Proceeding Paper

On the Efficiency of Ratio-product Estimator for Estimation of Finite Population Coefficient of Variation

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Abstract: Ratio product estimators have been proposed by several authors for the estimation of population mean, and population variance, but very few authors have proposed ratio product estimators for the estimation of population coefficient of variation. In this paper, we proposed a ratio product estimator for the estimation of the population coefficient of variation. The mean square error of the proposed estimator has been obtained up to the first order of approximation using the Taylor series technique. The numerical analysis was conducted and the results show that the proposed ratio product estimator is more efficient.

Keywords: Estimator; MSE; Coefficient of variation; Study variable; Finite population

1. Introduction

Various estimation strategies have been developed by many researchers in the field of sample surveys for the estimation of population parameters Zakari et al. (2020a), Muhammad et al. (2021), Zakari et al., (2020b), and Muhammad et al., (2022). Some of the estimation methods use auxiliary information for the precision of the estimate of the parameter. Auxiliary information is information on auxiliary variables, like population mean, population variance, sample mean, sample variance, and so on which are used to improve the efficiency of estimators. Authors such as Sahai and Ray (1980), Sisodia and Dwivedi (1981), Singh and Singh (2002), Srivastava and Jhajj (1981), Solanki and Singh (2015), Singh and Solanki (2012), Singh and Tailor (2005), Kumar and Adichwal (2016), Shabbir and Gupta (2016), Adichwal et al. (2017) have worked in that direction.

For estimating the population coefficient of variation, Das and Tripathi (1981) were the first to propose the estimator for the coefficient of variation when samples were selected using SRSWOR. Other works include that of Patel and Shah (2009), Rajyaguru and Gupta (2002, 2006), Archana and Rao (2014), Singh et al. (2018), Audu et al. (2021), and Yunusa et al. (2021).

In the current study, we proposed a ratio product estimator in the presence of population mean, population variance, sample mean, and sample variance of X for the estimation of the population coefficient of variation of the study variable Y, with the aim of obtaining a precise estimate of the parameter.

Following the introduction is Section 2, which contains the methodology and some existing estimators in literature while Section 3 presents the proposed estimator, bias, and MSE of the proposed estimator. Section 4 discusses the efficiency comparisons of the proposed estimator while the empirical study and conclusion are presented in Sections 5 and 6 respectively.

2. Methodology

Let us consider a simple random sample size n drawn from the given population of N units. Let the value of the study variable Y and the auxiliary variable X for the i^{th} units (i=1, 2,3, 4, …, N) of the population be denoted by Y_i and X_i and for the i^{th} unit in the sample (i=1, 2, 3, …, n) by y_i and x_i respectively

$\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i$ and $\bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i$ - are the population means of the study and auxiliary variables.

$S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i - \bar{Y})^2$ - is the population variance of the study variable.

$S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \bar{X})^2$ - is the population variance of the auxiliary variable.

$S_{xy} = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \bar{X})(Y_i - \bar{Y})$ - is the population covariance of the auxiliary and study variable.

$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ and $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ - are the sample mean of the study and auxiliary variables.

$s_y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2$ - is the sample variance of the study variable.

$s_x^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$ - is the sample variance of the auxiliary variable.

$s_{xy} = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})$ - is the sample covariance of the auxiliary and study variable.

Now, let us define sampling errors for both the mean and variance of Y and X:

$$e_0 = \bar{Y}^{-1}\left(\bar{y} - \bar{Y}\right), e_1 = \bar{X}^{-1}\left(\bar{x} - \bar{X}\right), e_2 = \left(S_y^2\right)^{-1}\left(s_y^2 - S_y^2\right), e_3 = \left(S_x^2\right)^{-1}\left(s_x^2 - S_x^2\right)$$

