[{}_{EK}\mathcal{D}_{0+}^{\lambda,\kappa}\, t^{\mu-1} M_{\nu,\mu}^{\eta,\gamma}(ut^{\nu})\right](x) = x^{\mu-1}\frac{\Gamma(\mu)}{\Gamma(\gamma)}\, {}_2\Psi_2\left[\begin{matrix}(\gamma,\eta),(\mu+\lambda+\kappa,\nu)\\ (\mu,\nu),(\mu+\kappa,\nu)\end{matrix}\middle| ux^{\nu}\right]. \tag{29}$$

Theorem 3. *Let $\lambda, \sigma, \kappa, \mu, \nu, \eta, \gamma \in \mathbf{C}$ and $x > 0, t > 0, \Re(\lambda) \geq 0, \Re(\nu) > 0$. Then the following right-sided hypergeometric fractional derivative formula holds true:*

$$\left[\mathcal{D}_{\infty-}^{\lambda,\sigma,\kappa}\, t^{\mu-1} M_{\nu,\mu}^{\eta,\gamma}\left(\frac{u}{t^{\nu}}\right)\right](x) = x^{\mu+\sigma-1}\frac{\Gamma(\mu)}{\Gamma(\gamma)}\, {}_3\Psi_3\left[\begin{matrix}(\gamma,\eta),(1-\sigma-\mu,\nu),(1-\mu+\lambda+\kappa,\nu)\\ (\mu,\nu),(1-\mu,\nu),(1-\mu+\kappa-\sigma,\nu)\end{matrix}\middle| \frac{u}{x^{\nu}}\right], \tag{30}$$

where $\Re(\mu) < 1 + \min\{-\Re(\sigma+\kappa), \Re(\lambda+\kappa)\}$

Proof. Using (7), we have

$$\left[\mathcal{D}_{\infty-}^{\lambda,\sigma,\kappa}\, t^{\mu-1} M_{\nu,\mu}^{\eta,\gamma}\left(\frac{u}{t^{\nu}}\right)\right](x) = \sum_{n=0}^{\infty}\frac{\Gamma(\mu)\Gamma(\eta n+\gamma)}{\Gamma(\gamma)\Gamma(\nu n+\mu)}\frac{u^n}{n!} \times \left(\mathcal{D}_{\infty-}^{\lambda,\sigma,\kappa}\, t^{\mu-\nu n-1}\right)(x). \tag{31}$$

Using (25), this leads to the right hand side of (30). □

Corollary 4. *Under the conditions already stated in Theorem 3 and setting $\sigma = -\lambda$ and $\sigma = 0$, the following Weyl fractional derivative formula and the right-sided Erdélyi-Kobar fractional derivative formulas holds true:*

$$\left[{}_{W}\mathcal{D}^{\lambda}_{\infty-}\ t^{\mu-1} M^{\eta,\gamma}_{\nu,\mu}\left(\frac{u}{t^{\nu}}\right) \right](x) = x^{\mu-\lambda-1} \frac{\Gamma(\mu)}{\Gamma(\gamma)}\ {}_2\Psi_2\left[\begin{matrix} (\gamma,\eta), (1+\lambda-\mu,\nu) \\ (\mu,\nu), (1-\mu,\nu) \end{matrix} \middle| \frac{u}{x^{\nu}} \right]. \tag{32}$$

$$\left[{}_{EK}\mathcal{D}^{\lambda,\kappa}_{\infty-}\ t^{\mu-1} M^{\eta,\gamma}_{\nu,\mu}\left(\frac{u}{t^{\nu}}\right) \right](x) = x^{\mu-1} \frac{\Gamma(\mu)}{\Gamma(\gamma)}\ {}_2\Psi_2\left[\begin{matrix} (\gamma,\eta), (1-\mu+\lambda+\kappa,\nu) \\ (\mu,\nu), (1-\mu+\kappa,\nu) \end{matrix} \middle| \frac{u}{x^{\nu}} \right]. \tag{33}$$

Funding:

Institutional Review Board Statement:

Informed Consent Statement:

Data Availability Statement:

Acknowledgments:

Conflicts of Interest:

References

1. Mittag-Leffler, G.M. Sur la nouvelle fonction $E_{\alpha}(x)$. *Comptes Rendus l'Acad. Sci. Paris* **1903**, *137*, 554–558.
2. Wiman, A. Über den fundamentalsatz in der Theorie der Funktionen $E_{\alpha}(x)$. *Acta Math.* **1905**, *29*, 191–201.
3. Prabhakar, T.R. A singular integral equation with a generalized Mittag-Leffler function in the kernel. *Yokohama Math. J.* **1971**, *19*, 7–15.
4. Wright, E.M. On the coefficients of power series having exponential singularities. *J. Lond. Math. Soc.* **1933**, *8*, 71–79.
5. Rainville, E.D. *Special Functions*; The Macmillan Company: New York, NY, USA, 1960.
6. Ghanim, F.; Al-Janaby, H.F. An analytical study on Mittag-Leffler confluent hypergeometric functions with fractional integral operator. *Math. Meth. Appl. Sci.* **2020**, *44*, 3605–3614.
7. Debnath, L.; Bhatta, D. *Integral Transforms and Their Applications*, 3rd ed.; Chapman and Hall (CRC Press), Taylor and Francis Group: London, UK; New York, NY, USA, 2014.
8. Erdélyi, A.; Magnus, W.; Oberhettinger, F.; Tricomi, F.G. *Higher Transcendental Functions*; McGraw-Hill: New York, NY, USA, 1953; Volume II.
9. Srivastava, H.M.; Manocha, H.L. *A Treatise on Generating Functions*; Halsted Press (Ellis Horwood Limited): Chichester, UK; John Wiley and Sons: New York, NY, USA; Chichester, UK; Brisbane, Australia; Toronto, ON, Canada, 1984.
10. Srivastava, H.M.; Agarwal, R.; Jain, S. Integral transform and fractional derivative formulas involving the extended generalized hypergeometric functions and probability distributions. *Math. Methods Appl. Sci.* **2017**, *40*, 255–273.
11. Srivastava, H.M.; Karlsson, P.W. *Mathematics and Its Applications*; Multiple Gaussian Hypergeometric Series; Ellis Horwood: Chichester, UK; Halsted Press (John Wiley & Sons): New York, NY, USA, 1985.
12. Kilbas, A.A.; Sebastian, N. Generalized fractional differentiation of Bessel function of the first kind. *Math. Balkanica (New Ser.)* **2008**, *22*, 323–346.