computer sciences and mathematics forum

MDPI

Proceeding Paper

A New Software Reliability Growth Model with Testing Coverage and Uncertainty of Operating Environment †

Sujit Kumar Pradhan [1,*,‡], Anil Kumar [1,‡] and Vijay Kumar [2,‡]

1 BITS Pilani K. K. Birla Goa Campus, Zuarinagar, 403726, Goa, India; anilpundir@goa.bits-pilani.ac.in
2 Amity Institute of Applied Sciences, Amity University Uttar Pradesh, Noida 201313, India; vijay_parashar@yahoo.com
* Correspondence: pradhansujit1994@gmail.com
† This paper is an extended version of our paper published in Presented at the 1st International Online Conference on Mathematics and Applications; Available online: https://iocma2023.sciforum.net/.
‡ Theseauthors contributed equally to this work.

Abstract: The number of software failures, software reliability, and failure rates can be measured and predicted by the software reliability growth model (SRGM). SRGM is developed and tested in a controlled environment where the operating environment is different. Many SRGMs have developed, assuming that the working and developing environments are the same. In this paper, we have developed a new SRGM incorporating the imperfect debugging and testing coverage function in the presence of a random environment. The proposed model's parameters are estimated from two real data sets and compared with some existing SRGMs based on five goodness-of-fit criteria. The results show that the proposed model gives better descriptive and predictive performance than the existing model.

Keywords: software reliability growth model (SRGM); mean value function (MVF); testing coverage; operating environment

check for updates

Citation: Pradhan, S.K.; Kumar, A.; Kumar, V. A New Software Reliability Growth Model with Testing Coverage and Uncertainty of Operating Environment. *Comput. Sci. Math. Forum* **2023**, *1*, 0. https://doi.org/

Academic Editor: Firstname Lastname

Published: 28 April 2023

1. Introduction

During the past four decades, various software reliability growth models (SRGM) [1–7] have been proposed to estimate reliability, predict the number of faults, determine the release time of the software, etc. Various proposed models have been developed based on different suppositions. For example, some models have discussed perfect debugging [1,7], and others have discussed imperfect debugging [3,4]. Some researchers have studied SRGM by considering a constant fault detection rate [1] or by the learning phenomenon [7]. During the testing and debugging process, various research papers have discussed resource allocation [8–10], testing effort [11,12], etc.

Most models have considered that the operating and testing environments are the same. In general, the software is implemented in the real working environment after the in-house testing process. In early 2000, researchers proposed different SRGMs incorporating uncertainty of the operating environment with new approaches. Teng and Pham [13] proposed a generalized SRGM considering the effects of the uncertainty of the working environment on software failure rate. Pham [6,14] presented an SRGM incorporating *V*tub-shaped and Loglog fault detection rates subject to random environments, respectively. Li and Pham [15] discussed an SRGM where fault removal efficiency and error generation are incorporated together with the uncertainty of the operating environment. Li et al. [16] proposed a generalized SRGM incorporating the uncertainty of the operating environment.

This paper presents an SRGM incorporating imperfect debugging and testing coverage functions under the effects of a random field environment. We have validated the goodness-of-fit and predictability of the proposed model on two datasets. The remaining part of the paper is as follows: in Section 2, the explicit solution of the mean value function is derived.

Numerical and data analysis is performed in Section 3. In Section 4, we summarize the paper's conclusions.

2. Software Reliability Growth Model

The cumulative number of detected software faults follows non-homogeneous Poisson process (NHPP) and express as follows

$$P\{N(t) = n\} = \frac{m(t)^n}{n!}\exp(-m(t)),\ \text{for } n = 1, 2, 3, \ldots. \tag{1}$$

The mean value function for the fault counting process is represented in terms of intensity function $\lambda(t)$ as

$$m(t) = \int_0^t \lambda(s)ds. \tag{2}$$

The following assumptions are taken for the proposed model

- The generation of fault in software follows the non-homogeneous Poisson process.
- Fault detection rate is proportional to the remaining faults in the software.
- After fault detection, the debugging process takes place immediately.
- During the testing process, new faults are introduced into the software.
- The testing coverage rate function is incorporated as the fault detection rate function.
- Random testing environment affects the fault detection rate.