Such that

$$\bar{y} = \bar{Y}\left(1+e_0\right), \bar{x} = \bar{X}\left(1+e_1\right), s_y = S_y\left(1+e_2\right)^{1/2}, s_x = S_x\left(1+e_3\right)^{1/2}, s_y^2 = S_y^2\left(1+e_2\right), s_x^2 = S_x^2\left(1+e_3\right)$$

$$E\left(e_0\right) = E\left(e_1\right) = E\left(e_2\right) = E\left(e_3\right) = 0$$

$$E\left(e_0^2\right) = \gamma C_y^2, E\left(e_1^2\right) = \gamma C_x^2, E\left(e_2^2\right) = \gamma\left(\lambda_{40} - 1\right), E\left(e_3^2\right) = \gamma\left(\lambda_{04} - 1\right),$$

$$E\left(e_0 e_1\right) = \gamma\rho C_y C_x, E\left(e_0 e_2\right) = \gamma C_y \lambda_{30}, E\left(e_0 e_3\right) = \gamma C_y \lambda_{12},$$

$E\left(e_1 e_2\right) = \gamma C_x \lambda_{21}, E\left(e_1 e_3\right) = \gamma C_x \lambda_{03}, E\left(e_2 e_3\right) = \gamma\left(\lambda_{22} - 1\right)$. Here $\gamma = n^{-1}\left(1 - f\right)$, $f = nN^{-1}$ sampling fraction. $C_y = \bar{Y}^{-1} S_y$ and $C_x = \bar{X}^{-1} S_x$ are the population coefficient of variation for the study variable Y and auxiliary variable X. Also ρ denotes the correlation coefficient between X and Y.

In general,

$\mu_{rs} = \left(n-1\right)^{-1}\sum_{i=1}^{n}(y_i - \bar{y})^r (x_i - \bar{x})^s$ and $\lambda_{rs} = \mu_{rs}\left(\mu_{20}^{r/2}\mu_{02}^{s/2}\right)^{-1}$ respectively.

2.1. Some existing estimators in literature

The estimator, for estimating population coefficient of variation in the absence of auxiliary variable is given by:

$$\hat{C}_y = \frac{s_y}{\bar{y}} \tag{2.1}$$

The mean square error (MSE) expression of the estimator $\hat{C}_y$ is given by:

$$MSE\left(\hat{C}_y\right) = C_y^2\gamma\left(C_y^2 + 0.25\left(\lambda_{40} - 1\right) - C_y\lambda_{30}\right) \tag{2.2}$$

Archana and Rao (2014) introduced estimators for calculating the finite population coefficient of variation. These estimators were designed specifically for estimating the coefficient of variation for one component of a bivariate normal distribution, considering prior knowledge about the second component. They established a Cramer-Rao-type lower bound based on the mean square error of these estimators. Through extensive simulations, they compared 28 estimators and found that eight of them exhibited higher relative efficiency compared to the sample coefficient of variation. They also provided the asymptotic mean square errors for the most effective estimators, offering valuable insights for users in calculating the coefficient of variation. Thus, the estimators are given as:

$$t_{AR1} = \hat{C}_y\left(\frac{\bar{X}}{\bar{x}}\right) \tag{2.3}$$

$$t_{AR2} = \hat{C}_y\left(\frac{\bar{x}}{\bar{X}}\right) \tag{2.4}$$

$$t_{AR3} = \hat{C}_y\left(\frac{S_x^2}{s_x^2}\right) \tag{2.5}$$

$$t_{AR4} = \hat{C}_y\left(\frac{s_x^2}{S_x^2}\right) \tag{2.6}$$

The mean square errors (MSEs) expression of the estimators is given by:

$$MSE\left(t_{AR1}\right) = C_y^2\gamma\left[C_y^2 + 0.25\left(\lambda_{40} - 1\right) + C_x^2 - C_x\lambda_{21} - C_y\lambda_{30} + 2\rho C_y C_x\right] \tag{2.7}$$

