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Finite Difference Simulation on Biomagnetic Fluid Flow and Heat Transfer with Gold Nanoparticles towards a Shrinking Sheet in the Presence of a Magnetic Dipole †

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Abstract: In this paper, we study the laminar, incompressible and steady flow of a biomagnetic fluid such as blood containing gold nanoparticles through a shrinking sheet in the presence of a magnetic dipole. This model is consistent with both the principles of magnetohydrodynamics (MHD) and ferro-hydrodynamics (FHD). An effective numerical method that is based on an iterative process, tridiagonal matrix manipulation, and a common finite difference method with central differencing is used to generate the numerical solution of obtained ODEs. The major numerical results show that the fluid velocity decreases as the ferromagnetic number increases whereas the skin friction coefficient shows the opposite behavior. As the ferromagnetic number increases, the rate of heat transfer with ferromagnetic interaction parameter is likewise observed and shown to be decreasing.

Keywords: biomagnetic fluid; gold nanoparticle; magnetohydrodynamic; ferrohydrodynamic; magnetic dipole; finite difference method

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1. Introduction

Study biomagnetic fluid has grown rapidly because it aims to uncover and develop remedies for several human body-related illnesses and disorders, such as producing artificial organs, making nanorobots for surgery, and creating cutting-edge imaging and signal processing methods for cancer, tumors, and other terminal illnesses. The field of medical imaging-based diagnostics (MRI, CT scan, ultrasound, etc.), targeted drug delivery, or hyperthermia therapies are only a few examples of how these study areas can directly affect the actual world [1–3]. Based on the principles of Ferrohydrodynamics, Haik et al. [9] first proposed a BFD model, the biomagnetic fluid is a Newtonian, electrically non-conducting magnetic fluid, according to this model. They stated that under the effect of high gradient magnetic fields, the fluid magnetization significantly affects the flow. This BFD model was further extended by Tzirtzilakis et al. [10] by combining both principles of FHD and MHD. Murtaza et al. [11] investigated both electrical conductivity and magnetization affected the BFD flow over a stretching sheet. In order to increase the qualities of nanoparticles, Choi et al. [12] invented a new fluid in 1995 called nanofluid that mixes them with a base fluid like blood, water, or oil. A nanofluid is a base fluid that has nanoparticles spread throughout it. The significance of magnetic dipole on heat characteristics of blood flow with $CoFe_2O_4$ particles towards an extended cylinder was investigated by Ferdows et al. [36] using MHD and FHD principles.

To the best of the authors' knowledge, none of the aforementioned research have yet examined the effect of a strong magnetic field on blood flow with gold nanoparticles toward a shrinking sheet. The governing boundary layer equations were resolved using a well-known finite difference technique.

2. Mathematical Flow Equations with Flow Geometry

The two-dimensional boundary layer flow, heat and mass transfer of a steady, viscous, laminar, an incompressible, and electrically conductive bio-magnetic fluid flow (namely blood) that contains gold nanoparticles in the presence of a magnetic dipole which towards a shrinking sheet is considered. This shrinking sheet kept at a constant temperature T_w and concentration C_w, at $y = 0$, Where the X-axis is taken in the direction of the flow and Y-axis is normal to it. It is assumed that the free stream velocity is $U_\infty(x) = bx$ and the plate is shrunk with the velocity $u_w(x) = cx$ where b and c are positive constants. It is also assumed that the constant mass flux velocity is v_0 with $v_0 < 0$ for suction and $v_0 > 0$ for injection. The ambient temperature is denoted by T_∞ and the concentration of nanoparticles by C_∞ respectively. In addition, a constant $B = B_0$ transverse magnetic field is applied to the flow, with the assumption that it is applied in the positive y-direction. As seen in Figure 1, the magnetic dipole is situated far below the sheet, creating a magnetic field strong enough to saturate the biomagnetic fluid.

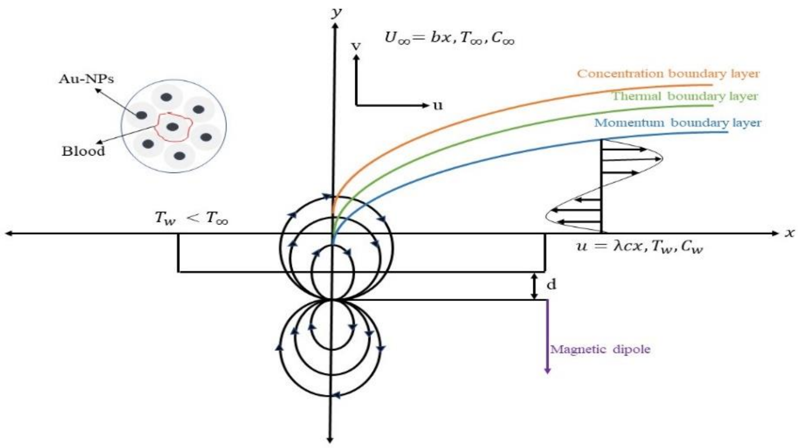

Figure 1. Schematic diagram of flow problem.

