

# Determination of the dynamic behavior of thin-walled hollow-box sandwich beams.

Hugo Miguel Silva<sup>1\*</sup>, Jerzy Wojewoda<sup>2</sup>

1. proMetheus, Instituto Politécnico de Viana do Castelo, Rua Escola Industrial e Comercial.

2. Division of Dynamics, Lodz University of Technology, Stefanowskiego 1/15, 90-924 Lodz, Poland

\* Correspondence: Correspondence: hugo.silva@hmsilva.science, [jerzy.wojewoda@p.lodz.pl](mailto:jerzy.wojewoda@p.lodz.pl)

**Abstract:** Sandwich geometries, mainly panels and beams are widely used in several transportation industries, namely aerospace, aeronautics and automotive. They are known for some advantages in structural applications: high specific stiffness, low weight, and possibility of design optimization prior to manufacturing. This study aims to discover the dynamic behavior of simply supported at its ends Finite Element Method (FEM) models, representing a novel type of sandwich beams. There are 12 examples of geometries discussed with the same base configuration. The models were previously subjected to a design optimization routine. Dynamic behavior of the initial models in relation to their final versions is considered. Influence of the geometry on the characteristic frequencies is discussed, as well as its improvement in relation to the initial models. It is shown that the statically optimized models represent a significant improvement over the initial ones. In some cases, the improvement surpasses 20%. It can, therefore, be concluded that the design optimization approach, developed for static analysis, might be moderately effective in improving the modal behavior of the studied beams.

**Keywords:** sandwich beams; dynamic analysis; Finite Element Method; FEM

## 1. Introduction

The Finite Element Method (FEM) functions by employing partial derivative equations to solve the discretization of the domain into many elements. In the context of dynamic analysis, the utilization of the Finite Element Method (FEM) necessitates the use of a computer system that possesses enough computational capabilities [1]. The literature encompasses several research that is relevant to the topic of the present work. The authors of reference [2] extended the conventional Vlasov theory for thin-walled beams with open and closed cross sections by incorporating distortional displacement forces. The engineering relevance of the eigenvalues identified through dynamic analysis lies in their ability to mitigate the amplification of distortional eigenmodes. The authors of [3] offer a beam that resembles a thin-walled hollow tube, which is reinforced by ribs. The objective of this study is to examine the impact behavior of the beam in order to enhance energy absorption and reduce the first peak force. Subsequently, the process of shape optimization design is undertaken. The research investigates the characteristics of free vibration in single-cell thin-walled tubes with regular convex polygonal cross-sections. The authors also conducted a comprehensive analysis of the many modes of these beams. In the study conducted by [4], a precise formulation of dynamic stiffness is presented using one-dimensional, higher-order theories. This formulation is subsequently employed to investigate the characteristics of free vibration in both solid and thin-walled structures. The objective of the study conducted by [5] was to expand the range of applications for the Generalized Beam Theory (GBT) formulation, which had been recently developed for conducting elastic linear buckling analyses of thin-walled components. This paper aims to provide data on the dynamic behavior of internally reinforced thin-walled beams through a

**Citation:** To be added by editorial staff during production.

Academic Editor: Firstname Last-name

Published: date



**Copyright:** © 2023 by the authors. Submitted for possible open access publication under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

