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Integrating the implied regularity into implied volatility models

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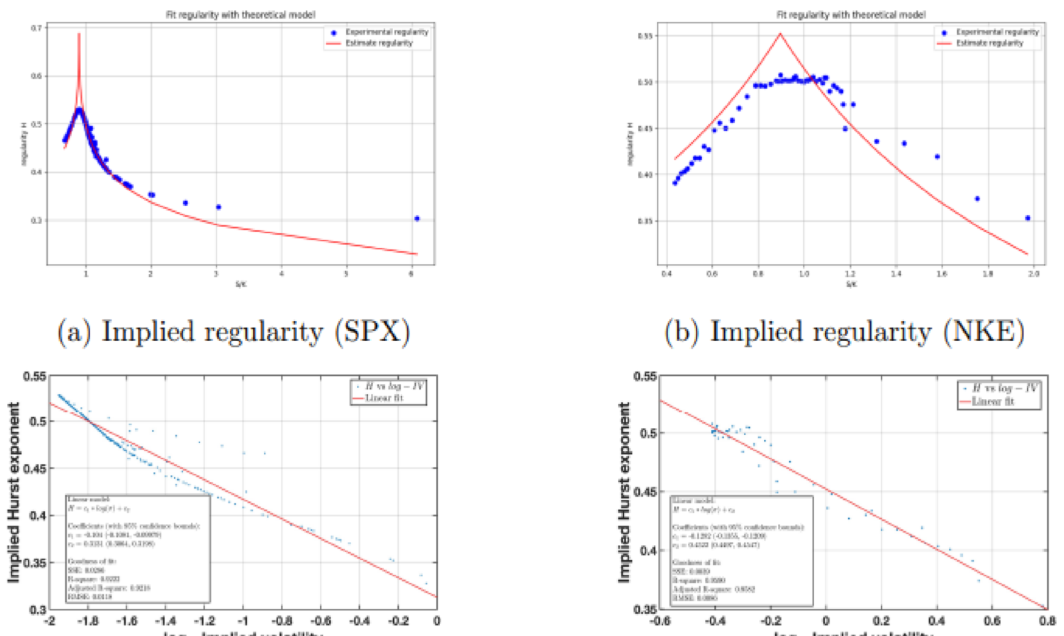
INTRODUCTION & AIM

In financial markets, implied volatility (IV) serves as a key indicator of expected future price fluctuations of an underlying asset. Traditionally, much research has explored how IV behaves in relation to moneyness. In this work, we examine the link between implied volatility and the implied Hurst exponent HH, focusing on how both vary with moneyness. A central finding is that H tends toward 0.5 when moneyness equals 1, indicating a potential point of market efficiency. To capture these dynamics more effectively, we propose a new IV model that incorporates the implied Hurst exponent. This model accounts for the interaction between HH and the ratio of the underlying asset's price to the strike price S/K which plays a key role in explaining variations in IV across different levels of moneyness. Tested across multiple market indexes and optimized using the Optuna algorithm, our model shows strong performance compared to existing frameworks such as SABR and fSABR. This enhanced modeling approach provides a deeper understanding of the relationship between IV and H, offering valuable tools for options pricing and volatility forecasting.



(a) Implied regularity (SPX) (b) Implied regularity (NKE)
(c) Implied regularity vs IV (d) Implied regularity vs IV

METHOD

The closed-form expression proposed in the Angelini–di Sciorio (AdS) model characterizes the dependence of implied volatility (σ) on the ratio between the underlying asset price (S) and the option's strike price (K), commonly referred to as ***moneyness*** in the financial literature. The model also incorporates the impact of memory—captured through the Hurst exponente (H)—and its sensitivity to moneyness. The formula is structured into two principal components, the first is given by:

$$\sigma\left(\frac{S}{K}\right) = \alpha\left(\frac{S}{K} - \frac{S}{K_{\min}}\right)^2 e^{-\beta H\left(\frac{S}{K}\right)\left(\frac{S}{K} - \frac{S}{K_{\min}}\right)} + \epsilon$$

This formulation reveals that implied volatility is influenced by the squared distance between the asset price and strike price, capturing the standard concave shape observed in volatility surfaces, especially in in-the-money (ITM) and out-of-the-money (OTM) regions. The memory effect is introduced via an exponential term, where the constant β and the Hurst exponent function H control how implied volatility decays as the strike price diverges from Kmin. A higher H (stronger memory) leads to a sharper drop in volatility with increasing distance, whereas a lower H leads to a flatter response.