Considering the above assumptions, the SRGM, with the uncertainty of the operating environment, is

$$\frac{dm(t)}{dt} = \eta \frac{c'(t)}{1 - c(t)} \Big\{ N(t) - m(t) \Big\},\ m(0) = 0. \tag{3}$$

where η is random variable, $c(t)$ is the testing coverage function, $N(t)$ is the total fault content at time t and $m(t)$ is cumulative number of software failure at time t.

The fault content function is

$$N(t) = N + dm(t) \tag{4}$$

where d is the fault introduction rate.

The general solution for the MVF $m_\eta(t)$ is given by

$$m_\eta(t) = \frac{N}{1 - d}\left(1 - e^{-\eta \int_0^t (1-d) \frac{c'(\tau)}{1-c(\tau)} d\tau}\right) \tag{5}$$

In order to find the mean value function $m(t)$, we have assume that, the random variable η follows Exponential distribution, i.e., $\eta \sim \exp(\alpha)$ and the probability density function of η is given by

$$f(\eta) = \alpha e^{-\alpha\eta},\quad \alpha > 0 \text{ and } \eta \geq 0. \tag{6}$$

An application of Laplace transformation of Equation (5) using Exponential distribution for random variable η, the mean value function $m(t)$ is given by

$$m(t) = \frac{N}{1 - d}\left\{1 - \left(\frac{\alpha}{\alpha + (1 - d)\int_0^t \frac{c'(\tau)}{1-c(\tau)} d\tau}\right)\right\} \tag{7}$$

In this paper, we have considered the following testing coverage rate function $c(t)$ as follows

$$c(t) = 1 - e^{1 - c^{t^b}},\ c > 1,\ b > 0 \tag{8}$$

After substituting $c(t)$ in Equation (8), we obtained the following closed form of the solution of the mean value function $m(t)$ as

$$m(t) = \frac{N}{1-d}\left\{1 - \left(\frac{\alpha}{\alpha + (1-d)(c^{t^b} - 1)}\right)\right\}. \tag{9}$$

Table 1 summarizes the MVF of the proposed model and other selected models, which are taken for comparison.

Table 1. Summary of SRGM.

No.	Model	MVF
1	Goel-Okumoto model [1]	$m(t) = a(1 - e^{-bt})$
2	Delayed S-shaped model [17]	$m(t) = a(1 - (1 + bt)e^{-bt})$
3	Yamada ID model-I [3]	$m(t) = \frac{ab}{\alpha+b}(e^{\alpha t} - e^{-bt})$
4	Yamada ID model-II [3]	$m(t) = a(1 - e^{-bt})(1 - \frac{\alpha}{b}) + \alpha at$
5	Yamada et al. (YExp) [18]	$m(t) = a\left\{1 - e^{-\gamma\alpha(1-e^{-\beta t})}\right\}$
6	Yamada et al. (YRay) [18]	$m(t) = a\left\{1 - e^{-\gamma\alpha(1-e^{-\beta t^2/2})}\right\}$
7	Pham-Zhang model [19]	$m(t) = \frac{1}{1+\beta e^{-bt}}\left((c+a)(1-e^{-bt}) - \frac{ab}{b-\alpha}(e^{-\alpha t} - e^{-bt})\right)$
8	Pham-Zhang ID model [20]	$m(t) = a(1 - e^{-bt})(1 + (b+d)t + bdt^2)$
9	Proposed model	$m(t) = \frac{N}{1-d}\left\{1 - \left(\frac{\alpha}{\alpha+(1-d)(c^{t^b}-1)}\right)\right\}$

3. Numerical and Data Analysis

3.1. Software Failure Data

The first data set (DS-I) discussed in this paper is collected from the online IBM entry software package [2]. During the testing process of 21 weeks, 46 failures are observed. The second data set (DS-II) is presented and collected from testing system T at AT&T [21]. The system takes a total of 14 weeks to perform testing. As a result, 22 number of faults are experienced during the testing weeks.

