Optimal Block Replacement Policies under Replacement First and Last Disciplines

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Outline

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Introduction

Preventive Maintenance (PM)

- Have been extensively studied to maintain system dependability.
- Include repair maintenance, age replacement, periodic replacement, and block replacement.
- Reduce the maintenance cost at failure time and improve system reliability.

Block Replacement

- A common method for maintaining the reliability of systems.
- Consist of a block or group of units.

Introduction

Cont-

- A failed item is replaced by a new item during the replacement interval T, and the schedualed replacement for the non-failed is performed at T.
- Systems become more complicated and random, random maintenance policy would fit more for the systems and should be done rather than a deterministic one.

Comparison Between Age Replacement and Block Replacement Model

- Age Replacement Model

$$C(T) = \frac{c_F F(T) + c_T \overline{F}(T)}{\int_0^T \overline{F}(t)\,dt}. \tag{1}$$

Introduction

Cont-

- Applied to these non-repairable systems.
- Block Replacement Model

$$C(T) = \frac{c_F M(T) + c_T}{T}. \quad (2)$$

- where
  - c_F: Failure replacement cost every time.
  - c_T: Preventive replacement cost every time.

Introduction

Opportunity is important !!!

- The restriction of delivery time lag of spare parts affects stable operations of supply chain management.
- Has the possibility to purchase spare parts with cheaper cost at unexpected timing through the networked supply chain systems.
- A lot of opportunity-based replacement models are developed, such as replacement first and replacement last models.

Related Works

- Nakagawa (2014) considered a unit works for a job with random working times and proposed the basic random block replacement models.
- Zhao and Nakagawa (2012) formulated replacement first and replacement last models.

Motivation

Our Contributions

- Derive the optimal opportunity-based block replacement policies taking account of replacement first and last discinples.
- Compare the opportunity-based block replacement policies by numerical illustrations.

Assumptions

The failure time X follows the general probability distribution with

- Probability mass function $f(t)$.
- Survivor function $\bar{F}(t)$.
- Failure rate function $r(t) = f(t)/\bar{F}(t)$.
- Renewal function $M(t)$.
- Renewal density function $m(t) = dM(t)/dt$.

The opportunity arrival time of preventive replacement, Y, obeys a common distribution with

- Probability mass function $g(t)$.
- Survivor function $\bar{G}(t)$.

Notation

Cont-

- Hazard rate function $h(t) = g(t)/\bar{G}(t)$.
- Reversed hazard rate $\hat{h}(t) = g(t)/G(t)$.

Where in general $\bar{\Phi}(.) \equiv 1 - \Phi(.)$.
The cost parameters are given by

- c_F : the failure replacement every time.
- c_P : the preventive replacement every time.
- c_Y : the opportunisic replacement every time.

Without loss of generality, $c_F > c_P \geq c_Y$.

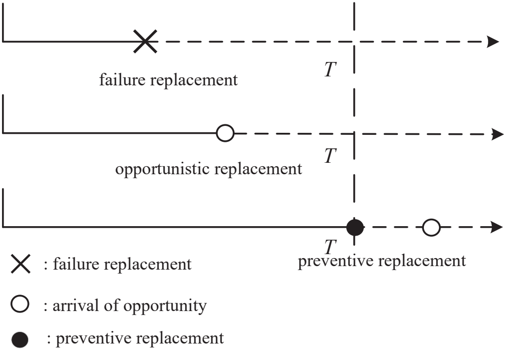

Figure 1: The configuration of RF discipline.

- If the failure occurs, then the failure replacement is made.
- The unfailed item is replaced at the time Y or T, whichever occurs first.