The Hurst function is defined as:

$$H\left(\frac{S}{K}\right) = \frac{1}{2}\left(1 + \left|1 - \frac{S}{K_{\min}}\right|^{\delta}\right)\frac{1}{\left(1 + \left|\frac{S}{K} - \frac{S}{K_{\min}}\right|^{\delta}\right)}$$

In the AdS model, H varies with the distance between current moneyness (S/K) and the reference , modulated by a shape parameter δ. This functional form gives rise to an **inverse smile effect** where the Hurst exponent equals =0.5 at-the-money (ATM) and decreases toward ITM and OTM regions. This behavior aligns with the Efficient Market Hypothesis (EMH) near ATM, where fractional Brownian motion converges to geometric Brownian motion. In contrast, memory effects become more pronounced in ITM and OTM areas, indicating increased market reactivity to past price movements. Thus, ATM options reflect informational efficiency, while others are more influenced by historical price paths.

RESULTS & DISCUSSION

The AdS model was applied to market data sourced from Yahoo Finance using the Python library yfinance, focusing on U.S. indices and individual stocks. The analysis was conducted on 30-day-to-expiration options, collecting data on strike prices, implied volatilities (IV), and the underlying asset's closing prices.

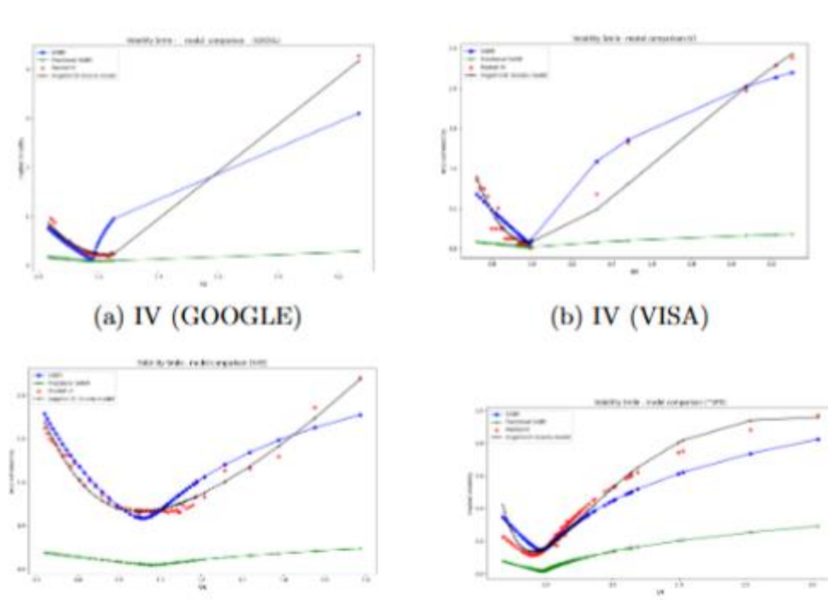
To evaluate model performance, the AdS model was compared against the standard SABR and fractional SABR (fSABR) models. All models were calibrated using Optuna (100 trials) by minimizing the root mean square error (RMSE). In addition to traditional metrics like mean squared error (MSE) and mean absolute error (MAE), the study introduced curvature-based metrics: ACE (Absolute Curvature Error) and RMSCE (Root Mean Square Curvature Error), which assess the second derivative of implied volatility with respect to moneyness.

