computer sciences and mathematics forum

MDPI

Proceeding Paper

Numerical Solution of The Effects of Variable Fluid Properties on Biomagnetic Fluid over an Unsteady Stretching Sheet †

M. G. Murtaza [1], Anik Gomes [1], Jahangir Alam [2], E. E. Tzirtzilakis [3] and M. Ferdows [2,*]

[1] Department of Mathematics, Comilla University, Comilla 3506, Bangladesh; murtaza@cou.ac.bd (M.G.M.); thomasanikgomes@gmail.com (A.G.)
[2] Research Group of Fluid Flow Modeling and Simulation, Department of Applied Mathematics, University of Dhaka, Dhaka 1000, Bangladesh; jahangircu1994@gmail.com
[3] Fluid Mechanics and Turbomachinery Laboratory, Department of Mechanical Engineering, University of the Peloponnese, 26334 Patras, Greece; etzirtzilakis@go.uop.gr
* Correspondence: ferdows@du.ac.bd

† Presented at the 1st International Online Conference on Mathematics and Applications; Available online: https://iocma2023.sciforum.net/.

Abstract: In this paper, the effects of various fluid properties on two dimensional unsteady Biomagnetic fluid flow (blood) and heat transfer over a stretching sheet under the appearance of magnetic dipole is investigated. The governing boundary layer equations are simplified by suitable transformation which are then solved by bvp4c function aaproach in MATLAB software. The results indicate that fluid velocity and temperature are greatly influenced for ferromagnetic interaction parameter. Where, for ferromagnetic number fluid velocity drops but temperature rises upward. It is also found that the coefficient of skin friction and Nusselt number increase with the increment values of thermal conductivity parameter. For certain parameter values, the results are also compared with previously published studies and found in acceptable agreement.

Keywords: Biomagnetic fluid; variable viscosity; variable thermal conductivity; magnetic field; unsteady; stretching sheet; skin friction coefficient; rate of heat transfer

Citation: Murtaza, M.G.; Gomes, A.; Alam, J.; Tzirtzilakis, E.E.; Ferdows, M. Numerical Solution of The Effects of Variable Fluid Properties on Biomagnetic Fluid over an Unsteady Stretching Sheet. *Comput. Sci. Math. Forum* **2023**, *6*, x. https://doi.org/10.3390/xxxxx

Academic Editor(s):

Published: 28 April 2023

1. Introduction

The study of two dimensional boundary layer flow and heat transfer of biomagnetic fluid over a stretching sheet with variable viscosity and thermal conductivity is very important due to its various applications in engineering, bio-medical, industrial disciplines. Crane [1] was first studied the steady two dimensional boundary layer flow of a flat elastic sheet. Since then the problem has been extensively studied by taking into account many different physical features either separately or in various combinations. Shedzad et al.[2] analyzed the effect of thermophoresis mechanisms on mixed convection flow with different flow and thermal conditions. Elbashbeshy and Bazid [3] analyzed a similarity solution for the boundary layer equations which describe the unsteady flow and heat transfer over a stretching sheet. Mukhopadhyay [4] presented a solutions for unsteady boundary layer flow past a stretching sheet with variable fluid viscosity and thermal diffusivity in presents of wall suction. The mathematical and numerical solutions for biomagnetic fluids dynamics (BFD) applications in cancear treatment is proposed by Misra et al. [5]. Bhatti et al.[6] analyzed the heat transfer properties and application of the blood clot with variable viscosity. Murtaza et al.[7] analyzed an extended BFD model with accompying the principles of ferohydrodynamic and megnetohydrodynamics. Misra et al.[8] investigated the biomagnetic fluid flow over a stretching sheet and consider the viscoelastic property of the fluid. Tzirtzilakis et al.[9] analyzed the study of two dimensional, steady, laminar and incompressible biomagnetic fluid past a stretching sheet with heat transfer. In that study, the magnetization of the fluid varied with the magnetic field strength and the temperature.

To authors best informations the investigations of blood flow affected by a magnetic field accompying with variable fluid properties over an unsteady stretching sheet has not yet studied. As we know that, the blood properties namely- viscosity and thermall conductivity have a fundamental relationship with temperature. Therefore, the aim of this this parper is to fill up this gap with mathematical assumptions and computational solutions. The results indicates that blood flow in boundary layer is appriciably influenced by ferromagnetic number by considering the values up to seven Tesla, which can be highly helpful in cancer treatment to kill cancer cells as well as practical in medication delivery.