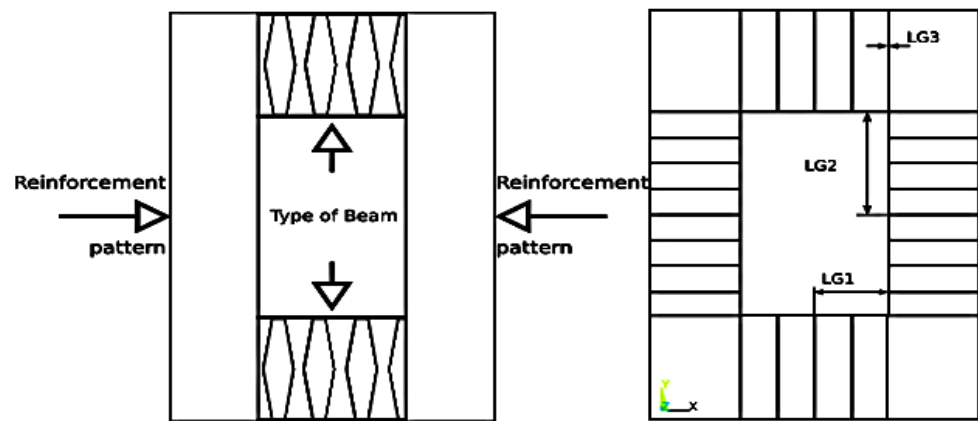
comparative analysis. The article [6] presents and studies a geometry stiffened honeycomb core to increase structural performance results in an upgraded honeycomb structure known as the stiffened honeycomb sandwich panel (stiffened HSP). The horizontal stiffened HSP has a lower natural frequency and a bigger buckling stress when compared to the standard and vertical stiffened HSPs [6]. The study [7] proposes a theoretical model of a stiffened plate with numerous dynamic vibration absorbers under various boundary restrictions. The model presented in [7] research improves the equivalent mass solution efficiency by 90% when compared to FEM [7]. In the reference [8], the vibration properties of sandwich panels with a sandwich core made of hierarchical composite honeycomb are presented. To offer an equivalent model (two-dimensional model), an orthotropic constitutive model of the hierarchical composite honeycomb sandwich core was used. The natural frequencies and mode shapes of the sandwich panels were predicted using modal testing, two-dimensional (2D) and three-dimensional (3D) finite element models. The comparable model's prediction results agreed with the findings of the 3D finite element analysis and the experiment [8]. In [9], the homogenized beam-like model for the transverse dynamics of reticulated structures is presented in a finite element formulation. The study deals with elastic periodic lattice structures, whose unit cell is composed of connected beams or plates and repeats itself in a single direction. Examples of these structures include foams, crystals, honeycombs, and multistory skyscrapers. The motion is given by a sixth-order differential equation, and the examined model is a one-dimensional enriched form of the fourth-order Timoshenko beam equation. It is demonstrated that the homogenized beam finite element solution presented approaches the whole detailed finite element structural model and recovers the analytical results [9].

The present study focuses on the modal analysis of internally reinforced beams that were designed and optimized for static loads. No study was found in which the geometries used in this study, or similar ones, were studied in modal analysis. Also, no studies were found that prove the effectiveness of design optimization, done in static analysis, on the improvement of the modal behavior of internally reinforced thin-walled beams. The twelve geometries presented in this article had already been studied in static analysis in [10-13]. The geometries are composed of sandwich panels on the top and on the bottom, in a total of 3 beams. The sandwich panels were already well present in the literature. At the sides, there are internal reinforcements, in a total of 4 patterns. The combination of the geometries of the sandwich panels, as well as the side reinforcement, is believed to be novel, until the publication of [10-13].

## 2. Numerical Procedure

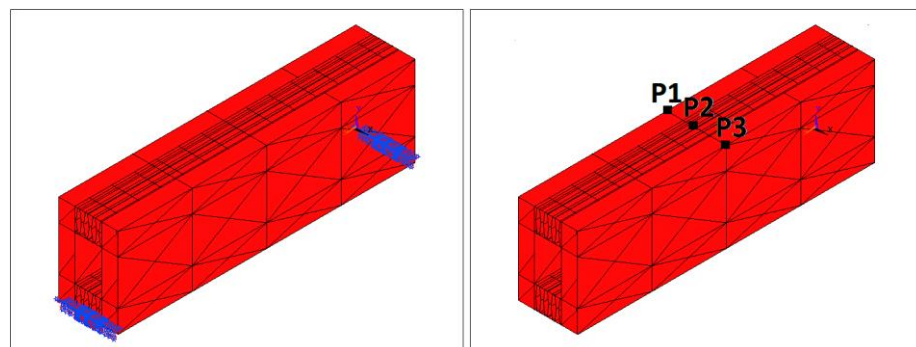
### 2.1. The FEM models.

For the purpose of analysis of dynamic behavior, 12 Finite Element Method (FEM) models were designed in the commercial FEM program ANSYS Mechanical APDL. These models represent different versions of the project of novel beams. They are composed of two sandwich panels on the top and on the bottom, and a reinforcement pattern on the sides, as shown in Fig. 1: Such models were earlier discussed by the authors in other application aspects [10-12]. Simple hollow-box beams, named hollow-solid sections, and abbreviated HSS in the results were also studied, using the same conditions as on the sandwich beams. Variable for the studied beams of the A type is shown in Fig. 1(right).




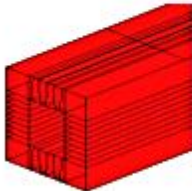
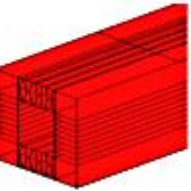
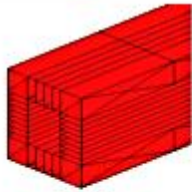
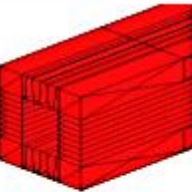
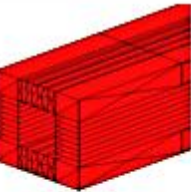
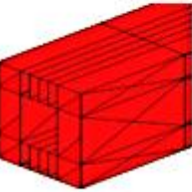
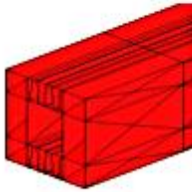
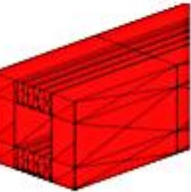
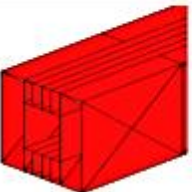
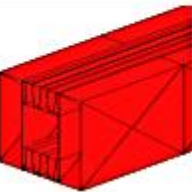

