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A Comprehensive Comparison of Hargreaves Isothermal Model with Schmidt Model for the Gamma Stirling Engine †

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Abstract: The Stirling engine, a type of external-combustion engine utilizing a compressible fluid as its working medium, holds promise as a highly efficient device for converting heat into mechanical work at Carnot efficiency. This research conducts a detailed analysis, comparing the Hargreaves isothermal model and the Schmidt model specifically for the gamma type Stirling engine. The study examines the impact of dead volume on the engine's performance, revealing that the engine network is solely influenced by these volumes. Furthermore, it highlights the effectiveness of Hargreaves model for the performance analysis of gamma type Stirling engines.

Keywords: Stirling engine; thermodynamics; power; efficiency

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1. Introduction

In 1816, a cleverly made Stirling engine was introduced. This special machine worked in a unique way that made it different from others. For almost 100 years, it proved itself as a dependable power source for many things. But as technology advanced, a different kind of engine, called the spark-ignition engine, became more popular and took its place [1].

Nowadays, things have changed. Stirling machines used to do a lot of work in industries, but now they have a new job. They are used as special heaters in businesses. They're really good at making things very cold and turning air into liquid in specific places where these things are needed. Even though the old-style Stirling engine isn't used as much as before, it's not forgotten. Instead, it's become an exciting area where people learn and make new things. Scientists and inventors keep studying it to find out all it can do. Recently, they've done some experiments that showed how powerful and efficient the engine can still be. These new ideas prove that the Stirling engine is still interesting and has a lot of potential [2–4].

The analysis of a Stirling machine that includes a space, which is often referred to as a "dead volume", can be approached through a method developed by Schmidt, as mentioned in reference [5]. This method is applicable when the engine's operation is considered to involve consistent temperature changes (isothermal evolutions) and an optimal process of restoring energy (ideal regeneration). Second-order design approaches initiate by first computing the theoretical maximum power that a system could generate. From this value, losses occurring independently due to factors like fluid friction and thermal inefficiencies are deducted. Subsequently, losses related to power transmission and conversion are subtracted, while also accounting for any additional losses during heat input or heat rejection processes. The model used to forecast this theoretical maximum power is typically more intricate compared to the simpler model employed in first-order analyses. There are many ways to estimate the performance of Stirling engine i.e., isothermal, adiabatic, finite time thermodynamics (FTT) [2], finite physical dimensions thermodynamics (FPDT) [6], but the most common and popular basic approach is through the isothermal method [7].

In this research work, the performance of gamma type Stirling engine is investigated by using Hargreaves Isothermal model and results of this model is compared with other thermodynamic models. The work done and power obtained by Hargreaves Isothermal model is 13.6 J and 200.8 W. The percentage error in work and power using this model is 81.6% and 80.03% respectively. The efficiency obtained by this model is equal to Carnot efficiency which is 30.6% and this efficiency is equal to Schmidt iso-thermal model. Moreover, the performance of the Stirling engine is also affected by dead volume. If the dead volume of the Stirling engine is large, the performance of Stirling engine will be low.

2. Engine Data

The engine under study is a gamma-type double-piston Stirling engine. In this engine, the exhaust gas of the diesel engine is used as the heat source. To heat the heater tubes of the Stirling engine, especially the cap is made for the Stirling engine. Further details of Engine data is given below [8].

Table 1. Engine data of gamma type double piston Stirling engine [8].

Engine Data	Values	Engine Data	Values
Cooler temperature	294	Pressure (bar)	3.58
Heater temperature	424	Rotational speed (rpm)	882
Cooler void volume	223×10^{-6}	Phase angle (degree)	88°
Heater void volume	87.28×10^{-6}	Regenerator void volume (m^3)	308.93
Expansion swept volume (m^3)	221×10^{-6}	Compression swept volume (m^3)	194×10^{-6}
Expansion clearance volume (m^3)	24×10^{-6}	Compression clearance volume (m^3)	35×10^{-6}

3. Schmidt Model and Hargreaves Model

In this Schmidt model, the equation for the expansion and compression space volume calculation is given below. Where as V_{DE}, V_{DC}, V_{SE}, V_{SC}, $\emptyset$ are the expansion dead volume, compression dead volume, expansion swept volume, compression swept volume and phase angle respectively [8].