2. Model Description

In Figure 1, the X -axis and Y -axis are taken along and perpendicular to the sheet respectively. We considered that for time $t<0$ the fluid and heat flows are steady. The unsteady fluid and heat flows start at $t=0$, the sheet is being stretched with the velocity $U_w(x,t)=ax(1-ct)^{-1}$ along X -axis, keeping the origin fixed. The temperature of the sheet $T_w(x,t)$, while the ambient blood temperature is T_∞. Due to applied magnetic field in boundary domain, a magetic dipole is produced and situtated at a distance from the sheet say d. Therefore, the modified mathematical equations for governing problem with the help of [10] can be expressed as:

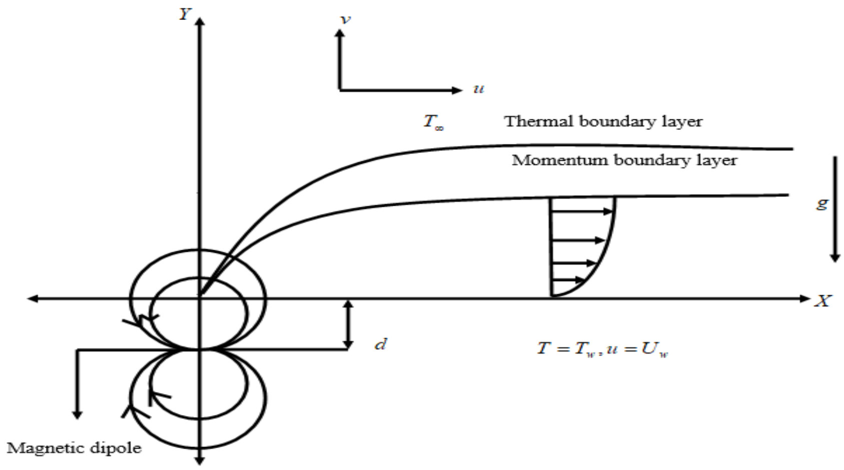


Figure 1. Sketch of the physical problem.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho_\infty}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)+\frac{\mu_0}{\rho_\infty}\mathrm{M}\frac{\partial \mathrm{H}}{\partial x}+g\beta^*(T-T\infty) \tag{2}$$

$$\rho_\infty C_p\left(\frac{\partial \mathrm{T}}{\partial t}+u\frac{\partial \mathrm{T}}{\partial x}+v\frac{\partial \mathrm{T}}{\partial y}\right)+\mu_0 T\frac{\partial \mathrm{M}}{\partial \mathrm{T}}\left(u\frac{\partial \mathrm{H}}{\partial x}+v\frac{\partial \mathrm{H}}{\partial y}\right)=\frac{\partial}{\partial y}\left(K\frac{\partial \mathrm{T}}{\partial y}\right) \tag{3}$$

With following boundary conditions:

$$\left.\begin{aligned} y=0 &: u=U_w, v=V_w(t), T=T_w \\ y\to\infty &: u\to 0, T\to T_\infty \end{aligned}\right\} \tag{4}$$

where u and v are the velocity components along the respectively directions. ρ_∞ is the fluid density, υ_∞ is the kinematic viscosity, g is the acceleration due to gravity, β^* is the coefficient of thermal expansion, T is the fluid temperature, C_p is the specific heat at constant pressure, μ is the co-efficient of viscosity, K is the variable thermal conductivity. Following, Lai and Kulacki [11] the variable fluid viscosity can be simplified as:

$$\frac{1}{\mu}=\frac{1}{\mu_\infty}\left[1+\gamma\left(T-T_\infty\right)\right] \tag{5}$$

And the thermal conductivity formula written by [12]:

$$K=K_\infty[1+\frac{\xi_1}{\Delta T}(T-T_\infty)] \tag{6}$$

Here, ξ_1 is a small parameter known as the variable thermal conductivity parameter. The magnetic field of intensity is given by [7,9]:

$\mathrm{H}(x,y)=[\mathrm{H}_x^2+\mathrm{H}_y^2]^{\frac{1}{2}}=\frac{\gamma}{2\pi}\left[\frac{1}{(y+d)^2}-\frac{x^2}{(y+d)^4}\right]$. Where, fluid magnetization with temperature is given by: $M=k\left(T-T_\infty\right)$, where k is a pyromagnetic coefficient constant.

