



Proceeding Paper

Fractional order thermoelastic model with voids in three-phase-lag thermoelasticity [†]

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Abstract: This paper deals with fractional order three-phase-lag (TPL) thermo-elasticity in a micropolar thermoelastic half space medium with voids. The subsequent non-dimensional coupled equations are solved by using the normal mode analysis and eigenvalue approach methods. By doing numerical computations of the physical fields for a substance that resembles a magnesium crystal in the presence of an electromagnetic field, the issue is proven to exist. The effect of the fractional order, the phase lags on the components of temperature, displacement, the stress, and changes in volume fraction field have been depicted graphically. Additionally, a graphic comparison of several types of models employing phase delays and the influence of the magnetic field is displayed.

Keywords: Fractional thermo-elasticity; Three-phase-lag thermo-elasticity; Eigen value approach; Normal mode analysis; Electro-magnetic field.

1. Introduction

Lord and Shulman(1967)[1] derived a new model to overcome the infinite speed of heat transportation which was established by Biot(1956)[2]. The heat equation of this theory is of the wave type, it automatically ensures finite speeds of propagation for heat and elastic waves. Linear model of thermoelasticity is incapable to characterize the behaviour of many new synthetic solids of the elastomer and polymer type. Eringen first time used the micropolar elasticity term to describe such materials.

Fractional calculus was first used by Abel in the solution of an integral equation that arises in the tautochrone problem. This area has grown rapidly, and applications have been found in several fields including solid mechanics, geophysics, physics, and mathematical biology. Many mathematical models in the fields of solid mechanics, bio-rheology, nonlinear dynamical systems in ecology, and so on, have been successfully modified using fractional calculus. The fractional derivative exhibits nonlocal properties, and global dependency is among the main reason to use. Kimmich considered anomalous diffusion and characterised it by the time-fractional diffusion wave equation using the Riemann-Liouville fractional integral. Povstenko [3,4] demonstrated the effect of fractional heat transportation in the presence of thermal stresses. Many authors [5–9] also discuss fractional calculus in thermoelasticity.

Recently, Tzou [10,11] developed dual-phase-lag heat equation theory by incorporating of two-phase-lags associated with temperature gradient heat flux vector. Two phase lag parameters are related to the fastest - effects of thermal inertial. This theory is known as dual-phase-lag (DPL) model. Later three-phase-lag (TPL) model was developed by Roychoudhury [12].

In the present article, we consider two dimensional problem under TPL heat conduction theory in present of electric and magnetic field. The governing equation are transformed by normal modes. Finally the displacement component and temperature distribution are



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fixed out analytically from the vector matrix differential equation by eigenvalue method in transformed domain.

2. Basic Equation

The system of governing equations of a rotating micropolar thermoelastic solid is given by[12,13]

$$\sigma_{ij,j} + F_i = \rho[\ddot{u}_i + \Omega \times (\Omega \times u)_i + (2\Omega \times \dot{u})_i] \tag{1}$$

$$m_{ij,i} + \epsilon_{ijr}\sigma_{ir} = J\rho[\ddot{\phi} + (\Omega \times \phi)_i] \tag{2}$$

$$\alpha\psi_{,ii} - \epsilon\psi - \omega\dot{\psi} - \beta^*u_{i,i} + m\theta = \rho\epsilon_1\dot{\psi} \tag{3}$$

The components of the force stress and couple stress tensors are

$$\sigma_{ij} = \lambda u_{r,r}\delta_{ij} + \mu(u_{i,j} + u_{j,i}) + k(u_{j,i} - \epsilon_{ijr}\phi_r) - \beta^*\psi\delta_{ij} - \gamma_T(\theta + v\frac{\partial\theta}{\partial t})\delta_{ij} \tag{4}$$

$$m_{ij} = \alpha\phi_{r,r}\delta_{ij} + \beta\phi_{i,j} + \gamma\phi_{j,i} \tag{5}$$