$$MSE\left(t_{AR2}\right) = C_y^2\gamma\left[C_y^2 + 0.25\left(\lambda_{40} - 1\right) + C_x^2 + C_x\lambda_{21} - C_y\lambda_{30} - 2\rho C_y C_x\right] \tag{2.9}$$

$$MSE\left(t_{AR3}\right) = C_y^2\gamma\left[C_y^2 + 0.25\left(\lambda_{40} - 1\right) + \left(\lambda_{04} - 1\right) - \left(\lambda_{22} - 1\right) - C_y\lambda_{30} + 2C_y\lambda_{12}\right] \tag{2.10}$$

$$MSE\left(t_{AR4}\right) = C_y^2\gamma\left[C_y^2 + 0.25\left(\lambda_{40} - 1\right) + \left(\lambda_{04} - 1\right) + \left(\lambda_{22} - 1\right) - C_y\lambda_{30} - 2C_y\lambda_{12}\right] \tag{2.11}$$

Audu et al. (2021) introduced three estimators that combine difference and ratio approaches for estimating the coefficient of variation in a finite population. These estimators utilize known population mean, population variance, and population coefficient of variation of an auxiliary variable. They also investigated the biases and mean square errors (MSEs) associated with these proposed estimators. By comparing their performance with existing estimators using information from two populations, they demonstrated that their proposed estimators were superior in efficiency compared to various other estimators, including unbiased, ratio type, exponential ratio type, and difference type estimators. Thus, the estimators are:

$$T_{M1} = \left[\frac{\hat{C}_y}{2}\left(\frac{\bar{X}}{\bar{x}} + \frac{\bar{x}}{\bar{X}}\right) + w_1\left(\bar{X} - \bar{x}\right) + w_2\hat{C}_y\right]\left(\frac{\bar{X}}{\bar{x}}\right) \tag{2.12}$$

$$T_{M2} = \left[\frac{\hat{C}_y}{2}\left(\frac{S_x^2}{s_x^2} + \frac{s_x^2}{S_x^2}\right) + w_3\left(S_x^2 - s_x^2\right) + w_4\hat{C}_y\right]\left(\frac{S_x^2}{s_x^2}\right) \tag{2.13}$$

The mean square errors of the estimators are given by

$$MSE\left(T_{M1}\right)=C_y^2\left(A+w_1^2B+w_2^2C+2w_1D-2w_2E-2w_1w_2F\right) \tag{2.14}$$

$$MSE\left(T_{M2}\right)=C_y^2\left(A_1+w_3^2B_1+w_4^2C_1+2w_3D_1-2w_4E_1-2w_3w_4F_1\right) \tag{2.15}$$

Where $A=\gamma\left(C_x^2+C_y^2+2\rho C_yC_x-C_x\lambda_{21}-C_y\lambda_{30}+\frac{\left(\lambda_{40}-1\right)}{4}\right)$, $B=\gamma\delta^2\left(\lambda_{04}-1\right)$ for $\delta=\frac{\bar{X}}{C_y}$

$C=1+\gamma\left(3C_x^2+3C_y^2+4\rho C_yC_x-2C_x\lambda_{21}-2C_y\lambda_{30}\right)$, $D=\gamma\delta\left(C_x^2+\rho C_yC_x-\frac{C_x\lambda_{21}}{2}\right)$

$E=\gamma\left(\frac{3C_x\lambda_{21}}{2}-3\rho C_yC_x-\frac{5C_x^2}{2}-2C_y^2+\frac{3C_y\lambda_{30}}{2}-\frac{\left(\lambda_{40}-1\right)}{8}\right)$, $F=\gamma\delta\left(\frac{C_x\lambda_{21}}{2}-\rho C_yC_x-2C_x^2\right)$

and $A_1=\gamma\left(\left(\lambda_{04}-1\right)+C_y^2+2C_y\lambda_{12}-\left(\lambda_{22}-1\right)-C_y\lambda_{30}+\frac{\left(\lambda_{40}-1\right)}{4}\right)$, $B_1=\gamma\delta_1^2\left(\lambda_{22}-1\right)$ for $\delta_1=\frac{S_x^2}{C_y}$