The following similarity variables are introduced:

$$\eta=\left(\frac{a}{(1-ct)\upsilon_\infty}\right)^{1/2}y;\ \psi=\left(\frac{a\upsilon_\infty}{(1-ct)}\right)^{1/2}xf(\eta);\ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty} \tag{7}$$

Therefore, Equations (2) and (3) reduced as:

$$f'''-\frac{1}{(\theta-\theta_r)}f''\theta'+\left(\frac{\theta-\theta_r}{\theta_r}\right)A\left(f'+\frac{\eta}{2}f''\right)+\left(\frac{\theta-\theta_r}{\theta_r}\right)\left(f'^2-ff''-\lambda_1\theta\right)+\left(\frac{\theta-\theta_r}{\theta_r}\right)\frac{2\beta\theta}{(\eta+\alpha_1)^4}=0 \tag{8}$$

$$\Pr\left(2A\theta+\frac{1}{2}\eta A\theta'+f'\theta-f\theta'\right)+\frac{2\beta\lambda(\theta+\varepsilon)}{(\eta+\alpha_1)^3}f=\xi_1\theta'^2+(1+\xi_1\theta)\theta'' \tag{9}$$

Subject to the boundary conditions

$$\left.\begin{aligned}&\eta=0: f'(\eta)=1,\ f(\eta)=f_w,\theta(\eta)=1\\&\eta\to\infty: f'(\eta)\to 0,\theta(\eta)\to 0\end{aligned}\right\} \tag{10}$$

where, $A=\frac{c}{a}$, $\Pr=\frac{C_p\rho_\infty\upsilon_\infty}{K_\infty}$, $\lambda_1=\frac{g\beta^*b}{a^2}$, $\lambda=\frac{a\mu_\infty^2}{\rho_\infty K_\infty(T_w-T_\infty)}$, $\varepsilon=\frac{T_\infty}{T_w-T_\infty}$, $\beta=\frac{\gamma}{2\pi}\frac{k\mu_0\rho_\infty(T_w-T_\infty)}{\mu_\infty^2}$, $\theta_r=-\frac{1}{\gamma(T_w-T_\infty)}$, $\alpha_1=\left(\frac{a}{\upsilon_\infty(1-ct)}\right)^{1/2}d$, $f_w=-\frac{\upsilon_0}{\sqrt{a\upsilon_\infty}}$.

The important characteristics of the flow are skin friction coefficient and the Nusselt number respectively defined by:

$$C_f=-2\left(\frac{\theta_r}{\theta-\theta_r}\right)\mathrm{Re}^{-1/2}f''(0);\ Nu_x=-\mathrm{Re}^{1/2}\theta'(0)$$

3. Results and Discussion

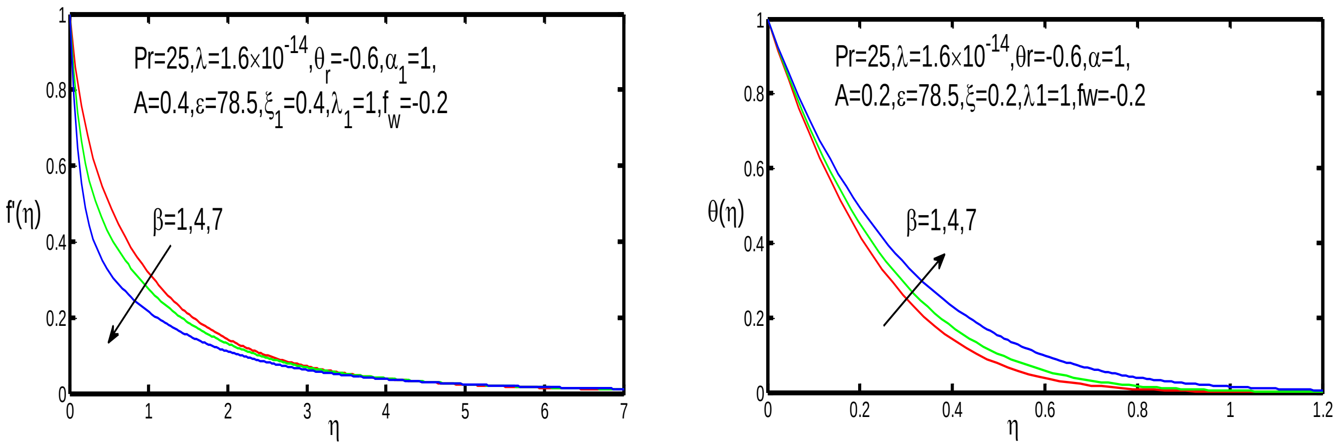
Computaional solutions of ODEs are obtained by well-known bvp4c technique in MATLAB. In order to assess the validity of the numerical analysis, a comparison has been conducted with Pop et al. [13] for various values of variable viscosity parameter when $\mathrm{Pr} = 0.7$. The comparison shows an excellent agreement as present in Table 1.