3.2. Parameter Estimation and Goodness-of-Fit Criteria

Usually, the parameters of the SRGMs are estimated using the least square estimation (LSE) or maximum likelihood estimation (MLE) methods. We have used the least square estimation method to estimate the parameters of the proposed model and the parameter estimation is shown in Table 2.

Table 2. Parameter estimation for DS-I [2] and DS-II [21].

No.	Model	Parameter Estimate (DSI)	Parameter Estimate (DSII)
1	Goel-Okumoto model	$\hat{a} = 192.3303$, $\hat{b} = 0.0121$	$\hat{a} = 23.0127, \hat{b} = 0.1884$
2	Delayed S-shaped model	$\hat{a} = 77.253$, $\hat{b} = 0.0966$	$\hat{a} = 20.0045$, $\hat{b} = 0.5198$
3	Yamada ID model-I	$\hat{a} = 38.1884$, $\hat{b} = 0.0439$, $\hat{\alpha} = 0.055$	$\hat{a} = 26.4815$, $\hat{b} = 0.13204$, $\hat{\alpha} = 0.00004$
4	Yamada ID model-II	$\hat{a} = 1.7105$, $\hat{b} = 0.2959$, $\hat{\alpha} = 1.5247$	$\hat{a} = 17.5163$, $\hat{b} = 0.2657$, $\hat{\alpha} = 0.0251$
5	Yamada et al. (YExp)	$\hat{a} = 1935.6515, \hat{\gamma} = 1.6636,$ $\hat{\alpha} = 2.4954, \hat{\beta} = 0.0003$	$\hat{a} = 34.7439, \hat{\gamma} = 4.2853,$ $\hat{\alpha} = 0.2857, \hat{\beta} = 0.1061$
6	Yamada et al. (YRay)	$\hat{a} = 77.8791, \hat{\gamma} = 2.5357,$ $\hat{\alpha} = 0.5513, \hat{\beta} = 0.0045$	$\hat{a} = 20.7737, \hat{\gamma} = 3.7091,$ $\hat{\alpha} = 0.7458, \hat{\beta} = 0.0520$
7	Pham-Zhang model	$\hat{a} = 83.1886, \hat{b} = 0.0747, \hat{\beta} = 0.0533$ $\hat{\alpha} = 0.0838, \hat{c} = 8.0682$	$\hat{a} = 22.7269, \hat{b} = 0.223, \hat{\beta} = 0.0011$ $\hat{\alpha} = 4.1802, \hat{c} = 0.1$
8	Pham-Zhang ID model	$\hat{a} = 75.7344, \hat{b} = 0.1001, \hat{d} = 0.001$	$\hat{a} = 19.4765, \hat{b} = 0.6924, \hat{d} = 0.113$
9	Proposed model	$\hat{N} = 3.1569, \hat{d} = 0.9527,$ $\hat{\alpha} = 6.5911, \hat{b} = 0.4806, \hat{c} = 3.8052,$	$\hat{N} = 29.773, \hat{d} = 0.001,$ $\hat{\alpha} = 0.0006, \hat{b} = 1.082, \hat{c} = 1.0001,$

Several goodness-of-fit criteria are available to predict the best-fit model in the literature. Out of those, the standard criteria used to compare with the existing selected model are mean-squared error (MSE), predictive ratio risk (PRR), bias, variance, and root mean square prediction error (RMSPE). The smaller value of all goodness-of-fit criteria gives a better fit of the model.

The MSE measures the average of the deviation between the predicted values with the actual data [22] and is represented as

$$\text{MSE} = \frac{1}{n}\sum_{i=1}^{n}(m(t_i) - y_i)^2,$$

where n is the number of observations in the model.