$C_1=1+\gamma\left(3\left(\lambda_{04}-1\right)+3C_y^2+4C_y\lambda_{12}-2\left(\lambda_{22}-1\right)-2C_y\lambda_{30}\right)$, $D_1=\gamma\delta_1\left(\left(\lambda_{04}-1\right)+C_y\lambda_{12}-\frac{\left(\lambda_{22}-1\right)}{2}\right)$

$E_1=\gamma\left(\frac{3\left(\lambda_{22}-1\right)}{2}-3C_y\lambda_{12}-\frac{5\left(\lambda_{04}-1\right)}{2}-2C_y^2+\frac{3C_y\lambda_{30}}{2}-\frac{\left(\lambda_{40}-1\right)}{8}\right)$, $F_1=\gamma\delta_1\left(\frac{\left(\lambda_{22}-1\right)}{2}-C_y\lambda_{12}-2\left(\lambda_{04}-1\right)\right)$

and $w_1=\frac{CD-EF}{F^2-BC}$, $w_2=\frac{DF-BE}{F^2-BC}$, $w_3=\frac{C_1D_1-E_1F_1}{F_1^2-B_1C_1}$ and $w_4=\frac{D_1F_1-B_1E_1}{F_1^2-B_1C_1}$.

The minimum mean square errors of the estimators are given by

$$MSE\left(T_{M1}\right)_{\min}=C_y^2\left[A+\frac{\left(CD^2+BE^2-2DEF\right)}{\left(F^2-BC\right)}\right] \tag{2.16}$$

$$MSE\left(T_{M2}\right)_{\min}=C_y^2\left[A_1+\frac{\left(C_1D_1^{\,2}+B_1E_1^{\,2}-2D_1E_1F_1\right)}{\left(F_1^2-B_1C_1\right)}\right] \tag{2.17}$$

3. Proposed estimator

Having studied the estimators developed by Archana and Rao (2014) and Audu et al. (2021), for the estimation of finite population coefficient of variation, we therefore, proposed a new ratio product estimator in the presence of population mean, population variance, sample mean, and sample variance of X for the estimation of the population coefficient of variation of the study variable Y, with the aim of obtaining a precise estimate of the parameter. As such, the proposed estimator is given as:

$$T_g=\hat{C}_y\left[k_1\left(\frac{\bar{X}}{\bar{x}}\right)\left(\frac{S_x^2}{s_x^2}\right)+k_2\left(\frac{\bar{x}}{\bar{X}}\right)\left(\frac{s_x^2}{S_x^2}\right)\right] \tag{3.1}$$

Where, k_1 and k_2 are unknown constants to be determined.

Expressing equation (3.1) in terms of error terms, we have,

$$T_g = \frac{S_y(1+e_2)^{1/2}}{\bar{Y}(1+e_0)}\left[k_1\left(\frac{S_x^2}{S_x^2(1+e_3)}\right)\left(\frac{\bar{X}}{\bar{X}(1+e_1)}\right)+k_2\left(\frac{S_x^2(1+e_3)}{S_x^2}\right)\left(\frac{\bar{X}(1+e_1)}{\bar{X}}\right)\right] \quad (3.2)$$

After simplifying equation (3.2) to first order of approximation, we obtain

$$T_g = C_y\left[k_1\left(\begin{array}{l} 1-e_3+e_3^2-e_1+e_1e_3+e_1^2-e_0 \\ +e_0e_3+e_0e_1+e_0^2+\frac{e_2}{2}-\frac{e_2e_3}{2} \\ -\frac{e_1e_2}{2}-\frac{e_0e_2}{2}+\frac{e_2^2}{8} \end{array}\right)+k_2\left(\begin{array}{l} 1+e_3+e_1+e_1e_3-e_0-e_0e_3 \\ -e_0e_1+e_0^2+\frac{e_2}{2}+\frac{e_2e_3}{2}+\frac{e_1e_2}{2} \\ -\frac{e_0e_2}{2}-\frac{e_2^2}{8} \end{array}\right)\right] \quad (3.3)$$