Fractional order Three-phase-lag Heat conduction without heat source as

$$\left[K^* \left(1 + \frac{\tau^\alpha_\theta}{\alpha!} \frac{\partial^\alpha}{\partial t^\alpha} \right) + K \left(1 + \frac{\tau^\alpha_\theta}{\alpha!} \frac{\partial^\alpha}{\partial t^\alpha} \right) \right] \theta_{ii} = \left(1 + \frac{\tau^\alpha_q}{\alpha!} \frac{\partial^\alpha}{\partial t^\alpha} + \frac{\tau^{2\alpha}_q}{2\alpha!} \frac{\partial^{2\alpha}}{\partial t^{2\alpha}} \right) (\rho C_E \frac{\partial\theta}{\partial t} + \beta T_0 \frac{\partial}{\partial t}(\dot{\epsilon}_{kk})) \tag{6}$$

and the strain components are

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \text{ and } e_{kk} = (\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}) \tag{7}$$

3. Formulation of the problem

$$\begin{aligned} & \frac{\partial^2 u}{\partial x^2} + (1 - a^2) \frac{\partial^2 w}{\partial x \partial z} + a^2 \frac{\partial^2 u}{\partial z^2} - \epsilon_3 \frac{\partial \phi_2}{\partial z} - \frac{1}{(\epsilon_1 + \epsilon_2)} (1 + v \frac{\partial}{\partial t}) \frac{\partial \theta}{\partial x} \\ & = (\frac{1}{\epsilon_1 + \epsilon_2}) \frac{\partial^2 u}{\partial t^2} - \frac{\Omega^2}{(\epsilon_1 + \epsilon_2)} u + \frac{2\Omega}{(\epsilon_1 + \epsilon_2)} \frac{\partial w}{\partial t} \end{aligned} \tag{8}$$

$$\begin{aligned} & \frac{\partial^2 w}{\partial z^2} + (1 - a^2) \frac{\partial^2 u}{\partial x \partial z} + a^2 \frac{\partial^2 w}{\partial x^2} - \epsilon_3 \frac{\partial \phi_2}{\partial x} - \frac{1}{(\epsilon_1 + \epsilon_2)} (1 + v \frac{\partial}{\partial t}) \frac{\partial \theta}{\partial z} \\ & = (\frac{1}{\epsilon_1 + \epsilon_2}) \frac{\partial^2 w}{\partial t^2} - \frac{\Omega^2}{(\epsilon_1 + \epsilon_2)} w + \frac{2\Omega}{(\epsilon_1 + \epsilon_2)} \frac{\partial u}{\partial t} \end{aligned} \tag{9}$$

$$\frac{\partial^2 \phi_2}{\partial x^2} + \frac{\partial^2 \phi_2}{\partial z^2} - 2\epsilon_7 \phi_2 + \epsilon_7 (\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}) = \epsilon_8 \frac{\partial^2 \phi_2}{\partial t^2} \tag{10}$$

$$(1 + \sum_{n=1}^N \frac{\tau^n_\theta}{n!} \frac{\partial^n}{\partial t^n}) (\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial z^2}) = p_l \left(\frac{\partial \theta}{\partial t} + \epsilon_9 \frac{\partial}{\partial t} (\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}) \right) \tag{11}$$

The components of stress tensor and force tensor are written in the below form:

$$\sigma_{xx} = (\epsilon_1 + \epsilon_2) \frac{\partial u}{\partial x} + (\epsilon_1 - 1) \frac{\partial w}{\partial x} - (1 + v \frac{\partial}{\partial t}) \theta \tag{12}$$

$$\sigma_{zz} = (\epsilon_1 + \epsilon_2) \frac{\partial w}{\partial z} + (\epsilon_1 - 1) \frac{\partial u}{\partial x} - (1 + \nu) \frac{\partial}{\partial t} \theta \tag{13}$$

$$\sigma_{xz} = \frac{\partial u}{\partial z} + \epsilon_2 \frac{\partial w}{\partial x} - (\epsilon_2 - 1) \phi_2 \tag{14}$$

$$\sigma_{zx} = \frac{\partial w}{\partial x} + \epsilon_2 \frac{\partial u}{\partial z} - (\epsilon_2 - 1) \phi_2 \tag{15}$$

$$m_{zy} = \frac{(\epsilon_2 - 1)}{\epsilon_4} \frac{\partial \phi_2}{\partial z} \tag{16}$$

$$m_{xy} = \frac{(\epsilon_2 - 1)}{\epsilon_4} \frac{\partial \phi_2}{\partial x} \tag{17}$$

where $\epsilon_i, i = 1(1)9$ are given in Appendices.