Expected Cost under RF Discipline

From the renewal reward argument,

- The expected cost per unit time in the steady state:

$$C(T) = \frac{B(T)}{A(T)}, \tag{3}$$

- The problem is to determine the optimal preventive replacement time T.
- where
  - $A(T)$ — Mean time length of one cycle.
  - $B(T)$ — Expected cost during one cycle.

$$A(T) = \int_0^T \bar{G}(t)\,dt. \tag{4}$$

Expected Cost under RF Discipline

- **Cont-**

$$\begin{aligned} B(T) &= c\int_0^T \bar{G}(t)m(t)\,dt + c_Y G(T) + c_P\bar{G}(T) \\ &= c_P + c_F\int_0^T \bar{G}(t)m(t)\,dt \\ &- (c_P - c_Y)\,G(T). \end{aligned} \tag{5}$$

Optimal RF Policy

For the problem $\min_T C(T)$**,**

- Taking the difference of $C(T)$ with respect T and setting it equal to zero, we have

$$c_F \int_0^T \bar{G}(t)\,[m(T) - m(t)]\,dt - (c_P - c_Y)\int_0^T \bar{G}(t)\,[r(T) - r(t)]\,dt = c_P, \tag{6}$$

- Let $Q_1(T)$denote the left-hand of above equation,

$$Q_1(T) = c_F \int_0^T \bar{G}(t)\,[m(T) - m(t)]\,dt - (c_P - c_Y)\int_0^T \bar{G}(t)\,[r(T) - r(t)]\,dt. \tag{7}$$

Optimal RF Policy

$$Q_1(0) = 0 < c_P. \tag{8}$$

Theorem 1 *Suppose that $m(t)$ is increasing in t and $h(t)$ is decreasing in t*

- *If $Q_1(\infty) \geq c_P$, then there exist a finite and unique T^* $(0 < T^* < \infty)$ and its resulting cost rate is*

$$C(T^*) = c_F m(T^*) - (c_P - c_Y)\, r(T^*). \tag{9}$$

- *If $Q_1(\infty) < c_P$, then the optimal replacement time is given by $T^* \to \infty$.*

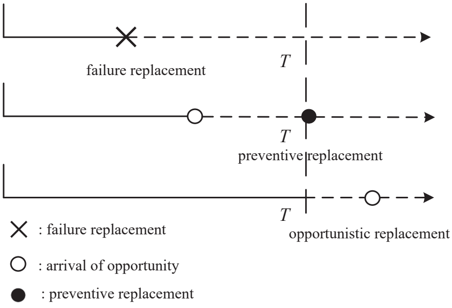

Figure 2: The configuration of RL discipline.

- If the failure occurs, then the failure replacement is made.
- The unfailed item is replaced at the time T or Y, whichever occurs last.

Expected Cost under RL Discipline

- The expected cost per unit time in the steady state:

$$C(T) = \frac{B(T)}{A(T)}, \tag{10}$$

- The problem is to determine the optimal preventive replacement T.
- where
  - $A(T)$ — Mean time length of one cycle.
  - $B(T)$ — Expected cost during one cycle.

$$\begin{aligned} A(T) &= TG(T) + \int_T^{\infty} tg(t)\,dt \\ &= T + \int_T^{\infty} \bar{G}(t)\,dt \end{aligned} \tag{11}$$

Expected Cost under RL Discipline

Cont-

$$\begin{aligned} B(T) &= c_P + c_F\left(M(T) + \int_T^\infty \bar{G}(t)m(t)\,dt\right) \\ &- (c_P - c_Y)\,\bar{G}(T). \end{aligned} \tag{12}$$

Optimal RL Policy

- Taking the difference of $C(T)$ with respect T and setting it equal to zero, we have

$$c_F\left[\int_0^T [m(T)-m(t)]\,dt+\int_T^\infty \bar{G}(t)\,[m(T)-m(t)]\,dt\right] + (c_P-c_Y)\left[\hat{r}(T)\,T+\hat{r}(T)\int_T^\infty \bar{G}(t)\,dt+\bar{G}(t)\right]=c_P. \tag{13}$$

- Let $Q_2(T)$denote the left-hand of above equation,

$$Q_2(0)=c_F\int_0^\infty \bar{G}(t)\,[m(0)-m(t)]\,dt + (c_P-c_Y)\left[\hat{r}(0)\int_0^\infty \bar{G}(t)\,dt+1\right]. \tag{14}$$

Optimal RL Policy

Theorem 2 Suppose that $Q_2(T)$ is increasing in T.

