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Proceeding Paper

Synergistic Application of Heat Source Reconstruction and Second-Harmonic Thermoelastic Stress Analysis for Rapid Metal Fatigue Characterisation †

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Abstract:
The rise of metal additive manufacturing demands faster fatigue assessment, as conventional testing remains time-consuming and costly. This study applies a combination of infrared thermography-based techniques: Heat Source Reconstruction (HSR) and Second-Harmonic Thermoelastic Stress Analysis (SHTSA), under constant and continuously varying stress amplitudes, for rapid fatigue characterisation. HSR quantifies mechanical dissipation, while SHTSA assesses nonlinear spectral responses beyond thermoelastic coupling. Tests on 5052-H32 aluminium and SS304 steel specimens validate the integrated approach.

Keywords: Heat Source Reconstruction HSR; Harmonic Analysis TSA; Fatigue self-heating; 5052-H32 aluminium alloy; SS304 steel

1. Introduction

Fatigue causes over 80% of structural failures [1], yet the creation of an S–N curve to provide indicative fatigue behaviour is time-intensive [2]. Thermographic methods provide a rapid alternative, correlating temperature with fatigue damage [3]. Under cyclic loading, two effects occur: temperature oscillation from thermoelastic coupling, and mean temperature rise (self-heating) from energy dissipation via internal friction and microstructural damage. About 90% of dissipated mechanical energy converts to heat [4], making surface temperature a reliable fatigue indicator.

Chrysochoos and Louche [5] introduced an infrared (IR) thermography method based on the ‘thermodynamics of irreversible processes’, linking surface temperature to internal heat sources via a heat diffusion model. In the elastic regime, stress and temperature vary linearly and inversely, forming the basis of Thermoelastic Stress Analysis (TSA), which enables the reverse calculation of surface stress fields from thermography.

At higher stress amplitudes, near or beyond the fatigue limit, nonlinear thermoelastic effects arise, manifesting as second and higher harmonic temperature components. The analysis of these components forms the foundation of Second-Harmonic TSA (SHTSA), a technique that enhances sensitivity to early fatigue damage and provides insight into material fatigue performance under cyclic loading.

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2. Heat Source Reconstruction (HSR)

HSR, developed by Chrysochoos and Louche [5], quantifies heat power density from measured surface temperatures using the heat diffusion equation Unlike finite element analysis, HSR infers "internal" heat generation (not related to environmental heat exchanges), enabling for instance analysis of the mechanical (or intrinsic) dissipation d_1 associated to material degradation (fatigue or plasticity) [2].

Following is the so-called zero-dimensional (0D) formulation of the heat equation, with the assumption that the heat source s_h is evenly spread across the gauge zone of the specimen at any given time:

$$\rho C\left(\frac{d\theta}{dt}+\frac{\theta}{\tau_{eq}}\right)=s_h(t)=s_{the}(t)+d_1(t) \quad (1)$$

In equation (1), the time constant τ_{eq} reflects global heat exchange (by convection with ambient air, by contact with the machine jaws, and by radiation) [2]. ρ and C are the density and specific heat of the material, respectively. The 0D approach involves spatial averaging of the temperature change θ over the specimen's gauge zone. By further applying temporal averaging over full load cycles (mean temperature change designated $\langle\theta\rangle$ in Eq. (2)), the thermoelastic heat source s_{the} averages to zero, isolating the mean mechanical dissipation component $\langle d_1\rangle$:

$$\langle d_1\rangle(t) \;=\; \rho C\left(\frac{d\langle\theta\rangle}{dt}+\frac{\langle\theta\rangle}{\tau_{eq}}\right) \quad (2)$$

3. Second-Harmonic TSA (SHTSA)

According to the fundamental TSA equation, $\Delta T=-K_m T_0\Delta$, the amplitude of the thermoelastic temperature oscillation ΔT is linearly proportional to the amplitude of the sum of the principal stresses $\Delta\sigma$ for isotropic materials, where $K_m=\alpha/(\rho C)$ is the thermoelastic constant. However, based on empirical studies, Wong et al. [6] incorporated the temperature dependence of modulus of elasticity (E) by introducing $\partial E/\partial T$, leading to a modified relation that accounts for nonlinearity.

