Explicit Expressions for Solar Panel Equivalent Circuit Parameters

Based on Analytical Formulation and the Lambert W-Function

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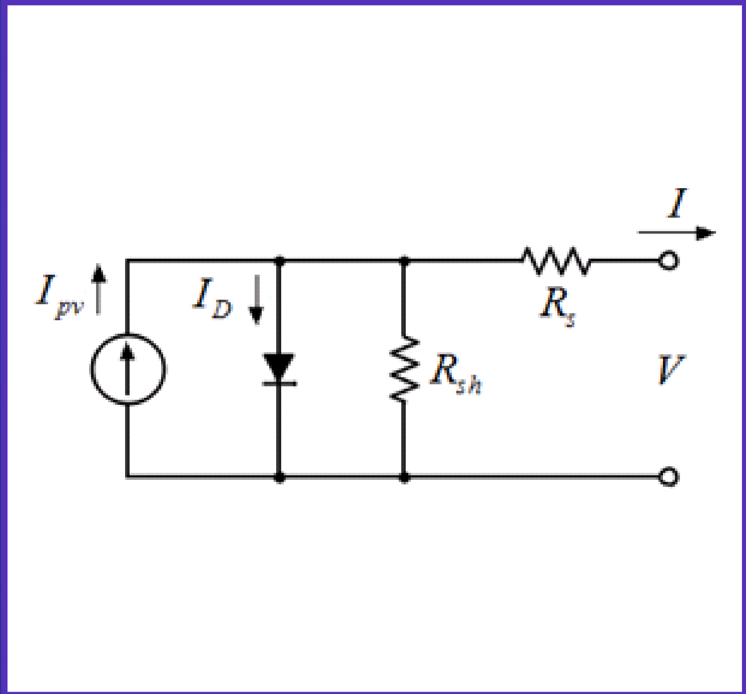
I Introduction

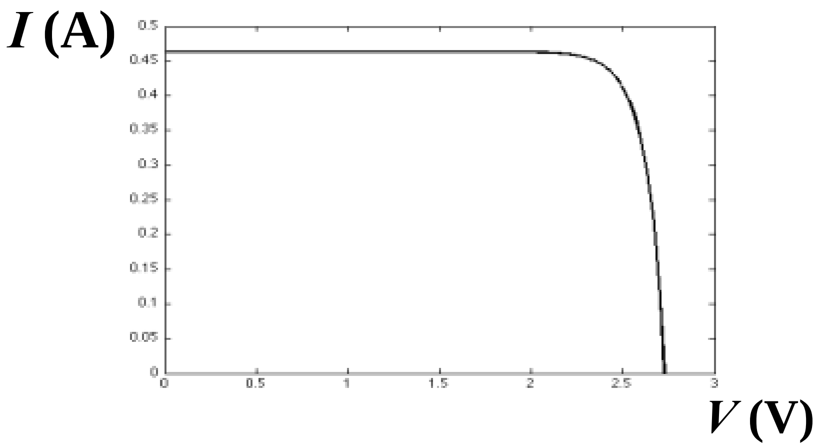
Modeling photovoltaic systems

- To obtain better performance is necessary to optimize electric systems.
- Modeling a system to reproduce different situations is a useful tool for optimization.
- Photovoltaic systems are a very variable energy source (Temperature, irradiance,...).
- Most common way of modeling of solar cells/panels is to calculate equivalent circuit.