The predictive ratio risk (PRR) gives the distance between the model estimates and actual data against the model estimates and is defined as [23]

$$\text{PRR} = \sum_{i=1}^{n}\left(\frac{(m(t_i) - y_i)}{m(t_i)}\right)^2$$

The bias is defined as the sum of the deviation of the model estimates testing curve from the actual data as [24]

$$\text{Bias} = \frac{1}{n}\sum_{i=1}^{n}(m(t_i) - y_i).$$

The variance is defined as [25]

$$\text{Variance} = \sqrt{\frac{\sum_{i=1}^{n}\left(y_i - m(t_i) - \text{Bias}\right)^2}{n-1}}.$$

The root mean square prediction error (RMSPE) is defined as [25]

$$\text{RMSPE} = \sqrt{\text{Variance}^2 + \text{Bias}^2}.$$

where $m(t_i)$ is the predict fault at time t_i and y_i is the observed fault at time t_i.

3.3. Model Comparison for DS-I

Table 3 shows that the proposed model performs better regarding MSE, PRR, Bias, Variance, and RMSE criteria. Figure 1a depicted the comparison between the proposed model and the existing selected model with observed failure data. Figure 1b shows the relative errors of the proposed model in terms of the test week. Overall, the proposed model is better than other selected existing models.

Table 3. Comparison criteria for DS-I.

No	MSE	PRR	Bais	Variance	RMSE
1	8.973872	1.019703	0.890848	3.452888	3.565957
2	1.480686	26.32063	−0.23211	1.313173	1.333528
4	4.057239	0.371741	−0.65302	2.06001	2.164841
5	1.458347	2.676588	−0.05303	1.241017	1.242149
6	6.245228	0.822893	0.844226	2.56076	2.696332
7	1.959292	55.68604	−0.39852	1.434314	1.488648
8	1.278945	2.729673	−0.0696	1.165397	1.167473
9	1.533603	40.66148	−0.25021	1.344427	1.367511
10	**1.168283**	**0.381616**	**−0.04598**	**1.105182**	**1.106138**

3.4. Model Comparison for DS-II

The performance of the proposed model is evaluated in terms of MSE, PRR, Bias, Variance, and RMSE and shown in Table 4. The comparison between the proposed and selected model's MVF is depicted in Figure 1c. Figure 1d shows the relative errors for different models.

Table 4. Comparison criteria for DS-II.

No	MSE	PRR	Bias	Variance	RMSE
1	0.837889	0.126591	0.294310	1.087284	1.126412
2	1.409615	0.385109	−0.12266	1.251662	1.257659
3	1.394307	0.144932	−0.16151	1.225382	1.235980
4	0.718011	0.146291	0.168120	0.929816	0.944893
5	0.746532	0.135072	0.061377	0.896637	0.898735
6	2.027924	1.355159	−0.17989	1.477809	1.488717
7	1.526415	0.118110	0.879679	2.035660	2.217600
8	1.969308	2.921449	−0.17227	1.488848	1.498781
9	**0.708826**	**0.117530**	**0.051773**	**0.878641**	**0.880165**