$$V_E = \frac{V_{SE}}{2}\left[1-\cos(\theta)\right]+V_{DE} \tag{1}$$

$$V_C = \frac{V_{SE}}{2}\left[1+\cos(\theta)\right]+\frac{V_{SC}}{2}\left[1-\cos(\theta-\phi)\right]+V_{DC} \tag{2}$$

The Shmidt model was modified by Abdul Rab et al. [8] for this specific gamma type Stirling engine. The mass of the Stirling engine is given by

$$M = m_C + m_{k1} + m_{k2} + m_r + m_h + m_E \tag{3}$$

$$M = \frac{p}{R}\left(\frac{V_C}{T_C}+\frac{V_{k1}}{T_C}+\frac{V_{k2}}{T_C}+\frac{V_r \log\left(\frac{T_E}{T_C}\right)}{T_E-T_C}+\frac{V_E}{T_E}+\frac{V_E}{T_E}\right) \tag{4}$$

The total workdone is defined by the following equation

$$W = W_E + W_C = \oint p\left(\frac{dV_E}{d\theta} + \frac{dV_C}{d\theta}\right)d\theta \tag{5}$$

The Hargreave isothermal model for the gamma type Stirling engine was modified for Wegnar et al. [9] in which heater, regenerater and cooler volume is considered as dead volume. The mass in the Stirling engine is given by

$$M = \sum m_i = m_D + m_E + m_C \tag{6}$$

$$M = \frac{\rho}{R}.\left[\frac{V_E}{T_E} + \frac{V_D}{T_D} + \frac{V_C}{T_C}\right] \tag{7}$$

(8)

$$t = \frac{T_C}{T_E} \text{ (Temperature ratio)} \tag{9}$$

$$v = \frac{V_{SC}}{V_{SE}} \text{ (Swept volume ratio)} \tag{10}$$

$$s = \frac{V_D}{V_{SE}}.\frac{T_C}{T_D} \text{ (Dead volume ratio)} \tag{11}$$

By using the above equation, the mass in the Stirling engine is re-arranged as

$$M = \frac{P}{R.T_C}\left[t.V_E + V_C + V_D.\frac{T_C}{T_D}\right] \tag{12}$$

The pressure in the Stirling engine is given as

$$P = \frac{P_m.\sqrt{1-c^2}}{1-c.\cos(\alpha-\delta)} \tag{13}$$

$$c = \frac{A}{B} \tag{14}$$

$$A = \sqrt{t^2 - 2.t + 1 + 2.(t-1).v.\cos\varphi + v^2} \tag{15}$$

$$B = t + 1 + v + 2.s \tag{16}$$

$$\delta = \arctan\left(\frac{v.\sin\varphi}{t-1+v.\cos\varphi}\right) \pm (0;\pi;2\pi;..........) \tag{17}$$

The expansion, compression and total workdone is given as

$$W_C = \int_0^{2\pi} P.dV_C \tag{18}$$

$$W_E = \int_0^{2\pi} P.dV_E \tag{19}$$

$$W = W_e + W_c \tag{20}$$

4. Results and Discussion

Model Validation

In this research work, the results of Hargreave model for gamma Stirling engine is compared with literature isothermal model, adiabatic and experimental model. The work obtained by Hargreave model 13.6 J and by Schmidt isothermal model is 9.08 J [8]. It is important to note that in the mentioned isothermal, the thermal or heat transfer losses are not included whereas in experimental results, all losses are included as referred to in literature. The experimental value of work obtained is 7.50 J [8]. When the Hargreave model is compared with experimental results it is found that it +81.6% error. Similar is the case in the power calculation, the power obtained by Hargreave model is 200.6 W which is very high as compared to experimental values which is 111.43 W [8]. The percentage error in power calculation by this model is 80.03%. But the efficiency of this engine by this model is equal to Carnot efficiency which is 30.6 and this efficiency is equal to other thermodynamics models. Table 2 shows the summary of thermodynamic models and its comparison Whereas Figure 1 shows the comparison of p-V diagram curves with other thermodynamics models. It can be seen from the p-V diagram curve that area under the curve of Hargreaves model is more than other thermal models. So, it is the case that indicated power using this model is more than other thermal models and experimental results.

Table 2. Summary of thermodynamic models.