$$\frac{\Delta T}{T_0}\rho C=-\left(\alpha-\frac{1}{E^2}\frac{\partial E}{\partial T}\sigma_m\right)\sigma_a\, sin\,\omega t-\frac{1}{4E^2}\frac{\partial E}{\partial T}(\sigma_a)^2\, cos\,2\,\omega t \quad (3)$$

with T_0: absolute reference temperature (K), α: coefficient of linear thermal expansion (K^{-1}), E: modulus of elasticity (N/m^2 or Pa), σ_m and σ_a: mean and amplitude of the applied sinusoidal stress (N/m^2 or Pa). The second-harmonic component $(\sigma_a)^2/(4E^2)\times\partial E/\partial T$, arises from nonlinear thermoelastic effects, particularly the temperature dependence of modulus of elasticity ($\partial E/\partial T$). Empirical studies have shown that the amplitude of the second-harmonic correlates with fatigue limit of the material [7].

4. Experiments

Aluminium 5052-H32 and stainless steel SS304, two engineering materials with distinct mechanical and thermoelastic properties, were chosen to validate the testing technique. Specimens underwent preliminary cyclic loading to achieve mechanical accommodation, eliminating initial plasticity and early fatigue effects, before tests with temperature measurement commenced. Conducted at 40 Hz, tests involved (a) constant σ_a (97 MPa for AL5052, 170 MPa for SS304) and (b) variable σ_a (0–132 MPa). In the latter, the testing machine followed ascending maximum and minimum loads while maintaining the load ratio, providing increasing stress amplitudes per cycle, as detailed in [2]. Temperature was measured using a FLIR X8501sc camera.

5. Results

<u>Constant Amplitude:</u> Figure 1 summarises the tests conducted under constant σ_a with two loading initiation conditions: starting at minimum stress (Test-1) and at maximum stress (Test-2). These initiation conditions influenced the specimen's mean temperature profile (Figure 1(A1)). In Test-1, an instantaneous temperature drop occurred due to thermoelastic coupling during the first half-cycle, whereas Test-2 exhibited an immediate temperature increase. In both cases, the temperature subsequently rose and stabilised above the initial level. A similar trend was observed in the SS304 tests; however, it is less apparent in Figure 1(A2) due to the broader temperature scale, which visually suppresses the trend. Higher temperatures in SS304 tests, compared to aluminium, resulted from testing above the fatigue limit. Note that, for all tests presented in this paper, a sliding-window analysis was applied to the temperature data to evaluate mechanical dissipation (d_1) and second-harmonic TSA (SHTSA) response. Both test conditions yielded consistent d_1 (HSR) and second-harmonic amplitude (SHTSA) measurements (Figure 1(B1), 1(C1)), confirming methodological robustness.