**Figure 1.** Configuration of the beam (left) [10-12] and Geometric variables of the FEM model used on the design optimization (right), [10,12].

In order to obtain an effective response to transversal beam deflection in terms of stiffness, 12 FEM models were built. The beams were subjected to modal analyses, by the Block-Lanczos method, simply supported at their ends. The supports are shown in Fig. 2 (left). The mesh is a quadrilateral free mesh, using SHELL63 elements and with a mean element size of 0.005 [m]. This construction is based on the principle that such a type of beam needs a zone along which accessories pass, such as compressed air tubes and electric cables. The central zone of the beam was chosen because that zone contains the neutral axis. In the peripheral zone, there are two lateral zones, and two other zones: one at the top and the other at the bottom. In these zones, a reinforcement is fundamental for increasing bending stiffness, while the lateral



**Figure 2.** Type of support applied to all FEM models – (left) [10-12] and points used to calculate displacements on optimization procedure (right) [10,12].

Table 1. Geometries of the FEM models [10-12]

	Beam A1	Beam A2	Beam A3
Pattern 1			
Pattern 2			
Pattern 3			
Pattern 4			

The ANSYS input file contains instructions to collect the displacements on the nodes attached to the keypoints indicated in Fig. 2 (one by each keypoint), which are situated at the edges and at the center, in order to gather the displacements on the same points in each iteration. These keypoints were selected because, even when the variable values change during optimization, their coordinates remain unchanged. Since the ribs at these sites provide considerable reinforcement, it is anticipated that the local deformation will not be significant for the thicknesses under consideration. In the present study, the following material properties were considered, typical of steel: Young's modulus ( $E$ ) of 210 GPa, density of 7890 kg/m<sup>3</sup>, and Poisson's ratio of 0.29 [-]. The modal analyses were done using simply supported at its ends boundary conditions.

## 2.2. Optimization

As seen in Fig. 3 the models were optimized with respect to their total mass and nodal displacements in the  $z$  direction, which were measured at three different places. The interaction between ANSYS and the MATLAB optimization software is seen in Figure 3. ANSYS and the MATLAB application collaborate in this process. In [10,12], the authors state that ANSYS computes the FEM models while MATLAB manages the optimization by means of a programming code.

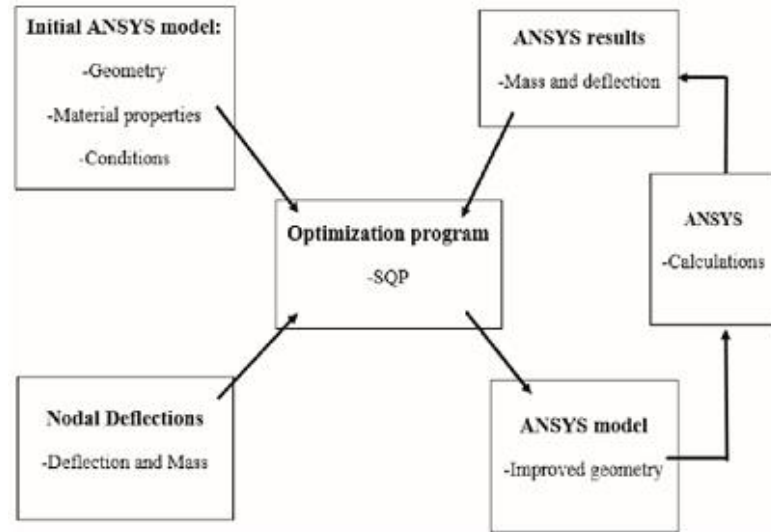


Figure 3. Functional flow chart of the optimization methodology (left) [10,12], adapted from [13].

Despite the variance in the geometric variables, these spots were selected at locations where all coordinates remain constant. By doing this, the findings are not directly impacted when the design factors are changed. The approach used in this work's MATLAB-based Finite Element Model Updating tool was first created in [13] for structural dynamic analysis. Additionally, in [14,15], it was modified for structural static analysis. In this work, eq. (1) served as the goal function that the MATLAB code used to optimize the models, as in [10,12].

$$O(m, \delta) = W_1 \frac{\sum_{j=1}^n M_j}{\sum_{j=1}^n M_j^i} + W_2 \frac{\sum_{j=1}^n |\delta_j|}{\sum_{j=1}^n |\delta_j^i|} \tag{1}$$

Where

$\delta_j$  is the nodal deflection obtained in each nodal point and in each iteration,  $\delta_j^i$  is the nodal deflection obtained in each nodal point in the initial model.