The results show that the AdS model outperforms both SABR and fSABR, particularly in the in-the-money (ITM) and out-of-the-money (OTM) regions. These regions are harder to model due to their sensitivity to complex market dynamics and reflect structural inefficiencies and future expectations. The AdS model captures these behaviors more accurately thanks to its dynamic flexibility: the Hurst exponent HH varies with moneyness (S/K).

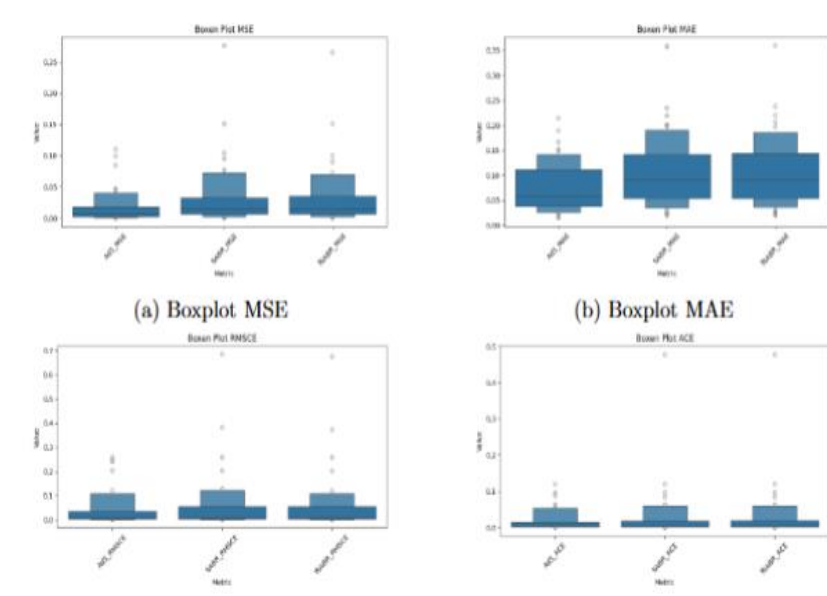
•Near at-the-money (ATM), H≈0.5, which aligns with the Efficient Market Hypothesis (EMH) and standard geometric Brownian motion.
•In ITM and OTM regions, H<0.5, indicating weaker memory a
•nd more stochastic price behavior.
Key features of the AdS model include:

1. A **quadratic term** that naturally reflects the curvature of real volatility surfaces, especially in ITM and OTM regions.
2. A **memory-dependent exponential decay** which adjusts how volatility decays based on the market's memory.
3. In ITM regions: a slower volatility decay when H is low, indicating higher sensitivity to intrinsic value.
4. In OTM regions: a steeper decay, consistent with higher uncertainty and lower liquidity.

The dynamic Hurst function H(S/K)H(S/K) allows the AdS model to adapt to structural changes in the market and investor behavior across different moneyness levels. This adaptability enables the AdS model to capture implied volatility patterns and market inefficiencies more effectively than the more rigid SABR and fSABR models.



(a) IV (GOOGLE) (b) IV (VISA)
(c) IV (NKE) (d) IV (SPX)



(a) Boxplot MSE (b) Boxplot MAE
(c) Boxplot RMSCE (d) Boxplot ACE

CONCLUSION

This research introduces a new implied volatility model that incorporates long-term memory effects. A notable feature is the behavior of H at-the-money (ATM), where S/K=1 and H = 0.5, aligning with the concept of a random walk (Brownian motion). This reflects maximum uncertainty and aligns with the Efficient Market Hypothesis.
Importantly, the model preserves **no-arbitrage conditions**, meaning it does not allow for risk-free profits based on the implied volatility surface. This ensures theoretical consistency with financial economics and reinforces the model's practical viability in efficient markets.

FUTURE WORK / REFERENCES

Integration with the Black-Scholes Framework
Extension to Dynamic Pricing Models
Estimation and Forecasting of the Hurst Exponent