Subtracting C_y from both sides of equation (3.3), we have

$$T_g - C_y = C_y\left[k_1\left(\begin{array}{l} 1-e_3+e_3^2-e_1+e_1e_3+e_1^2-e_0 \\ +e_0e_3+e_0e_1+e_0^2+\frac{e_2}{2}-\frac{e_2e_3}{2} \\ -\frac{e_1e_2}{2}-\frac{e_0e_2}{2}+\frac{e_2^2}{8} \end{array}\right)+k_2\left(\begin{array}{l} 1+e_3+e_1+e_1e_3-e_0-e_0e_3 \\ -e_0e_1+e_0^2+\frac{e_2}{2}+\frac{e_2e_3}{2}+\frac{e_1e_2}{2} \\ -\frac{e_0e_2}{2}-\frac{e_2^2}{8} \end{array}\right)-1\right] \quad (3.4)$$

Taking expectation on both sides of equation (3.4) to obtain the bias of the estimator as

$$Bias(T_g) = C_y\left[k_1\left(1+\gamma\left(\begin{array}{l} (\lambda_{04}-1)+C_x\lambda_{03}+C_x^2 \\ +C_y\lambda_{12}+\rho C_yC_x+C_y^2 \\ -\frac{(\lambda_{22}-1)}{2}-\frac{C_x\lambda_{21}}{2} \\ -\frac{C_y\lambda_{30}}{2}-\frac{(\lambda_{40}-1)}{8} \end{array}\right)\right)+k_2\left(1+\gamma\left(\begin{array}{l} C_x\lambda_{03}-C_y\lambda_{12} \\ -\rho C_yC_x+C_y^2 \\ +\frac{(\lambda_{22}-1)}{2}+\frac{C_x\lambda_{21}}{2} \\ -\frac{C_y\lambda_{30}}{2}-\frac{(\lambda_{40}-1)}{8} \end{array}\right)\right)-1\right] \quad (3.5)$$

Squaring and taking expectation on both sides of equation (3.4) to obtain the mean square error (MSE) of the estimator as

$$MSE(T_g) = C_y^2\left(1+k_1^2A_2+k_2^2B_2+2k_1k_2C_2-2k_1D_2-2k_2E_2\right) \quad (3.6)$$

Where, $A_2 = 1+\gamma\left(3(\lambda_{04}-1)+4C_x\lambda_{03}+3C_x^2+4C_y\lambda_{12}+4\rho C_yC_x+3C_y^2-2(\lambda_{22}-1)-2C_x\lambda_{21}-2C_y\lambda_{30}\right)$,

$B_2 = 1+\gamma\left(4C_x\lambda_{03}-4C_y\lambda_{12}-4\rho C_yC_x+3C_y^2+2(\lambda_{22}-1)+2C_x\lambda_{21}-2C_y\lambda_{30}+(\lambda_{04}-1)+C_x^2\right)$,

$C_2 = 1+\gamma\left(3C_y^2-2C_y\lambda_{30}\right)$,

$$D_2 = 1+\gamma\begin{pmatrix} (\lambda_{04}-1)+C_x\lambda_{03}+C_x^2+C_y\lambda_{12}+\rho C_yC_x+C_y^2-\frac{1}{2}(\lambda_{22}-1)-\frac{1}{2}C_x\lambda_{21}-\frac{1}{2}C_y\lambda_{30} \\ -\frac{1}{8}(\lambda_{40}-1) \end{pmatrix},$$

$$E_2 = 1+\gamma\left(C_x\lambda_{03}-C_y\lambda_{12}-\rho C_yC_x+C_y^2+\frac{1}{2}(\lambda_{22}-1)+\frac{1}{2}C_x\lambda_{21}-\frac{1}{2}C_y\lambda_{30}-\frac{1}{8}(\lambda_{40}-1)\right).$$