Under these considerations, we extend the work of [9,10] and governing boundary equations are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{dy} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U_\infty\frac{dU_\infty}{dx} + \upsilon_{nf}\frac{\partial^2 u}{\partial y^2} + \frac{\sigma_{nf}B_0^2}{\rho_{nf}}(U_\infty - u) + \frac{\mu_0}{\rho_{nf}}\,M_1\,\frac{\partial H}{\partial x} \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + \frac{\mu_0}{(\rho c_p)_{nf}}\,T\,\frac{\partial M_1}{\partial T}\left(u\frac{\partial H}{\partial x} + v\frac{\partial H}{\partial y}\right) = \frac{k_{nf}}{(\rho c_p)_{nf}}\frac{\partial^2 T}{\partial y^2} + \sigma_{nf}\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right] \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} \tag{4}$$

The equations are subjected to the boundary Conditions [9]:

$$u = v = 0, T = T_\infty, C = C_\infty \text{ For any } x, y$$
$$v = v_0, u = \lambda u_w(x), T = T_w, C = C_w \; at \; y = 0 \tag{5}$$
$$u \to U_\infty(x), T \to T_\infty, C \to C_\infty \; as \; y \to \infty$$

where, u and v are the velocity components along the x and y direction respectively. The meaning of the symbols found in [2,5,6,9]. The magnetic field of intensity is given by [5,6]: $H(x,y) = \frac{\gamma}{2\pi}\left[\frac{1}{(y+d)^2} - \frac{x^2}{(y+d)^4}\right]$. Where, fluid magnetization with temperature is given by: $M_1 = KH(T_\infty - T)$, where k is a pyromagnetic coefficient constant. Thermophysical corelation of nanofluid is given by [2]:

$$\mu_{nf} = \mu_f(1-\varphi)^{-2.5}, (\rho c_p)_{nf} = (1-\varphi)(\rho c_p)_f + \varphi(\rho c_p)_s,$$

$$\rho_{nf} = (1-\varphi)\rho_f + \varphi\rho_s, \upsilon_{nf} = \frac{\mu_{nf}}{\rho_{nf}} = \frac{\mu_f}{(1-\varphi)^{2.5}\left[(1-\varphi)\rho_f + \varphi\rho_s\right]},$$

$$\sigma_{nf} = (1-\varphi)\sigma_f + \varphi\sigma_s, \frac{k_{nf}}{k_f} = \left[\frac{(k_s + 2k_f) - 2\varphi(k_f - k_s)}{(k_s + 2k_f) + \varphi(k_f - k_s)}\right]$$

The following transformations are introduced [9]:

$$\eta = \sqrt{\frac{c}{\upsilon_f}}y, \psi = \sqrt{c\upsilon_f}xf(\eta), \theta(\eta) = \frac{(T - T_\infty)}{(T_w - T_\infty)}, \phi(\eta) = \frac{(C - C_\infty)}{(C_w - C_\infty)} \tag{6}$$

Therefore, the reduced form of the above equations are:

$$f''' + ff''(1-\varphi)^{2.5}\left[(1-\varphi) + \varphi\frac{\rho_s}{\rho_f}\right] - f'^2 (1-\varphi)^{2.5}\left[(1-\varphi) + \varphi\frac{\rho_s}{\rho_f}\right] + MA(1-\varphi)^{2.5}\left[(1-\varphi) + \varphi\frac{\sigma_s}{\sigma_f}\right] - Mf'(1-\varphi)^{2.5}\left[(1-\varphi) + \varphi\frac{\sigma_s}{\sigma_f}\right] + A^2(1-\varphi)^{2.5}\left[(1-\varphi) + \varphi\frac{\rho_s}{\rho_f}\right] - \frac{2\beta\theta\alpha^2(1-\varphi)^{2.5}}{(\eta+\alpha)^6} = 0 \tag{7}$$

