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Proceeding Paper

Mathematical Analysis of a Discrete System Modeling COVID-19 †

Meriem El-Batoul Keddar * and Omar Belhamiti

Department of Mathematics, University of Mostaganem, Mostaganem city, Algeria; omar.belhamiti@univ-mosta.dz
* Correspondence: meriem.keddar.etu@univ-mosta.dz
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Abstract: COVID-19 is one of the worst pandemics ever; it is spreading rapidly and creating a health crisis around the world. This disease continues to seriously threaten human life and has caused more than 759 million infections and over 6.8 million deaths worldwide. In this work, we propose a discrete mathematical model of COVID-19 to predict the evolution of the dynamics of this pandemic. We calculate the basic reproduction number and the equilibrium points, and then perform the stability analysis theoretically and numerically. Finally, we conclude with local sensitivity analysis.

Keywords: Discrete system; modeling in epidemiology; COVID-19; sensitivity analysis; stability analysis; the basic reproduction number

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1. Introduction

In December 2019, a mysterious pneumonia appeared in the city of Wuhan, China. By 3 January 2020, 44 cases of this disease had been diagnosed. A month later, it was determined that this was caused by a new coronavirus, which was named SARS-CoV-2 and whose disease is known as COVID-19 (coronavirus disease 2019). In March 2020, the World Health Organization declared COVID-19 a pandemic due to its spread to most continents, creating a global health crisis with over 118 thousand confirmed cases and 4292 deaths worldwide. By the end of the following month, the number of infected had exceeded 3 billion with 217,769 deaths. COVID-19 had affected almost every corner of the world. Many countries announced the closure of places vulnerable to the spread of the epidemic, such as schools, universities, markets, and places of entertainment. They also went into lockdown and closed borders to reduce the spread of the virus. However, the number of infected cases continued to increase, and no ways to end the pandemic have been found. Three years later, this pandemic, which initially raised few concerns, has killed over 6.8 million people and changed the world forever.

Since the beginning of the pandemic, all sectors have mobilized to fight against the highly transmissible SARS-CoV-2, including mathematics, specifically modeling. Many models have been proposed to predict the evolution of the dynamics and behavior of this disease. These models allow for the estimation of lethality and the basic reproduction number, especially to find ways to minimize the number of infected and deaths and to control scenarios responsible for transmission. In this work, a discrete SEIR model was proposed to predict the dynamics of COVID-19, including asymptomatic infected, infected with moderate symptoms, and those developing a severe format that requires hospitalization. The model calculates the basic reproduction rate R_0 and the equilibrium points. Stability analysis was performed, followed by a local sensitivity analysis.

2. Materials and Methods

We developed a compartmental model for COVID-19 using a discrete deterministic system based on epidemiological data. The model was used to analyze the disease and predict its evolution. We calculated the basic reproduction number using the next-generation method. The local stability theorem of Lyapunov was used for stability analysis. To perform the local sensitivity analysis, we calculated the sensitivity indexes for R_0.

3. Results and Discussion

3.1. Stability Analysis

After the stability analysis of our system, we found that it has two equilibrium points; the disease-free equilibrium $\mathbf{E_0}$ which is locally asymptotically stable when $\mathbf{R_0} < \mathbf{1}$, and the endemic equilibrium $\mathbf{E_1}$ which is locally asymptotically stable when $\mathbf{R_0} > \mathbf{1}$.

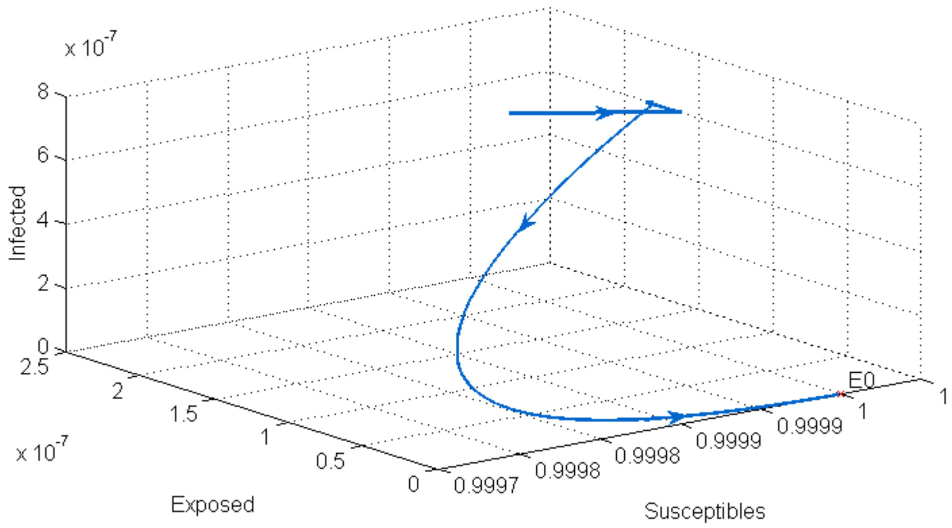

Figure 1. Stability of the disease-free point E_0.

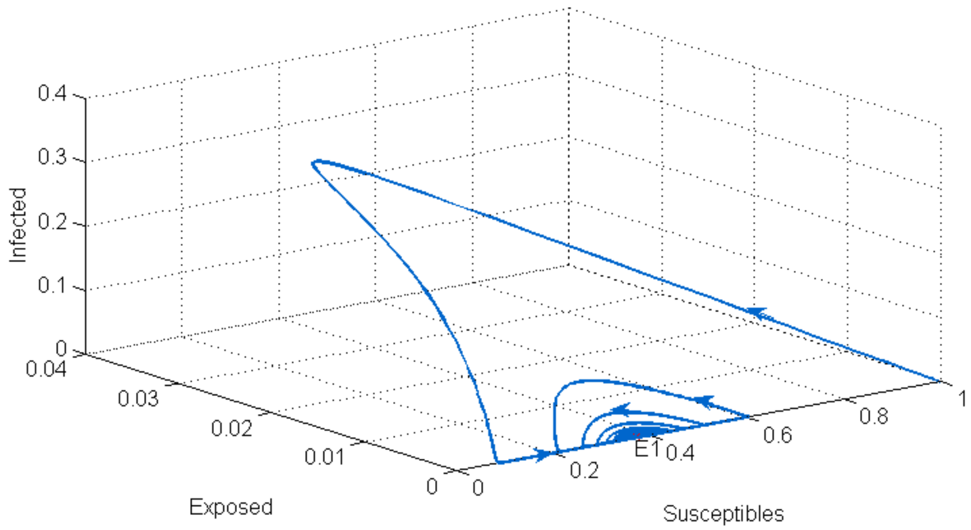

Figure 2. Stability of the endemic equilibrium point E_1.

3.2. Sensitivity Analysis

We performed a sensitivity analysis by calculating the sensitivity indices of $\mathbf{R_0}$. Our analysis showed that R_0 is most sensitive to the transmission rate of COVID-19 and the recovery rate of those infected with moderate symptoms. An increase in the transmission rate leads to an increase in R_0, while an increase in the recovery rate leads to a decrease in R_0.

Table 1. Sensitivity indices of R_0.

Parameter	Sensitivity Index
α	+1.0000
γ_1	−0.6139
γ_2	−0.3346
β_3	−0.1500
β_2	+0.0855
ϖ	−0.0711
β_1	+0.0647
λ_1	+0.0268

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