- *If* $Q_2(0) < c_P$ *and* $Q_2(\infty) > c_P$, *then there exist a finite and unique* T^* $(0 < T^* < \infty)$ *and its resulting cost rate is*

$$C(T^*) = c_F m(T^*) + (c_P - c_Y)\,\hat{r}(T^*). \tag{15}$$

- *If* $Q_2(0) \geq c_P$, *then the optimal replacement time is given by* $T^* = 0$.
- *If* $Q_2(\infty) \leq c_P$, *then the optimal replacement time is given by* $T^* \to \infty$.

Numerical Illustrations

Parameter setting

- $f(t)$ obeys the Gamma distribution.
- $\alpha = 2,\ \lambda = 1,\ \bar{F}(t) = t/(t+1)$.
- $g(t)$ obeys the Exponential distribution.
- $\sigma = 2, \bar{G}(t) = 2$.
- $c_P = 10, 12, 14, 16, 18, 20, 22, 24, 26$.
- $c_F = 200,\ c_Y = 5, 10$.

Numerical Illustrations

Table 1: The optimal preventive replacement time and associated expected cost of RF and RL, when $\lambda = 1$, $\sigma = 2$, $c_F = 200$, $c_Y = 10$.

	RF		RL	
c_P	T^*	$C(T^*)$	T^*	$C(T^*)$
10	0.5005	63.25	0.5034	63.46
12	0.5902	69.28	0.5442	66.32
14	0.6898	74.83	0.5868	69.07
16	0.8047	80.00	0.6315	71.72
18	0.9437	84.85	0.6784	74.25
20	1.1242	89.44	0.7278	76.67
22	1.3910	93.81	0.7800	78.98
24	1.9509	97.98	0.8355	81.19
26	∞	200	0.8946	83.29

Numerical Illustrations

Table 2: The optimal preventive replacement time and associated expected cost of *RF* and *RL*, when $\lambda = 1$, $\sigma = 2$, $c_F = 200$, $c_Y = 5$.

	RF		*RL*	
c_P	T^*	$C(T^*)$	T^*	$C(T^*)$
10	0.5005	53.25	0.3986	63.14
12	0.5902	59.28	0.5203	66.02
14	0.6898	64.83	0.6945	68.79
16	0.8047	70.00	0.7933	71.45
18	0.9437	74.85	0.9607	74.02
20	1.1242	79.44	1.1164	76.47
22	1.3910	83.81	1.4426	78.81
24	1.9509	87.98	1.8901	81.04
26	∞	200	2.4520	83.16

Lessons from the Tables

Lessons from the Tables

- Table 1 presents the optimal preventive time T^* and its minimum expected cost $C(T^*)$ under replacement first and replacement last disciplines with $c_Y = 10$. the replacement last is better than replacement first in most cases except the $c_Y = c_P = 10$.
- Table 2 presents the optimal preventive time T^* and its minimum expected cost $C(T^*)$ with $c_Y = 5$. When c_P is small, the replacememt first is better than replacement last. Conversely, if c_P is big enough, the replacememt last is better than replacement first.

Conclusions

Summary

- Introduced the opportunity-based block replacement models.
- Developed the cost models under RF and RL disciples.
  - Derive the optimal replacemnent policies under some conditions.
- Compared RF policy with RL policy.
  - When c_P is small, RF policy is better than RL policy.
  - When c_P is big enough, RL policy is better than RF policy.

Future Work

Future Work

- Zheng, Okamura and Dohi focuse (2021) on the age replacement problems under the assumption that the arrival of opportunities follows a stochastic point process.
- The opportunity-based age replacement models by Dekker and Dijkstra (1992) and Zhao and Nakagawa (2012) are extended in the environment where the opportunity process is in accordance with a Markovian arrival process (MAP).
- We will formulate the block models where the arrival of opportunity obeys Markov process.

Thank you very much!!