$M_j$  is the element mass obtained in each nodal point and in each iteration,  $M_j^i$  is the element mass obtained in each nodal point in the initial model. The same logic also applies to the term  $f(\delta)$

### 2.3. Improvement of the models

Each beam model in its initial state was parametrized in ANSYS APDL and has the same values as the variables LG1, LG2 and LG3. These variables are shown in Fig. 1 (right). Their initial values are: LG1=45, LG2=75 and LG3=2 [mm]. The outer section dimensions are kept, by principle, unaltered. The models were statically optimized earlier, in [10,12] and, as such, the values of the variables changed during the optimization routine. The final values of them are shown in Table 2

Table 2. Final variable and objective function values obtained on the optimized models.

Bending	A1	A2	A3		A1	A2	A3
	Pattern 1				Pattern 3		
LG1f	4.86	1.80	1.80		LG1f	4.50	2.17
LG2f	7.73	9.26	8.32	LG2f	7.51	9.02	9.52
LG3f	3.74	2.79	2.63	LG3f	3.61	2.76	2.58

Final objective	0.98	0.83	0.79	Final objective	0.97	0.86	0.80
	Pattern 2				Pattern 4		
LG1f	1.80	2.17	1.80	LG1f	1.80	2.17	
LG2f	10.15	7.61	11.90	LG2f	8.05	7.70	8.36
LG3f	2.79	2.99	2.65	LG1f	2.75	3.55	2.76
Final objective	0.87	0.89	0.81	Final objective	0.80	0.85	0.77

The analyzed models were previously subjected to bending and torsion modelled as uncoupled loads, in [10-12]. The results regarding the dynamic behavior presented in Figs. 5 and 6, were compared by means of an improvement factor:

$$Impfreq = \frac{\omega_f - \omega_i}{\omega_i} * 100\% \tag{2}$$

where Impfreq is the improvement of the final models in relation to the initial models, in terms of characteristic frequencies,  $\omega_i$  is the frequency of the initial models and  $\omega_f$  is the frequency of the final models.

### 3. Results and discussion

Mode shapes for the model A1 and Pattern 4 are shown in Fig. 4 as an example. In Fig. 4, both the displaced and initial mode shapes are presented.

#### 3.1. Mesh convergence analysis

To select a mesh size that originates accurate results, a mesh convergence analysis was done to the model beam 3 pattern 3. As this model is the most complex, it is expected that, if the mesh convergence yields accurate results for this geometry, the other models would originate accurate results, as well. Table 3 presents the mesh convergence study, showing the mean element size  $Esize$ , the frequency of the 1<sup>st</sup> mode  $\omega_1$  and the error obtained by the application of Eq. 3:

**Table 3.** : Mesh convergence study.

	$\omega_1$	Error [%]
40	145,88	
20	144,01	1,28
10	142,01	1,39
5	141,55	0,32
2,5	141,61	0,04

The error shown in Table 3 was calculated by using eq. (3):

$$Error[\%] = \frac{|\omega_i - \omega_{i+1}|}{\omega_1} * 100 \tag{3}$$

Because the element size of 2.5 mm originates the most accurate results, in comparison to other element sizes, with a value of 0,04%, that element size was used in the simulations.

#### 3.2. Analysis of the frequencies

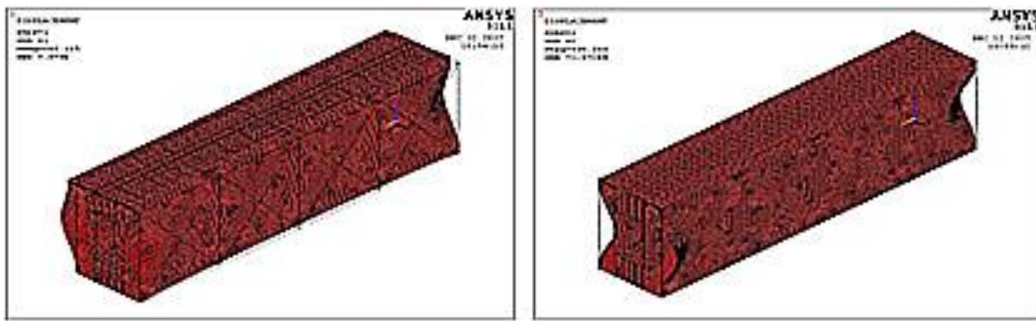


Figure 4. Mode shapes for the mode 1 at 175.631 Hz (left) and mode 2 at 450.587 Hz (right).