Parameters	Isothermal [8]	Experimental [8]	Hargreave Model
Work (J)	9.08	7.50	13.6
Power (W)	133.8	111.43	200.8
Efficiency (%)	30.70	24.70	30.6

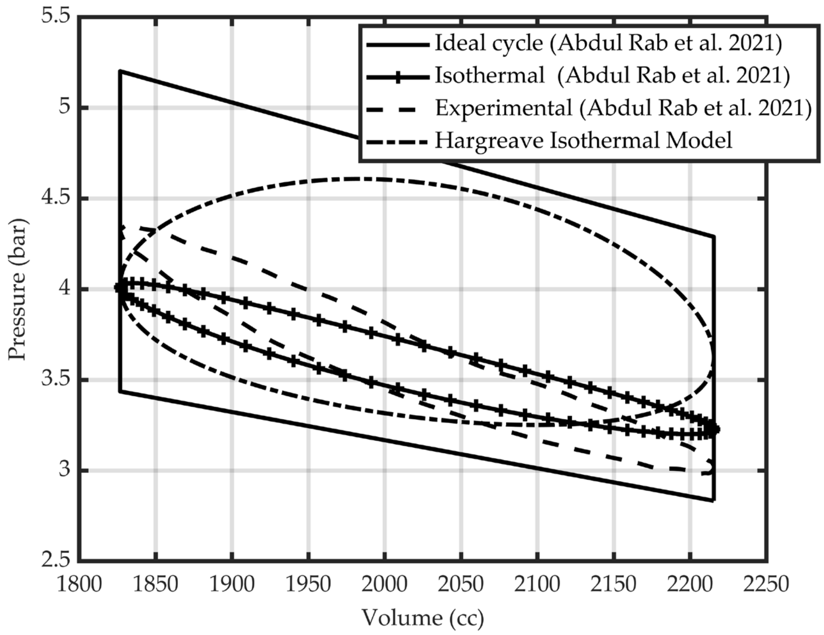

Figure 1. Comparison of PV diagram with other thermodynamic models.

The Figure 2 represents the derivative work done in the expansion, compression and total work done using Hargreaves model. The peak positive value of derivative curve of total work done is approximately touching to 100 J and negative peak value is to −60 J. The total work done is calculated by taking the average values of work done at every crank angle. Moreover, the dead volume of the Stirling engine plays a very important role for designing of Stirling engine. If the dead volume is large the performance of the Stirling engine will be low.

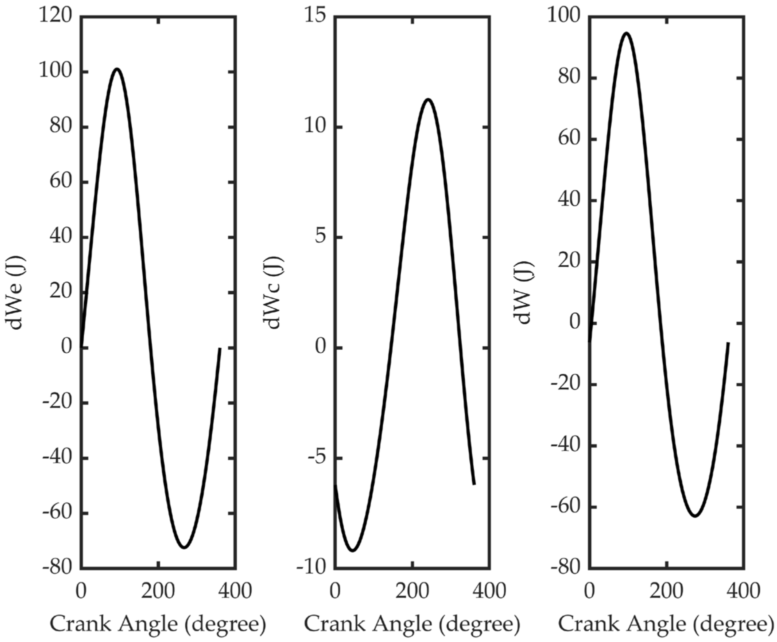

Figure 2. Derivative of expansion, compression and total work done by Hargreaves Model.

5. Conclusions

This research focuses on evaluating the performance of a gamma-type Stirling engine through the utilization of the Hargreaves Isothermal model. A comparison is made between the outcomes of this model and those of alternative thermodynamic models. According to the Hargreaves Isothermal model, the achieved work and power stand at 13.6 J and 200.8 W, respectively. The corresponding percentage errors for work and power using this model are 81.6% and 80.03%, respectively. Notably, the efficiency attained from this model aligns with the Carnot efficiency, registering at 30.6%, and coincides with the Schmidt Isothermal model's efficiency. Additionally, the Stirling engine's effectiveness is influenced by its dead volume, a larger dead volume results in diminished engine performance.

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