Table 1. Comparison values of $-f''(0)$ and $-\theta'(0)$ for $\mathrm{Pr} = 0.7$.

Viscosity	Present Results		Pop et al. [13]	
θ_r	$-f''(0)$	$-\theta'(0)$	$f''(0)$	$\theta'(0)$
−4	−0.507688	−0.344253	−0.5077877	−0.3442274
−2	−0.562720	−0.3350554	−0.5628924	−0.3348913
2	−0.278501	−0.380666	−0.2783288	−0.3806688
4	−0.369565	−0.366999	−0.3698711	−0.3667289

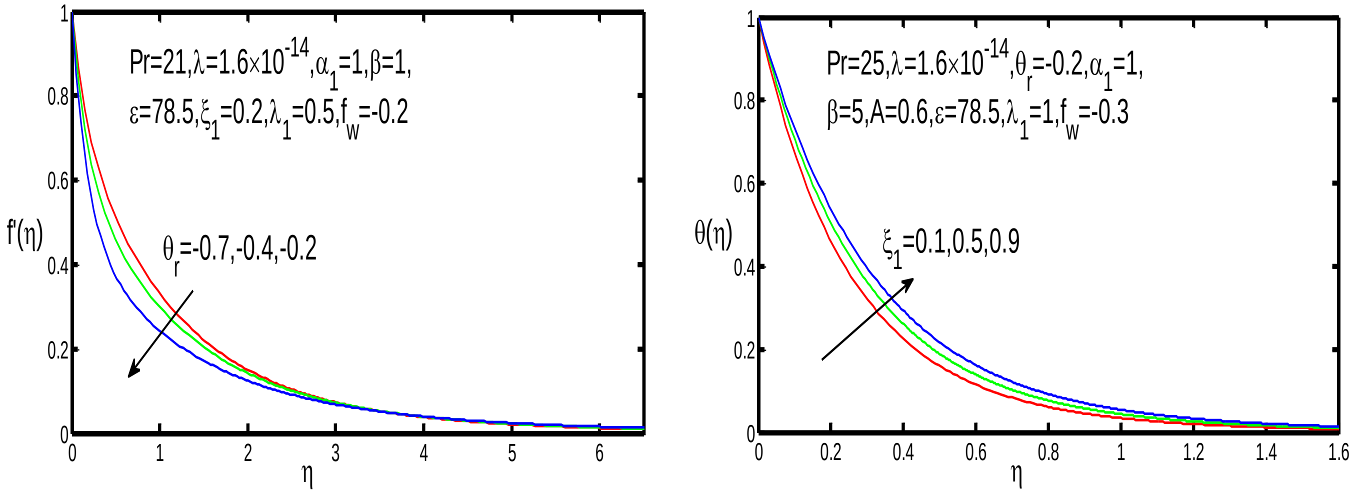
The impacts of ferromagnetic interaction parameter on velcity and temperature profile shown by Figure 2a,b. It is observed that with rising values of ferromagnetic number blood velocity decreases but temperature accelerate. This is causes because of polarization of blood. Since by applying a strong magnetic field on boundary domain there has been produced a resist force which known as Kelvn force. As a results, flow in boundary layer slow down.

Figure 3a,b shows the variations of variable viscosity parameter and thermal conductivity parameter on velocity and temperature profiles, respectively. It is seen that blood velocity gradually decreased for incrementing values of viscosity parameter. Physically, larger values of viscosity parameter suggests a greater temperature difference between the blood surface and the surrounding air. Where, temperature enhanced with thermal conductivity parameter (see Figure 3b).



(**a**) (**b**)

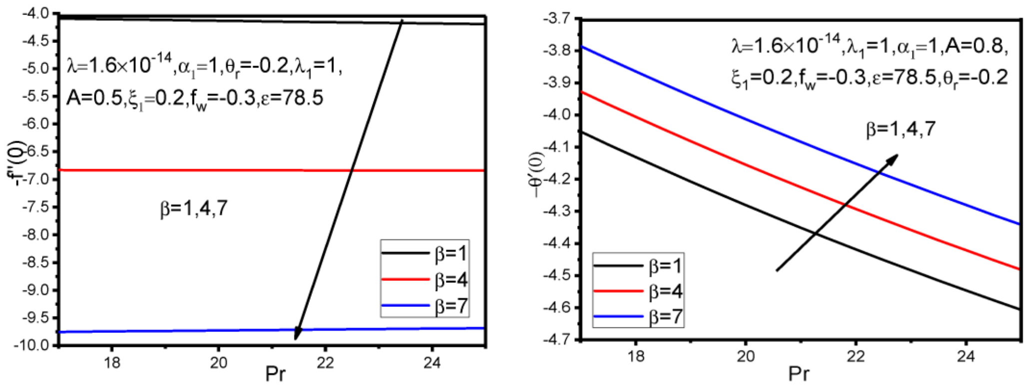
Figure 2. Variations of ferromagnetic number on (**a**) velocity; (**b**) temperature profiles.