Differentiating equation (3.6) partially with respect to k_1 and k_2 and equate the terms obtained to zero, we get: $A_2k_1+C_2k_2=D_2$ and $C_2k_1+B_2k_2=E_2$, solving these simultaneously we get the optimum values of k_1 and k_2 as:

$k_1=\dfrac{B_2D_2-C_2E_2}{A_2B_2-C_2^2}$ and $k_2=\dfrac{A_2E_2-C_2D_2}{A_2B_2-C_2^2}$, putting these expressions into equation (3.6) gives the minimum mean square error $(MSE)_{min}$ as:

$$MSE(T_g)_{\min} = C_y^2\left[1-\frac{\left(A_2E_2^2+B_2D_2^2-2C_2D\,E_2\right)}{\left(A_2B_2-C_2^2\right)}\right] \tag{3.7}$$

4. Efficiency comparisons

In this section, efficiency conditions of T_g over sample coefficient of variation $\hat{C}_y$, $t_{AR1}, t_{AR2}, t_{AR3}, t_{AR4}, T_{M1}$ and T_{M2} were established.

i. T_g is more efficient than $\hat{C}_y$ if:

$$MSE(T_g)_{\min} < MSE(\hat{C}_y) \tag{4.1}$$

$$\left(1-\frac{\left(A_2E_2^2+B_2D_2^2-2C_2D_2E_2\right)}{\left(A_2B_2-C_2^2\right)}\right) < \gamma\left(C_y^2+0.25(\lambda_{40}-1)-C_y\lambda_{30}\right) \tag{4.2}$$

ii. T_g is more efficient than t_{AR1} if:

$$MSE(T_g)_{\min} < MSE(t_{AR1}) \tag{4.3}$$

$$\left(1-\frac{\left(A_2E_2^2+B_2D_2^2-2C_2D_2E_2\right)}{\left(A_2B_2-C_2^2\right)}\right) < \gamma\left(C_y^2+0.25(\lambda_{40}-1)+C_x^2-C_x\lambda_{21}-C_y\lambda_{30}+2\rho C_yC_x\right) \tag{4.4}$$

iii. T_g is more efficient than t_{AR2} if:

$$MSE(T_g)_{\min} < MSE(t_{AR2}) \tag{4.5}$$

$$\left(1-\frac{\left(A_2E_2^2+B_2D_2^2-2C_2D_2E_2\right)}{\left(A_2B_2-C_2^2\right)}\right) < \gamma\left(C_y^2+0.25(\lambda_{40}-1)+C_x^2+C_x\lambda_{21}-C_y\lambda_{30}-2\rho C_yC_x\right) \tag{4.6}$$

iv. T_g is more efficient than t_{AR3} if:

$$MSE(T_g)_{\min} < MSE(t_{AR3}) \tag{4.7}$$

$$\left(1-\frac{\left(A_2E_2^2+B_2D_2^2-2C_2D_2E_2\right)}{\left(A_2B_2-C_2^2\right)}\right)<\gamma\begin{pmatrix}C_y^2+0.25\left(\lambda_{40}-1\right)+\left(\lambda_{04}-1\right)-\left(\lambda_{22}-1\right)\\-C_y\lambda_{30}+2C_y\lambda_{12}\end{pmatrix} \qquad (4.8).$$

v. T_g is more efficient than t_{AR4} if:

$$MSE\left(T_g\right)_{\min}<MSE\left(t_{AR4}\right) \qquad (4.9)$$

$$\left(1-\frac{\left(A_2E_2^2+B_2D_2^2-2C_2D_2E_2\right)}{\left(A_2B_2-C_2^2\right)}\right)<\gamma\begin{pmatrix}C_y^2+0.25\left(\lambda_{40}-1\right)+\left(\lambda_{04}-1\right)+\left(\lambda_{22}-1\right)\\-C_y\lambda_{30}-2C_y\lambda_{12}\end{pmatrix} \qquad (4.10).$$