To simplify the equations (1)-(7) we use the non-dimensional variables as below

$$\begin{aligned} (x^*, z^*) &= \frac{\eta_0}{c_0} (x, z), (t^*, t_0^*, \tau_0^*, \tau_q^*, \tau_T^*, v^*) = \eta_0 (t, t_0, \tau_0, \tau_q, \tau_T, v), \Omega^* = \frac{\Omega}{\eta_0} \\ (u^*, w^*) &= \frac{\rho c_0 \eta_0}{\gamma_T T_0} (u, w), \theta^* = \frac{\theta}{T_0}, \sigma_{ij}^* = \frac{\sigma_{ij}}{\gamma_T T_0}, m_{ij}^* = \frac{\eta_0 m_{ij}}{c_0 \gamma_T T_0}, \phi_2^* = \frac{\rho c_0^2 \phi_2}{\gamma_T T_0}. \end{aligned} \tag{18}$$

Where

$$c_0^2 = \frac{\mu}{\rho}, \beta = \sqrt{\frac{\lambda + 2\mu}{\mu}}, \eta_0 = \frac{\rho c_E c_0^2}{k}, p_l = (\delta + \tau_0) \frac{\partial}{\partial t} + \sum_{n=1}^N \frac{\tau_q^{n+1}}{(n+1)!} \frac{\partial^{n+1}}{\partial t^{n+1}}.$$

Equations (8)-(17) takes the above form dropping the * for convenience.

4. Normal mode analysis

The decomposition of the solution of the physical variables under consideration has the following form in the normal mode

$$(u, w, \psi, \theta, \phi_2, \sigma_{jk})(x, z, t) = (\bar{u}, \bar{w}, \bar{\psi}, \bar{\theta}, \bar{\phi}_2, \bar{\sigma}_{jk})(x) e^{(st+iqz)}; j, k = x, z \tag{19}$$

where $\bar{u}, \bar{w}, \bar{\theta}, \bar{\phi}_2, \bar{\sigma}_{jk}, \bar{Q}$ are the amplitudes of the functions, s is a complex constant, $i = \sqrt{-1}$ and q is the wave numbers in the z -direction.

Using above equations we get the vector matrix differential equation as

$$\frac{d\bar{z}}{dx} = A\bar{z} \tag{20}$$

where

$$\bar{z} = (\bar{u}, \bar{w}, \bar{\phi}_2, \bar{\theta}, \bar{\psi}, \frac{d\bar{u}}{dx}, \frac{d\bar{w}}{dx}, \frac{d\bar{\phi}_2}{dx}, \frac{d\bar{\theta}}{dx}, \frac{d\bar{\psi}}{dx})^T \tag{21}$$

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ a_{51} & a_{52} & a_{53} & 0 & 0 & 0 & a_{56} & 0 & a_{58} & 0 \\ a_{61} & a_{62} & 0 & a_{64} & 0 & a_{65} & 0 & a_{67} & 0 & 0 \\ a_{71} & 0 & a_{73} & 0 & 0 & 0 & a_{76} & a_{77} & 0 & 0 \\ 0 & a_{82} & 0 & a_{84} & 0 & a_{85} & a_{86} & a_{87} & a_{88} & 0 \\ 0 & a_{92} & 0 & a_{94} & a_{95} & a_{96} & 0 & 0 & 0 & 0 \end{bmatrix} \tag{22}$$

Where $p_{ls} = \frac{(\delta + \tau_0 s + \sum_{n=1}^N \frac{\tau_q^{n+1}}{(n+1)!} s^{n+1})}{(1 + \sum_{n=1}^N \frac{\tau_\theta^n}{n!} s^n)}$ and all the components of matrix A in equation-(22) are given in Appendices.