In order to study the dynamic response of the beams, modal analyses were done. The modal extraction method was Block-Lanczos, with frequency range between 0–20000 Hz. The first 20 modes were expanded, and their eigenvalues collected. The results are shown in Figs. 5–10.

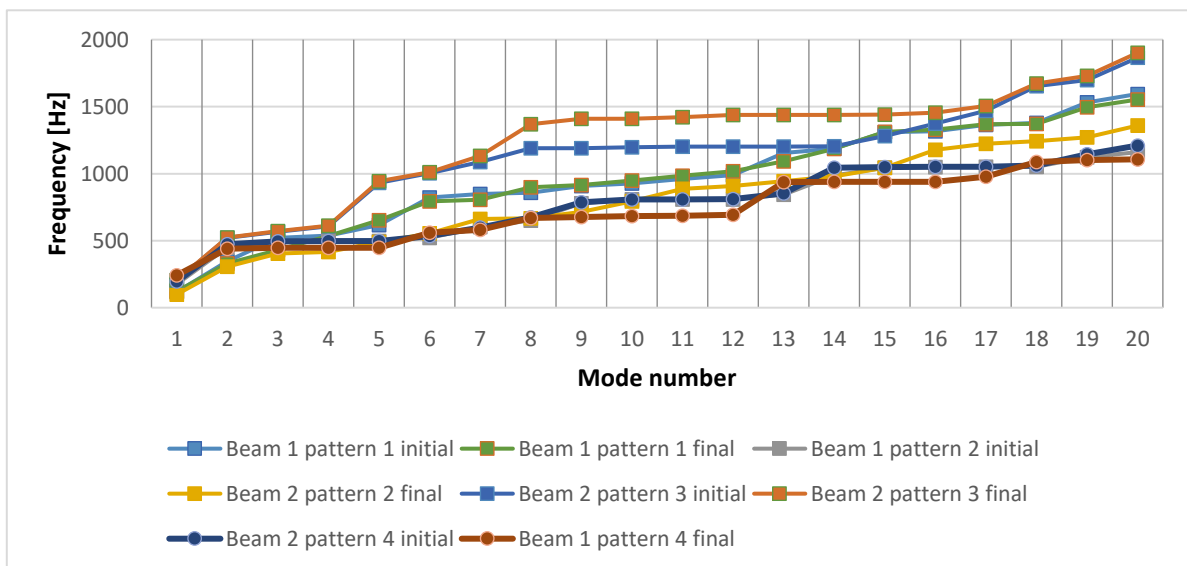


Figure 5. Frequency vs. mode number for the beam A1.

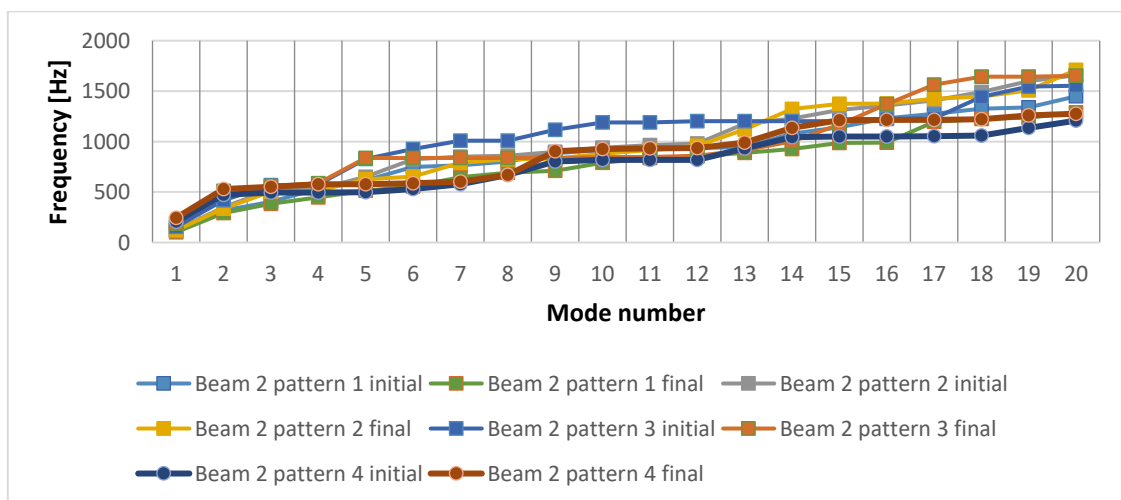


Figure 6. Frequency vs. mode number for the beam A2.