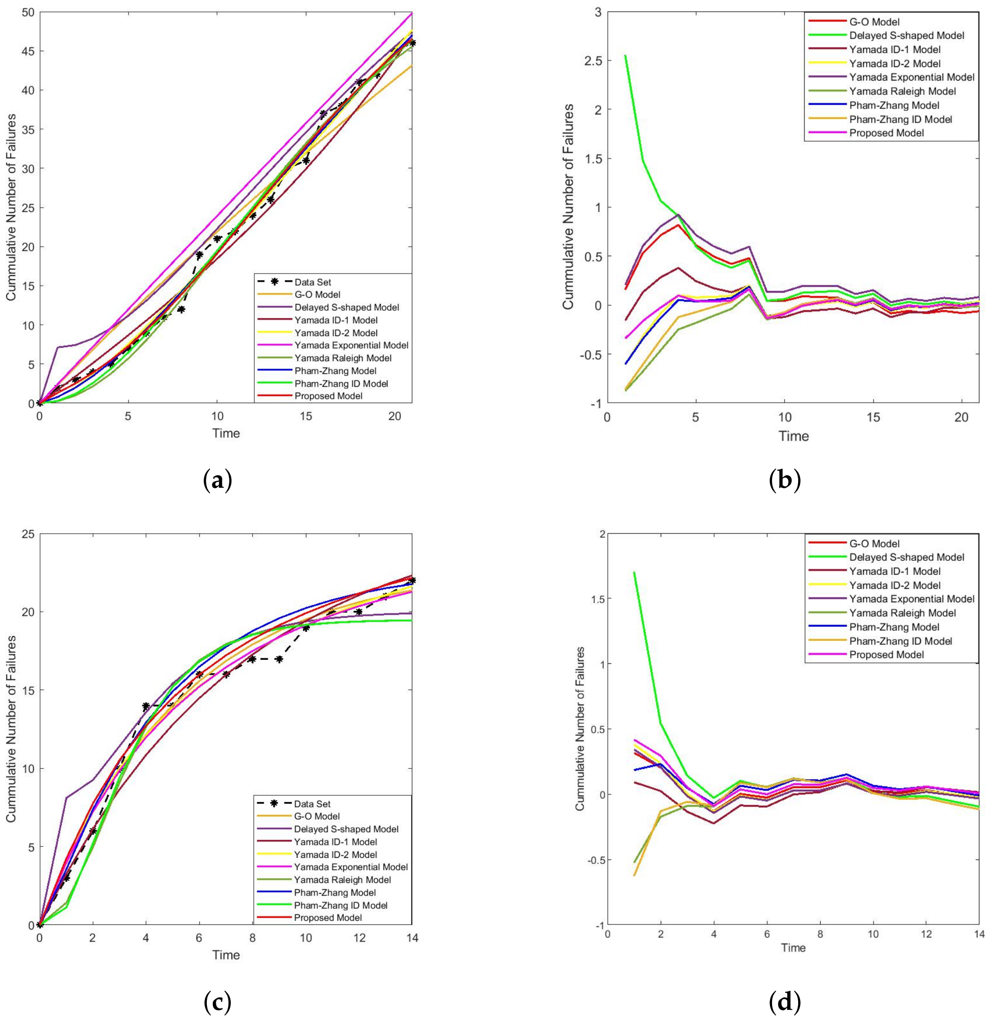

Figure 1. (**a**) Estimated MVFs for different selected and proposed model (DS-I). (**b**) Relative errors curve for different selected and proposed model (DS-I). (**c**) Estimated MVFs for different selected and proposed model (DS-II). (**d**) Relative errors curve for different selected and proposed model (DS-II).

4. Conclusions

Many SRGMs have been proposed on different realistic issues. This paper has incorporated imperfect debugging, the testing coverage rate function, and random field environment in the model. The main contribution of the model is implementing a random variable, which follows Exponential distribution. The proposed model's parameter is estimated using two datasets and validated over five goodness-of-fit criteria. The results show that the proposed model gives a better fit than other selected models.

Author Contributions: All the authors have contributed almost equally to developing the proposed model and manuscript preparation. All authors have read and agreed to the published version of the manuscript.

Funding: The authors didn't receive any funding from any organizations.

Institutional Review Board Statement:

Informed Consent Statement:

Data Availability Statement: We confirm that the data supporting the findings of this study are cited and available within the article.

Conflicts of Interest: The authors declare that they have no conflict of interest.