5. Solution:

The following equation represents the matrix A’s characteristic equation:

$$\det(A - \Lambda I) = 0 \tag{23}$$

The eigenvalues of the characteristic equation (20) are taken in the form $\Lambda = \pm \Lambda_i$ ($i = 1, 2, 3, 4$)

The eigen vector \tilde{X} corresponding to the eigen value Λ calculated as

$$\tilde{X} = \left[\Gamma_1 \quad \Gamma_2 \quad \Gamma_3 \quad \Gamma_4 \quad \Lambda \Gamma_1 \quad \Lambda \Gamma_2 \quad \Lambda \Gamma_3 \quad \Lambda \Gamma_4 \right]^T \tag{24}$$

where Γ_i ($i, j = 1, 2, 3, 4$) are given in Appendix. We construct the inverse of the matrix $V = (X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8) = (X_{ij})_{8 \times 8}, i, j = 1(1)8$ as

$$V^{-1} = (\omega_{ij}), \quad i, j = 1, 2, 3, 4, 5, 6, 7, 8$$

Then the solution of the differential Equation (23) is [14,15]

$$\bar{z} = \sum_{j=1}^4 X_j y_j \tag{25}$$

$$y_r = C_r e^{\lambda_r x} + e^{\lambda_r x} \int_{-\infty}^{\infty} Q_r e^{-\lambda_r x} \text{ and } Q_r = \sum_{j=1}^8 \omega_{rj} f_j \tag{26}$$

where C_r is an arbitrary constant which is to be evaluated using initial and boundary conditions.

$$u = e^{(st+iqy)} \sum_{j=1}^4 X_{1j} (C_j e^{\Lambda_j x} - \frac{Q_j}{\Lambda_j}) \tag{27}$$

$$w = e^{(st+iqy)} \sum_{j=1}^4 X_{2j} (C_j e^{\Lambda_j x} - \frac{Q_j}{\Lambda_j}) \tag{28}$$

$$\theta = e^{(\omega t + iby)} \sum_{j=1}^4 X_{3j} (C_j e^{\Lambda_j x} - \frac{Q_j}{\Lambda_j}) \tag{29}$$

Using equations (27)-(29) simplifying the above equations we get the stress components as follows-

$$\sigma_{zz} = e^{(\omega t + ibz)} \left(\sum_{j=1}^4 C_j S_{1j}(x) e^{\Lambda_j x} - M_1 \right), \quad \sigma_{zx} = e^{(\omega t + ibz)} \left(\sum_{j=1}^4 C_j S_{2j}(x) e^{\Lambda_j x} - M_2 \right) \tag{30}$$

$$\sigma_{xx} = e^{(\omega t + ibz)} \left(\sum_{j=1}^4 C_j S_{4j}(x) e^{\Lambda_j x} - M_4 \right), \quad \sigma_{xz} = e^{(\omega t + ibz)} \left(\sum_{j=1}^4 C_j S_{3j}(x) e^{\Lambda_j x} - M_3 \right) \tag{31}$$

$$m_{xy} = e^{(\omega t + ibz)} \left(\sum_{j=1}^4 C_j S_{6j} e^{\Lambda_j x} - M_6 \right), \quad m_{zy} = e^{(\omega t + ibz)} \left(\sum_{j=1}^4 C_j S_{5j} e^{\Lambda_j x} - M_5 \right) \quad (32)$$

where C_j , $S_{ij}(x)$ and M_k ($i=1,2,3,4,5,6,7$; $j=1,2,3,4$; $k=1,2,3,4,5,6,7$) are given in the appendix.

6. Discussion

We discuss TPL model in figures 1, 2 and 3. In [figure 1](#) the fractional order parameter α has a significant effect on the temperature distribution, where increasing on α causes increasing on and the rate of change of with respect to x also increases when α increases at fixed $t=0.1$.

In [figure 2](#) When fractional order $\alpha=0.6$, time $t=0.1$ are fixed and other constants are same, then u displacement distribution starts with negative values for DPL, LS, CT theory. u - distribution upto $x=1.2$ for LS model is greater than DPL model and CT model. After $x=1.2$ the result is reversed.