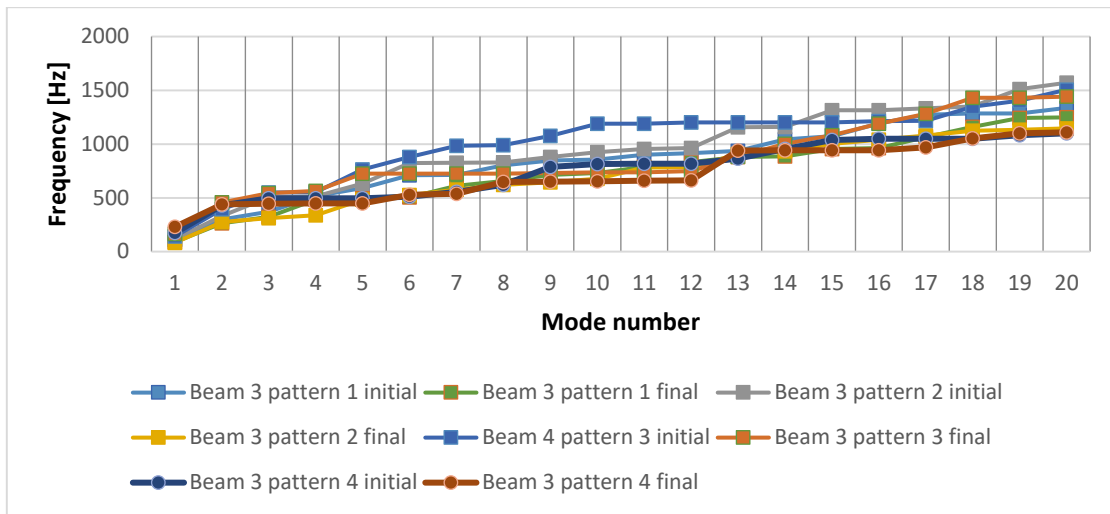


Figure 7. Frequency vs. mode number for the beam A3.

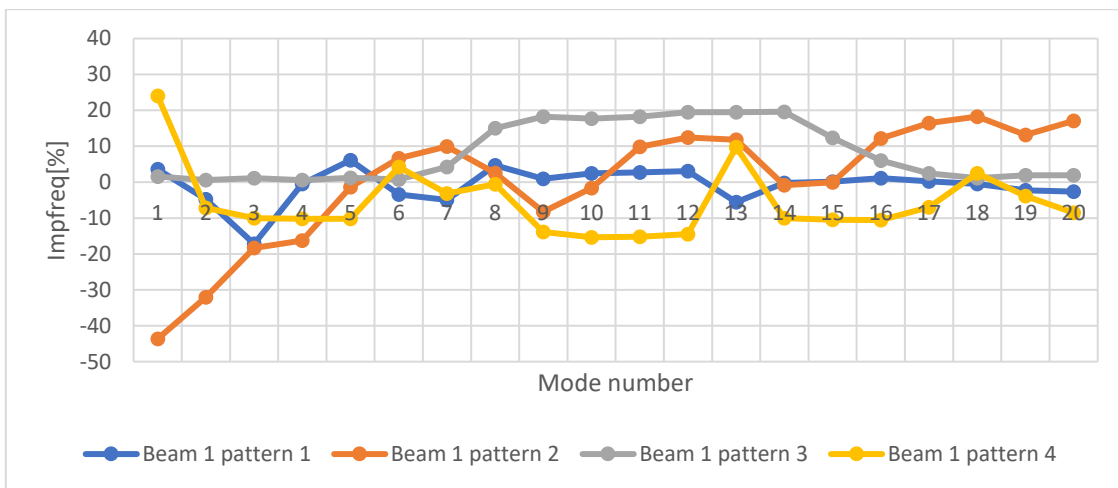


Figure 8. Improvement of statically optimized models in relation to the initial models, for beams A1.