$$\theta'' + f\,\theta' Pr\left[\frac{(k_s + 2k_f) + \varphi(k_f - k_s)}{(k_s + 2k_f) - 2\varphi(k_f - k_s)}\right]\left((1-\varphi) + \varphi\frac{(\rho c_p)_s}{(\rho c_p)_f}\right) + \left((1-\varphi) + \varphi\frac{\sigma_s}{\sigma_f}\right)Pr\left[\frac{(k_s + 2k_f) + \varphi(k_f - k_s)}{(k_s + 2k_f) - 2\varphi(k_f - k_s)}\right]\left((1-\varphi) + \varphi\frac{(\rho c_p)_s}{(\rho c_p)_f}\right)\left[Nb\phi'\theta' + Nt\theta'^2\right] - \frac{2\beta\lambda_a\, k_f\, f\, \alpha^2(\varepsilon - \theta)}{k_{nf}\;(\eta+\alpha)^5} = 0 \tag{8}$$

$$\phi'' + Le\, f\phi' + \frac{Nt}{Nb}\theta'' = 0 \tag{9}$$

The transformed boundary conditions are:

$$\text{At } \eta \to 0: \; f = S, f' = \lambda, \theta = 1, \phi = 1 \tag{10}$$

$$\text{At } \eta \to \infty: \; f' \to A, \theta \to 0, \phi \to 0 \tag{11}$$

Here, $Pr = \frac{\upsilon_f}{\alpha_f}$, $M = \frac{\sigma_f B_0^2}{\rho_f c}$, $A = \frac{b}{c}$, $\varepsilon = \frac{T_\infty}{T_\infty - T_w}$, $\beta = \frac{\gamma}{2\pi}\frac{\mu_0 KH(0,0)(T_\infty - T_w)\rho_f}{\mu_f^2}$, $\lambda_a = \frac{c\mu_f^2}{\rho_f k_f(T_\infty - T_w)}$, $\alpha = \sqrt{\frac{c}{\upsilon_f}}d$, $S = \frac{-v_0}{\sqrt{c\upsilon_f}}$, $Nt = \frac{\sigma_f D_T(T_w - T_\infty)}{\upsilon_f T_\infty}$, $Nb = \frac{\sigma_f D_B(C_w - C_\infty)}{\upsilon_f}$, $Le = \frac{\upsilon_f}{D_B}$.

3. Results and Discussion

An effective numerical finite difference technique is used to solve ODEs numerically that proposed by [2,6]. According to authors informations, this technique based on the following features: (1) It is based on the common finite difference method with central differencing, (2) On a tridiagonal matrix manipulation, and (3) On an iterative procedure. A comparison between present study with earlier studies also calculated to show the accuracy of the applied code and present in Table 1. Which encourages to continue the work. Table 2 shows the thermo-physical values of blood and gold.

Table 1. Comparison values of $-f''(0)$ for different values of A when $\beta = 0, M = 0, \varphi = 0, S = 0, \lambda = 1, Pr = 1$.

A	Present Results	Ibrahim et al. [10]	Mahapatra et al. [11]	Hayat et al. [12]
0.1	−0.969405	−0.9694	−0.9694	−0.96954
0.2	−0.918135	−0.9181	−0.9181	−0.91813
0.5	−0.667262	−0.6673	−0.6673	−0.66735

Table 2. Thermophysical values of blood and gold nanoparticles [8,13].

Thermophysical Properties	Blood	Gold
$C_p(j/kgk)$	3.9×10^3	129
$\rho(kg/m^3)$	1050	19,300
$\sigma(S/m)$	0.8	4.1×10^{-7}
$k(W/mK)$	0.5	318

In Figure 2, the effect of ferromagnetic interaction parameters on velocity, temperature, and concentration profiles is depicted. In Figure 2, $\varphi = 0$ indicates pure blood and $\varphi = 0.1$ indicates Au (gold)-pure blood. In Figure 2a, it is observed that the increasing values of ferromagnetic interaction parameter magnetic fluid velocity decreases. This is happening due to the correlation between the ferromagnetic number and Kelvin force. In Figure 2b. signifies the impact of ferromagnetic interaction parameter on the temperature profile. It is observed that the temperature profile is decreased with increasing values of β. This is due to the fact that the applied magnetic field caused by the magnetic dipole generates the Kelvin, also known as the resistive force. The interaction of blood velocity and the applied magnetic field is to responsible for this. In Figure 2c, The influence of ferromagnetic interaction parameter on concentration profile graph is as the values of ferromagnetic interaction parameter increase, the concentration boundary layer thickness is increasing in both cases for pure blood and gold pure blood.

Finally, Figure 2d–f depict the variation of skin friction coefficient $-f''(0)$, local Nusselt Number $\theta'(0)$ and local Sherwood Number $\varphi'(0)$ for different values of ferromagnetic parameter βagainst Magnetic parameter M. From the figures it is seen that, $-f''(0)$ and $\varphi'(0)$ both are increases with the increasing values of β; wheras reverse trend is observed in $\theta'(0)$.