(**a**) (**b**)

Figure 3. Variations of variable viscosity parameter on (**a**) velocity; and thermal conductivity parameter on (**b**) temperature profiles.

Finally, the variations of skin friction coefficient and rate of heat transfer for various values of ferromagnetic number against Prandtl number presented in Figure 4a,b. It is seen that for ferromagnetic number, skin friction decline whereas rate of heat trancfer enhanced.



(**a**) (**b**)

Figure 4. Variations of ferromagnetic number on (**a**) skin friction coefficient; (**b**) rate of heat transfer.

4. Conclusions

On the basis of computational outcomes, we can encapsulate our results with following statements:

(i) Fluid velocity reduced for enlarging values of ferromagnetic number and viscosity variation parameter.

(ii) For increasing values of ferromagnetic number and thermal conductivity parameter, temperature distributions increased

(iii) Skin friction coefficient decreases for enhancement of ferromagnetic number; whereas reverse phenomena is observed in rate of heat transfer.

References

1. Crane, L.J. Flow past a stretching plate. *Z. Fur Angew. Phys.* **1970**, *21*, 645–647.
2. Shedzad; Abbasi, F.M.; Hayat, T.; Alsaadi, F. MHD mixed convection peristaltic motion of nanofluid with Joule heating and thermophoresis effects. *PLoS ONE* **2014**, *9*, e111417

3. Elbashbeshy, E.M.A.; Bazid, M.A.A. Heat transfer over an unsteady stretching surface with internal heat generation. *Appl. Math. Comput.* **2003**, *138*, 239–245.
4. Mukhopadhyay, S. Unsteady boundary layer flow and heat transfer past a porous stretching sheet in presence of variable viscosity and thermal diffusivity. *Int. J. Heat Mass Transf.* **2009**, *52*, 5213–5217.
5. Misra, J.C.; Sinha, J.A.; Shit, G.C. Flow of a biomagnetic viscoelastic fluid: Application to estimation of blood flow in arteries during electromagnetic hyperthermia, a therapeutic procedure for cancer treatment. *Appl. Math. Mech.* **2010**, *31*, 1405–1420.
6. Bhatti, M.M.; Zeeshan, M.; Ellahi, R. Heat transfer analysis on peristaltically induced motion of particle-fluid suspension with variable viscosity: Colt blood model. *Comput. Meth. Prog. Bio.* **2016**, *137*, 115–124.
7. Murtaza, M.G.; Tzirtzilakis, E.E.; Ferdows, M. Effect of electrical conductivity and magnetization on the biomagnetic fluid flow over a stretching sheet. *Z. Angew. Math. Phy.(ZAMP)* **2017**, *68*, 93.
8. Misra, J.C.; Shit, J.C. Biomagnetic viscoelastic fluid flow over a stretching sheet. *Appl. Math. Comput.* **2009**, *210*, 350–361.
9. Tzirtzilakis, E.E.; Tanoudis, G.B. Numerical study of Biomagnetic fluid flow over a stretching sheet with heat transfer. *Int. J. Numer. Methods Heat Fluid Flow* **2003**, *13*, 830–848.
10. Vajravelu, K.; Prasad, K.V.; Chin-On, N. Unsteady convective boundary layer flow of a viscous fluid at a vertical surface with variable fluid properties. *Nonlinear Anal. Real World Appl.* **2013**, *14*, 455–464.
11. Lai, F.C.; Kulacki, F.A. The effect of variable viscosity on conductive heat transfer along a vertical surface in a saturated porous medium. *Int. J. Heat Mass Transf.* **1990**, *33*, 1028–1031.
12. Chiam, T.C. Heat transfer with variable thermal conductivity in a stagnation point flow towards a stretching sheet. *Int. Commun. Heat Mass Transf.* **1996**, *23*, 239–248.
13. Pop, I.; Subba, R.; Glora, R.; Rashidi, M. The effect of variable Viscosity on flow and Heat Transfer to a Continuous on moving flat plate. *Int. J. Eng. Sci.* **1992**, *30*, 1–6.