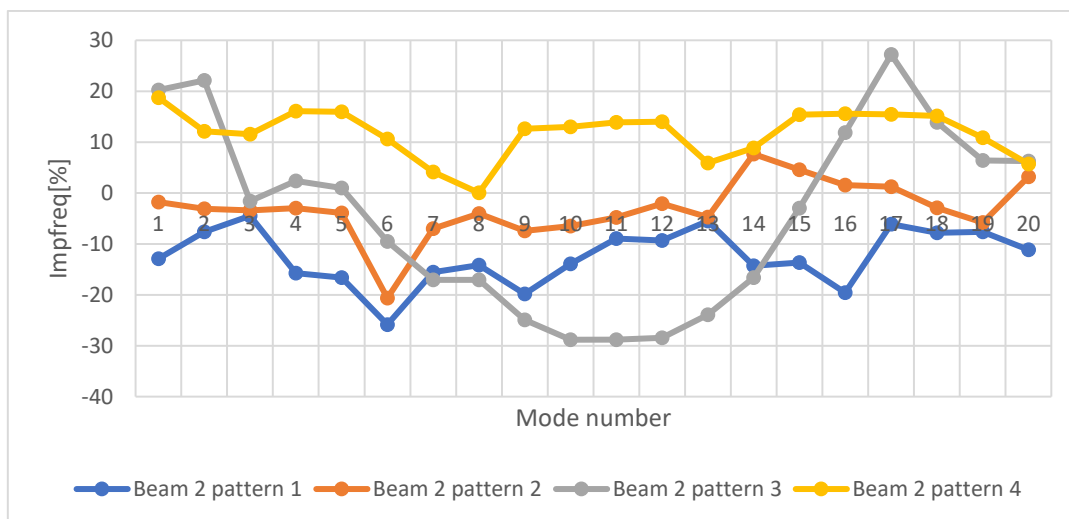
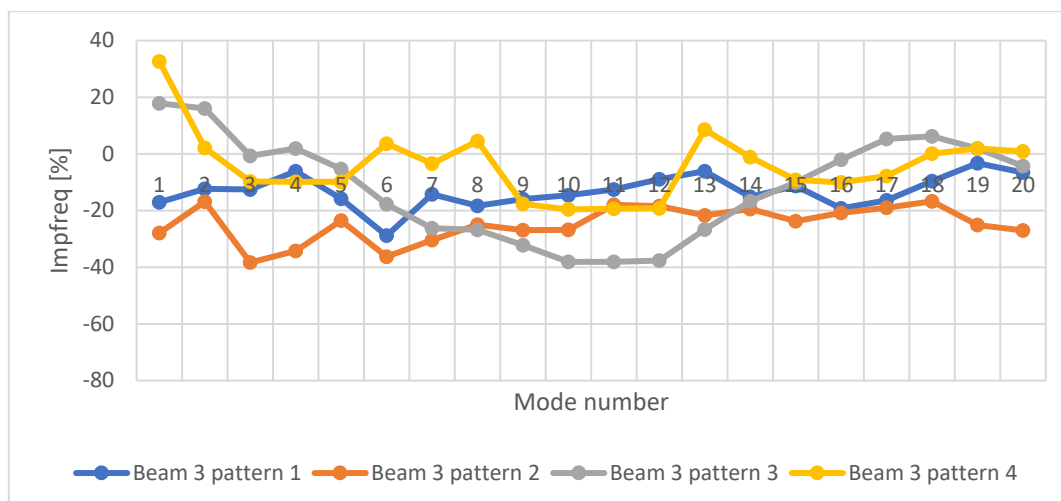


Figure 9. Improvement of statically optimized models in relation to the initial models, for beams A2.





**Figure 10.** Improvement of statically optimized models in relation to the initial models for beams A3.

It can be seen that all the beams behave similarly in terms of their dynamic response. The frequency range surpasses 1800 Hz for the Beam 1 pattern 3 final B. It is the maximum value of frequency obtained for all the models. The beam A3 pattern 4 is the model that behaves best, as the highest maximization of frequencies can be obtained considering all models. The dynamic response of a structure, in terms of natural frequencies, is very important in structural analysis. It is known that lower frequencies are more energetic, and, therefore, they are known to be more prone to disrupt the adequate operation on applications involving high accelerations and comprising lightweight mobile parts. The dynamic behavior can be improved if the natural frequencies of the optimized models can be maximized in comparison with initial ones. It can be seen from beam A1, whose results are shown in Fig. 8, there is an improvement for Patterns 1 and 3. Pattern 4 presents an advantage only for some modes, and Pattern 2 is causing an overall worsening. For beams of the A2 type, shown in Fig. 9, there exists an overall improvement for Pattern 4 while Pattern 3 gives that only for some modes, and 280 Patterns 1 and 2 present an overall worsening. Beams of the A3 type shown in Fig. 10 present an improvement for some modes for Patterns 3 and 4 and overall worsening for Patterns 1 and 2.

#### 4. Conclusions

Although the internal reinforcements are useful in improving the static behavior, as shown in [10-12], the improvement they originate do not appear to be worth the price of increasing the mass. All the 12 studied beams were already subjected to optimization routines for the improvement of the static behavior [10,12]. The initial and optimized models were then subjected to modal analysis. When comparing the modal behavior of A3 pattern 4 with a simple hollow-box beam, it can be seen that the improvements are quite good. It is shown that overall, the improvement of the dynamic behavior originated by the static optimization is significant, although mild.