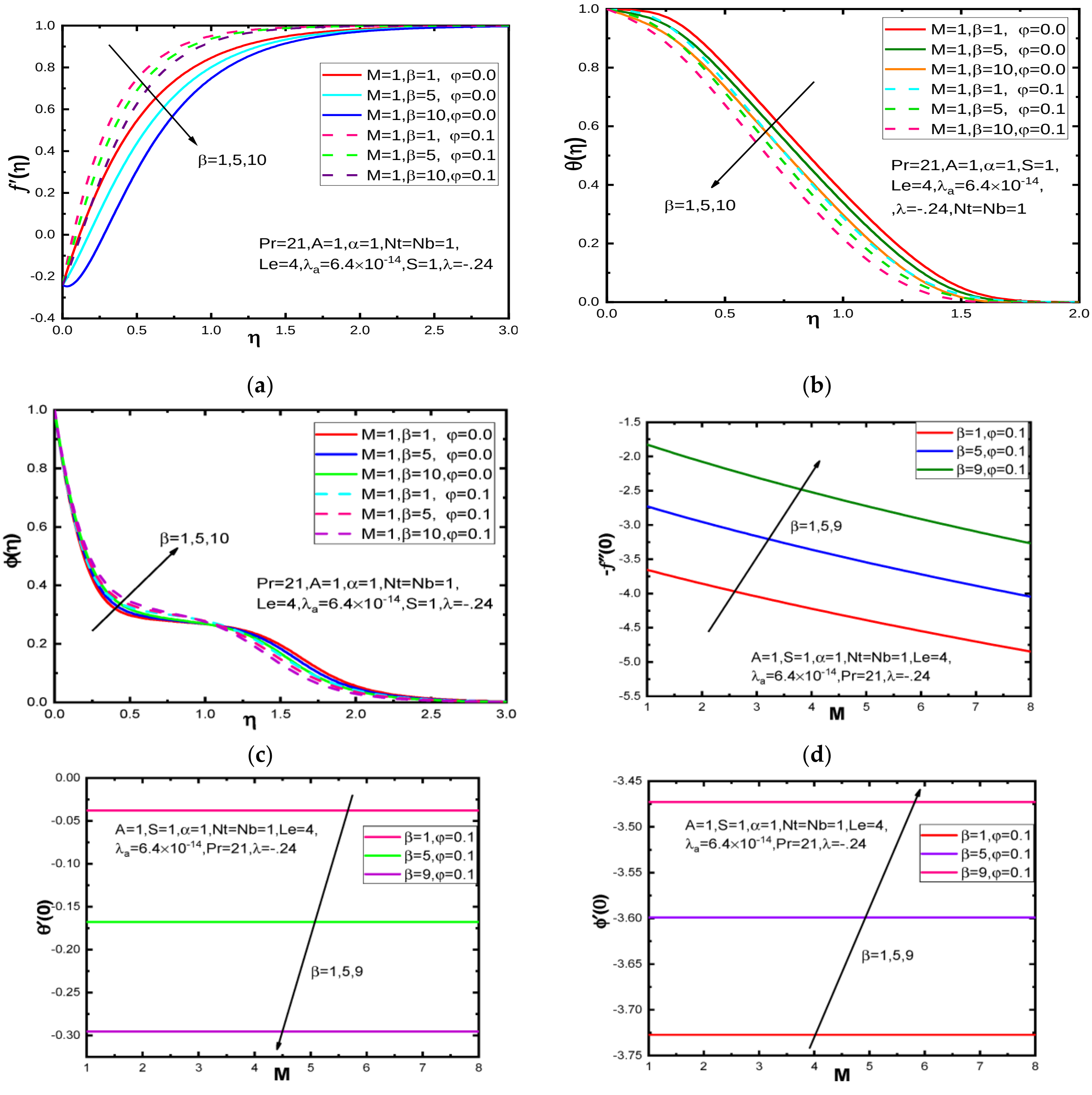

Figure 2. Variations of ferromagnetic interaction parameter on (**a**) velocity; (**b**) temperature profiles. (**c**) concentration profile; (**d**) skin friction coefficient. (**e**) rate of heat transfer; (**f**) local Sherwood Number.

4. Conclusions

Based on numerical results, the following sentences can be used to summarize our findings:

(i) For enlarging values of ferromagnetic number, fluid velocity and temperature decreases but reverse phenomena is observed in concentration profile.

(ii) A significant improvement is observed for blood temperature and velocity profile when gold nanoparticles are mixed with blood compare to that of conventional regular fluid.

(iii) Local Sherwood number as well as skin friction coefficient enhanced with augmenting values of ferromagnetic number; while major reduction is observed for rate of heat transfer.

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