vi. T_g is more efficient than T_{M1} if:

$$MSE\left(T_g\right)_{\min}<MSE\left(T_{M1}\right) \qquad (4.11)$$

$$\left(1-\frac{\left(A_2E_2^2+B_2D_2^2-2C_2D_2E_2\right)}{\left(A_2B_2-C_2^2\right)}\right)<\left(A+\frac{\left(CD^2+BE^2-2DEF\right)}{\left(F^2-BC\right)}\right) \qquad (4.12)$$

vii. T_g is more efficient than T_{M2} if:

$$MSE\left(T_g\right)_{\min}<MSE\left(T_{M2}\right) \qquad (4.13)$$

$$\left(1-\frac{\left(A_2E_2^2+B_2D_2^2-2C_2D_2E_2\right)}{\left(A_2B_2-C_2^2\right)}\right)<\left(A+\frac{\left(C_1D_1^2+B_1E_1^2-2D_1E_1F_1\right)}{\left(F_1^2-B_1C_1\right)}\right) \qquad (4.14)$$

5. Empirical Study

In this section, empirical study will carry out to demonstrate the performance of the proposed estimator over existing ones. Data from the book Murthy (1967) and Sarjinder Singh (2003) will used be used.

Population 1: [Source: Murthy (1967), p.399]

X: Area under wheat in 1963, Y: Area under wheat in 1964

$$N=34, n=15, \bar{X}=208.88, \bar{Y}=199.44, C_x=0.72, C_y=0.75, \rho=0.98, \lambda_{21}=1.0045, \lambda_{12}=0.9406,$$
$$\lambda_{40}=3.6161, \lambda_{04}=2.8266, \lambda_{30}=1.1128, \lambda_{03}=0.9206, \lambda_{22}=3.0133$$

Population 1: [Source: Sarjinder Singh (2003), p.1116]

X: Number of fish caught in year 1993, Y: Number of fish caught in year 1995

$$N=69, n=40, \bar{X}=4591.07, \bar{Y}=4514.89, C_x=1.38, C_y=1.35, \rho=0.96, \lambda_{21}=2.19, \lambda_{12}=2.30,$$
$$\lambda_{40}=7.66, \lambda_{04}=9.84, \lambda_{30}=1.11, \lambda_{03}=2.52, \lambda_{22}=8.19$$

Table 1. MSEs and PREs of Proposed and Existing estimators.

	Population1	**Population 1**	**Population 2**	**Population 2**
Estimators	MSE	PRE	MSE	PRE
$\hat{C}_y$	0.008003575	100	0.03808827	100
t_{AR1}	0.02589068	30.91296	0.08517984	44.71512
t_{AR2}	0.01184353	67.57761	0.06393314	59.57516

t_{AR3}	0.03365777	23.77928	0.188603	20.19494
t_{AR4}	0.05890541	13.58716	0.2261359	16.84309
T_{M1}	0.006737495	118.7916	0.03533973	107.77748
T_{M2}	0.006013652	133.09009	0.02810758	135.5089
T_g	**0.004943499**	**161.901**	**0.01718988**	**221.5738**

Table1 above shows the mean square error (MSE) and the percentage relative efficiency (PRE) of the proposed estimator. The results revealed that the proposed estimator has minimum mean square error and higher percentage relative efficiency. This implies that the suggested estimator is more efficient than the existing ones.

6. Conclusion

In this study, we proposed ratio product estimator for estimation of finite population coefficient of variation. This estimator utilized information on the sample and population mean as well as the sample and population variance of the auxiliary variable X. From the numerical analysis, the results show that the proposed estimator is more efficient than the conventional estimators with the evidence of having minimum mean square error, hence, it should in real life situation for estimation.

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