**Author Contributions:** Conceptualization, Hugo Miguel Silva; methodology, Hugo Miguel Silva; software, Hugo Miguel Silva; validation, Hugo Miguel Silva; formal analysis, Hugo Miguel Silva; investigation, Hugo Miguel Silva; resources, Hugo Miguel Silva; data curation, Hugo Miguel Silva; writing—original draft preparation, Hugo Miguel Silva; writing—review and editing, Jerzy Wojewoda; visualization, Hugo Miguel Silva; supervision, Jerzy Wojewoda; project administration, Jerzy Wojewoda; funding acquisition, Jerzy Wojewoda. All authors have read and agreed to the published version of the manuscript.

**Funding:** H. M. Silva gratefully acknowledge the support provided by the Foundation for Science and Technology (FCT) of Portugal, within the scope of the project of the Research Unit on Materials, Energy and Environment for Sustainability (proMetheus), Ref. UID/05975/2020, financed by national funds through the FCT/MCTES. This research was also supported by a Dean's grant at the Lodz University of Technology for PhD studies, whose recipient was Hugo Silva

**Data Availability Statement:** The datasets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

**Conflicts of Interest:** Authors declare no conflict of interest.

## References

- [1] Meirovitch, L., *Fundamentals of Vibrations*, reprinted Edition, McGraw Hill, 2001.
- [2] Jonsson, J., Andreassen, M. Distortional eigenmodes and homogeneous solutions for semi-discretized thin-walled beams, *Thin-Walled Structures* 49 (2011) 691–707.
- [3] Gonçalves, R., Camotim, D., The vibration behaviour of thin-walled regular polygonal tubes, *Thin-Walled Structures* 84 (2014) 77–188.
- [4] Pagani, A., Boscolo, M., Banerjee, J., Carrera, E., Exact dynamic stiffness elements based on one-dimensional higher-order theories for free vibration analysis of solid and thin-walled structures, *Journal of Sound and Vibration* 332 (2013) 6104–6127.
- [5] Bebianoa, R., Basagliab, C., Camotima, D., Gonçalves, R., GBT buckling analysis of generally loaded thin-walled members with arbitrary flat-walled cross-sections, *Thin-Walled Structures* 123 (2018) 11–24.
- [6] Yang, F., Zhong, Y., Liu, R., Cao, H., 2023, Effective performance analysis of stiffened honeycomb sandwich panels using VAM-based equivalent model, *Thin-Walled Structures*, Volume 185, 110590.
- [7] Du, Y., Zou, T., Pang, F., Hu, C., Ma, Y., Li, H., 2023, Design method for distributed dynamic vibration absorbers of stiffened plate under different boundary constraints, *Thin-Walled Structures*, Volume 185, 110494.
- [8] Wang, Y-j., Zhang, Z-j., Xue, X-m., Zhang, L., 2019, Free vibration analysis of composite sandwich panels with hierarchical honeycomb sandwich core, *Thin-Walled Structures*, Volume 145, 106425.
- [9] Franco, C., Chesnais, C., Semblat, J-F., Giry, C., Desprez, C., 2022, Finite element formulation of a homogenized beam for reticulated structure dynamics, *Computers & Structures*, Volumes 261–262, 106729.
- [10] Silva, H.M., 2018, "Optimization of the Mechanical Behavior of hollow-box beams", PhD Thesis, Lodz University of Technology, Poland
- [11] Silva, H.M., Wojewoda, J., 2019, "Sensitivity analysis of internally reinforced thin-walled hollow-box beams subjected to bending and torsion loadings, *Mechanics and Mechanical Engineering*, 23:1–8.
- [12] Silva, H. M., Meireles, J.F., 2018, "Structural Optimization of internally reinforced beams subjected to coupled and uncoupled bending and torsion loadings for industrial applications"- *Mechanics and Mechanical Engineering*, 2017, Vol. 21, No. 2, 329-351.
- [13] Meireles, J.F., 2008, *Análise Dinâmica de Estruturas por Modelos de Elementos Finitos Identificados Experimentalmente*, PhD thesis, University of Minho, Portugal.
- [14] Silva, H.M., 2011 "Determination of the Material/geometry of the section most adequate for a static loaded beam subjected to a combination of bending and torsion", MSc. Thesis, University of Minho, Portugal.
- [15] Silva, H.M., Meireles, J.F., 2012, "Determination of material/geometry of the section most adequate for a static loaded beam subjected to a combination of bending and torsion ", *Materials Science Forum* Vols. 730-732 pp 507-512 Trans Tech Publications. DOI: doi.org/10.4028/www.scientific.net/MSF.730.

**Disclaimer/Publisher's Note:** The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.