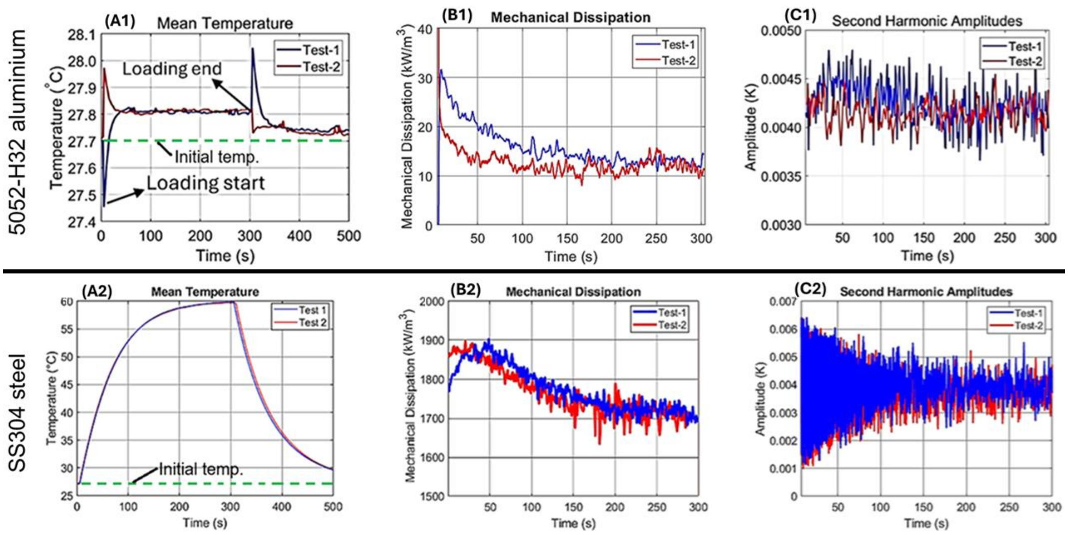

Figure 1. (A1, A2) Mean temperature; (B1, B2) mechanical dissipation (d_1); (C1, C2) SH amplitude. Plots B and C show data during cyclic loading (3.5–300 s). Test parameters: σ_a (AL5052) = 97.2 MPa, σ_a(SS304) = 170.6 MPa, stress ratio = 0.1, 40 Hz loading, 203 Hz sampling frequency.

Figure 1(B1, B2) shows consistent mechanical dissipation levels for Test-1 and Test-2, especially in the steady-state temperature regime, validating the method's reliability. The initial apparent decrease in mechanical dissipation is an HSR technique artefact, not a material change. Similarly, for the case of second-harmonic Figure 1(C1, C2), amplitudes remained stable for all the tests. Again, the initial variability in the steel results, in comparison to the aluminium, is because of the difference in the applied stress amplitude and resulting temperature change. The stable results verify the expectations as the tests are conducted with constant amplitudes on specimens which are preliminary cyclically loaded for mechanical accommodation.

<u>Continuously Varying Stress Amplitude:</u> Figure 2 presents results from two SS304 tests under continuously varying σ_a conditions. Both tests were performed using identical parameters to evaluate the robustness of the testing methodology. The stress amplitude profiles and their corresponding θ responses exhibit a high degree of similarity and therefore appear superimposed in the figure. The mechanical dissipation data for the SS304 specimens reveal distinct material responses under variable loading. Mechanical dissipation increases markedly with rising stress amplitude, especially beyond 50 MPa, where dissipation escalates sharply, reaching values of up to approximately 800 kW/m^3 at

132 MPa. This trend is consistently observed across both Test-1 and Test-2, thereby reinforcing the reliability of the continuously varying amplitude methodology.

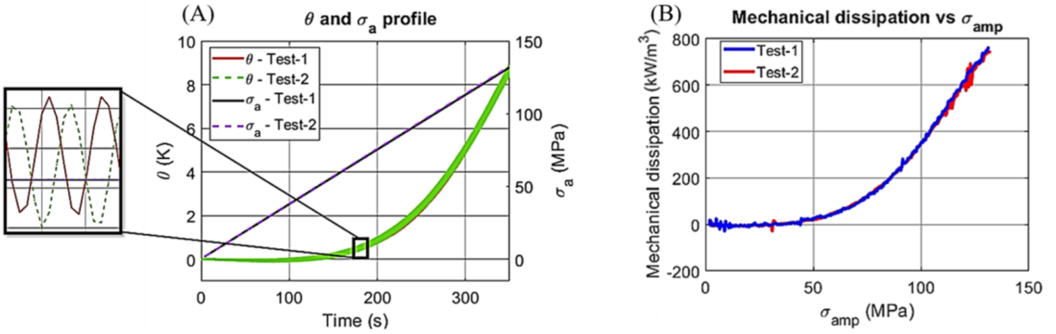

Figure 2: Material: SS304. Cyclic loading with varying σ_a (0 to 132 MPa), 40 Hz loading, 203 Hz sampling frequency. (A) σ_a and θ profiles; (B) mechanical dissipation (d_1) vs. σ_a.

6. Conclusion:

Tests at constant stress amplitude showed repeatable results, with stable mechanical dissipation and second-harmonic response in steel and aluminium specimens, aligning with theoretical expectations. Varying stress amplitude techniques show promise for improving thermography-based rapid fatigue testing efficiency. The SHTSA technique's response to varying amplitudes is under investigation.

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