References

1. Goel, A.L.; Okumoto, K. Time-dependent error-detection rate model for software reliability and other performance measures. *IEEE Trans. Reliab.* **1979**, *28*, 206–211.
2. Ohba, M. Software reliability analysis models. *IBM J. Res. Dev.* **1984**, *28*, 428–443.
3. Yamada, S.; Tokuno, K.; Osaki, S. Imperfect debugging models with fault introduction rate for software reliability assessment. *Int. J. Syst. Sci.* **1992**, *23*, 2241–2252.
4. Pham, H. An imperfect-debugging fault-detection dependent-parameter software. *Int. J. Autom. Comput.* **2007**, *4*, 325.
5. Pham, H. A software cost model with imperfect debugging, random life cycle and penalty cost. *Int. J. Syst. Sci.* **1996**, *27*, 455–463.
6. Pham, H. Loglog fault-detection rate and testing coverage software reliability models subject to random environments. *Vietnam. J. Comput. Sci.* **2014**, *1*, 39–45.
7. Yamada, S.; Ohba, M.; Osaki, S. S-shaped reliability growth modeling for software error detection. *IEEE Trans. Reliab.* **1983**, *32*, 475–484.
8. Pradhan, S.K.; Kumar, A.; Kumar, V. An Optimal Resource Allocation Model Considering Two-Phase Software Reliability Growth Model with Testing Effort and Imperfect Debugging. *Reliab. Theory Appl.* **2021**, pp. 241–255.
9. Pradhan, S.K.; Kumar, A.; Kumar, V. An Effort Allocation Model for a Three Stage Software Reliability Growth Model. In *Predictive Analytics in System Reliability*; Springer: Cham, Switzerland, 2023; pp. 263–282.
10. Pradhan, S.K.; Kumar, A.; Kumar, V. An optimal software enhancement and customer growth model: A control-theoretic approach. *Int. J. Qual. Reliab. Manag.* **2023**. https://doi.org/10.1108/IJQRM-08-2022-0240.
11. Kapur, P.; Goswami, D.; Bardhan, A. A general software reliability growth model with testing effort dependent learning process. *Int. J. Model. Simul.* **2007**, *27*, 340–346.
12. Samal, U.; Kushwaha, S.; Kumar, A. A Testing-Effort Based Srgm Incorporating Imperfect Debugging and Change Point. *Reliab. Theory Appl.* **2023**, *18*, 86–93.
13. Teng, X.; Pham, H. A new methodology for predicting software reliability in the random field environments. *IEEE Trans. Reliab.* **2006**, *55*, 458–468.
14. Pham, H. A new software reliability model with Vtub-shaped fault-detection rate and the uncertainty of operating environments. *Optimization* **2014**, *63*, 1481–1490.
15. Li, Q.; Pham, H. NHPP software reliability model considering the uncertainty of operating environments with imperfect debugging and testing coverage. *Appl. Math. Model.* **2017**, *51*, 68–85.
16. Li, Q.; Pham, H. A generalized software reliability growth model with consideration of the uncertainty of operating environments. *IEEE Access* **2019**, *7*, 84253–84267.
17. Ohba, M.; Yamada, S. S-shaped software reliability growth models. In Proceedings of the 4th International Colloquium on Reliability and Maintainability, Tregastel, France, day month 1984; pp. 430–436.
18. Yamada, S.; Ohtera, H.; Narihisa, H. Software reliability growth models with testing-effort. *IEEE Trans. Reliab.* **1986**, *35*, 19–23.
19. Pham, H.; Zhang, X. An NHPP software reliability model and its comparison. *Int. J. Reliab. Qual. Saf. Eng.* **1997**, *4*, 269–282.
20. Pham, H. *System Software Reliability*; Springer Science & Business Media: Berlin/Heidelberg, Germany, 2007.
21. Ehrlich, W.; Prasanna, B.; Stampfel, J.; Wu, J. Determining the cost of a stop-test decision (software reliability). *IEEE Softw.* **1993**, *10*, 33–42.

22. Kapur, P.; Goswami, D.; Bardhan, A.; Singh, O. Flexible software reliability growth model with testing effort dependent learning process. *Appl. Math. Model.* **2008**, *32*, 1298–1307.
23. Pham, H.; Deng, C. Predictive-ratio risk criterion for selecting software reliability models. In Proceedings of the 9th International Conference on Reliability and Quality in Design, Waikiki, HI, USA, 6–8 August 2003; pp. 17–21.
24. Pillai, K.; Nair, V.S. A model for software development effort and cost estimation. *IEEE Trans. Softw. Eng.* **1997**, *23*, 485–497.
25. Kapur, P.; Pham, H.; Anand, S.; Yadav, K. A unified approach for developing software reliability growth models in the presence of imperfect debugging and error generation. *IEEE Trans. Reliab.* **2011**, *60*, 331–340.