In [figures 3](#) the fractional parameter has significant effects on the stress σ_{xx}, σ_{xy} distribution. Both starts from zero initially which shows that they satisfy boundary condition. Increasing of fractional parameter α causes decreasing the absolute values of the stresses, and the rate of change of them with respect to x also increase when α increases. For the fixed value of x the stress σ_{xy} distribution has the lowest value for the fractional parameter $\alpha=0.2$ in the range $0 \leq x \leq 1.0$. For $x \geq 1.0$ the distribution gradually increases towards zero.

7. Conclusions

In this work, the effect of the heat source, fractional order and phase lag parameters on the temperature distribution, displacement components, the stress components have been studying for a two-dimensional problem in a half space micropolar thermoelastic material is considered with the context of the fractional order thermoelasticity theory caring a heat source. We found that, the fractional order parameter has significant effects on all the studied fields and the results supporting the definition of the classification of the thermal conductivity of the materials. We also found significant effects of phase lag parameters.

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Sample Availability:

Abbreviations

The following abbreviations are used in this manuscript:

TPL Three phase lag

LS Lord-Shulman

CT Coupled Thermoelasticity theory

Appendix A

Appendix A.1

$$\begin{aligned}
 a_{51} &= \frac{\epsilon_4 s^2 + a^2 q^2 + \frac{s^2}{\epsilon_1 + \epsilon_2}}{1 + \epsilon_5}, a_{52} = \frac{i q \epsilon_3}{1 + \epsilon_5}, a_{53} = \frac{i q (a^2 - 1) - i q \epsilon_5}{1 + \epsilon_5}, a_{56} = \frac{(1 + \nu s)}{(\epsilon_1 + \epsilon_2)(1 + \epsilon_5)}, a_{58} = -b_3 \\
 a_{61} &= \frac{q^2 + \epsilon_4 s^2 + \epsilon_5 q^2 + \frac{s^2}{\epsilon_1 + \epsilon_2}}{a^2}, a_{62} = \frac{i q (1 + \nu s)}{a^2 (\epsilon_1 + \epsilon_2)}, a_{64} = -\frac{i q b_3}{a^2}, a_{65} = \frac{i q (a^2 - 1) - \epsilon_5}{a^2}, a_{67} = -\frac{\epsilon_3}{a^2} \\
 a_{71} &= -i q \epsilon_7, a_{73} = q^2 + \epsilon_8 s^2 + 2 \epsilon_7, a_{77} = \epsilon_7 \\
 a_{82} &= i q p_{1s} \epsilon_9, a_{84} = (p_{1s} + q^2), a_{85} = p_{1s} \epsilon_{10}, a_{86} = p_{1s} \epsilon_9 s, \\
 a_{92} &= i q \epsilon_{11}, a_{93} = 0, a_{94} = -\epsilon_{13}, a_{95} = q^2 + \epsilon_{14} s^2 + s \epsilon_6 + \epsilon_{12}, a_{96} = \epsilon_{11} \\
 \epsilon_1 &= \frac{\lambda + \mu}{\rho c_0^2}, \epsilon_2 = \frac{\mu + k}{\rho c_0^2}, \epsilon_3 = \frac{k / \rho c_0^2}{(\epsilon_1 + \epsilon_2)}, \epsilon_4 = \frac{\epsilon_0 \mu_0^2 H_0^2}{\rho (\epsilon_1 + \epsilon_2)}, \epsilon_5 = \frac{\mu_0^2 H_0^2}{\rho c_0^2 (\epsilon_1 + \epsilon_2)}, \epsilon_7 = \frac{k c_0^2}{\gamma \eta_0^2}, \\
 \epsilon_8 &= \frac{\rho l \eta_0^2}{\gamma}, \epsilon_9 = \frac{\beta T_0 \gamma_T}{\rho K \eta_0}, \epsilon_{10} = \frac{m T_0 \gamma_T}{\rho K \eta_0} \\
 \epsilon_6 &= \frac{\omega c_0^2}{\beta \eta_0}, \epsilon_{11} = \frac{b c_0^2}{\beta \eta_0^2}, \epsilon_{12} = \frac{\epsilon c_0^2}{\beta \eta_0}, \epsilon_{13} = \frac{m \rho c_0^4}{\beta \gamma_T \eta_0^2}, \epsilon_{14} = \frac{\rho \xi c_0^2}{\beta}
 \end{aligned}$$

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