II
Solar Cell Modeling



- Easy and realistic way of simulate the solar cell behavior



- Current source
- One diode
- One series resistance
- One shunt resistance

One diode model

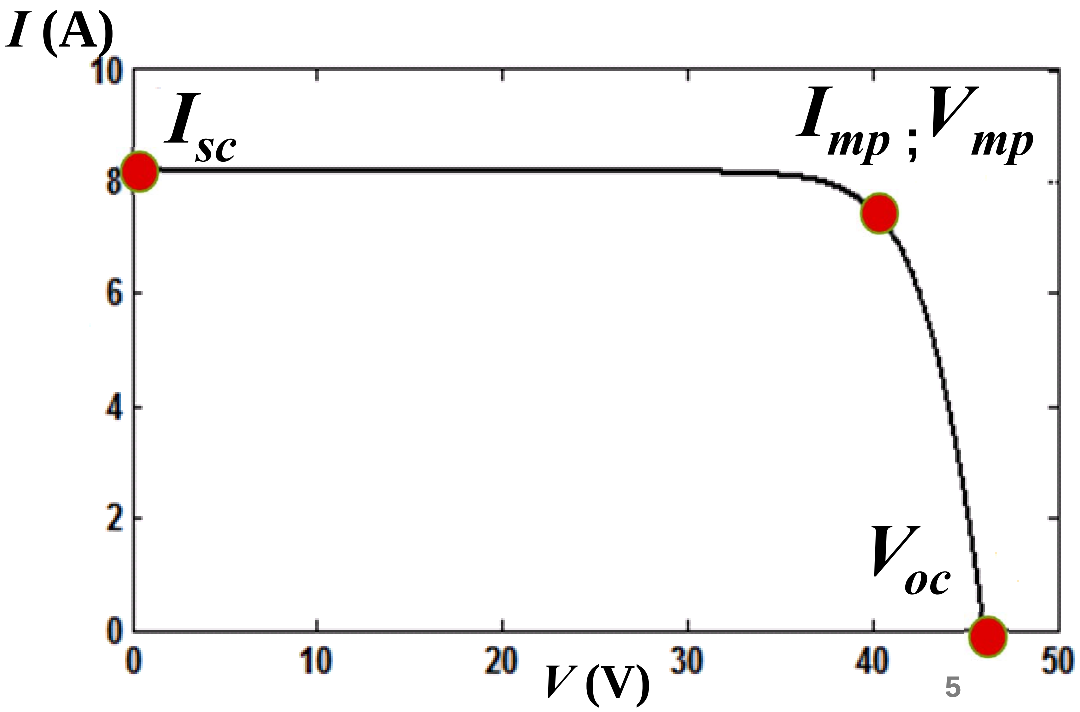
Equation

$$I = I_{pv} - I_0\left[\exp\left(\frac{V + IR_s}{aV_T}\right) - 1\right] - \frac{V + IR_s}{R_{sh}}$$

- I, current
- V, voltage
- n, number of cells
- V_T, termal voltage
  - $$V_T = n\frac{kT}{q}$$
- I_{pv}, constant current
- I_0, sat. current of diode
- a, ideality factor of diode
- R_s, series resistance
- R_{sh}, shunt resistance

I-V Curve and characteristic points

- Example of the current-voltage curve of a typical solar panel.



- Solar cell *IV* behaviour
- Short circuit point
  - $I = I_{sc}$; $V = 0$
- Open circuit point
  - $I = 0$; $V = V_{oc}$
- Maximum power point
  - $I = I_{mp}$; $V = V_{mp}$

Exampe of Data included in manufacturer datasheet

- Manufacturer information (AM1.5g; 25ºC)

MSP290AS-36.EU (multicrystalline)				MSMD290AS-36.EU (monocrystaline)			
n	72	T_r (ºC)	25	n	72	T_r (ºC)	25
P_{mp} (W)	290	γ (%/ºC)	-0.45	P_{mp} (W)	290	γ (%/ºC)	-0.44
I_{mp} (A)	7.82	αI_{mp} (%/ºC)	-	I_{mp} (A)	7.70	αI_{mp} (%/ºC)	-
V_{mp} (V)	37.08	βV_{mp} (%/ºC)	-0.35	V_{mp} (V)	37.66	βV_{mp} (%/ºC)	-0.35
I_{sc} (A)	8.37	αI_{sc} (%/ºC)	+0.04	I_{sc} (A)	8.24	αI_{sc} (%/ºC)	+0.04
V_{oc} (V)	44.32	βV_{oc} (mV/ºC)	-0.33	V_{oc} (V)	44.68	βV_{oc} (mV/ºC)	-0.31

- Objetive:
  - Design an equivalent circuit that meets all that specification

III Parameter Calculation

- I_{pv}, constant current
- I_0, sat. current of diode
- a, ideality factor of diode
- R_s, series resistance
- R_{sh}, shunt resistance

- Main disadvantage of equivalent circuit models is the determination of the parameters
- Dependent of external conditions
  - Temperature
  - Illumination
  - …
- Available information
  - Experimental data
    - Many *I-V* curve points
  - Manufacturer data
    - Characteristic points
      - Numerical
      - Analytical

Manufacturer data

- 4 Equations from boundary conditions
- Short circuit

$$I_{sc} = I_{pv} - I_0\left[\exp\left(\frac{I_{sc}R_s}{aV_T}\right) - 1\right] - \frac{I_{sc}R_s}{R_{sh}}$$

- Open circuit

$$0 = I_{pv} - I_0\left[\exp\left(\frac{V_{oc}}{aV_T}\right) - 1\right] - \frac{V_{oc}}{R_{sh}}$$

- Maximum power point [I_{mp}; V_{mp}]

$$I_{mp} = I_{pv} - I_0\left[\exp\left(\frac{V_{mp} + I_{mp}R_s}{aV_T}\right) - 1\right] - \frac{V_{mp} + I_{mp}R_s}{R_{sh}}$$

- Maximum power at [I_{mp}; V_{mp}]

$$\frac{\partial P}{\partial V} = V\frac{\partial I}{\partial V} + I = 0$$

- 5 param.

One has to be estimated, $\boldsymbol{a}$ is the most delimited

- $\boldsymbol{a} \in [1,\ 1.5]$
  - $\boldsymbol{a = 1.1}$
- $\boldsymbol{R_s}$
- $\boldsymbol{R_{sh}}$
- $\boldsymbol{I_0}$
- $\boldsymbol{I_{pv}}$

New analytical method

- The use of a new analytical method is proposed.
- New methodology, first analytical model that only uses manufacturer data.
- Using Lambert-W function explicit ecuations for the parameters of the equivalent circuit are achieved.
- The method calculates parameters analytically only from manufacturer data.
  - Non-iterative
  - Accurate
  - Straight forward

Solving sequence

1 Estimate a → $a \in [1, 1.5]$

2

$$R_s = \frac{aV_T}{I_{mp}}\left(W_{-1}\left(-\frac{V_{mp}\left(2I_{mp}-I_{sc}\right)}{\left(V_{mp}I_{sc}+V_{oc}\left(I_{mp}-I_{sc}\right)\right)}\exp\left(-\frac{2V_{mp}-V_{oc}}{aV_T}+\frac{\left(V_{mp}I_{sc}-V_{oc}I_{mp}\right)}{\left(V_{mp}I_{sc}+V_{oc}\left(I_{mp}-I_{sc}\right)\right)}\right)\right)-\left(\frac{V_{mp}-V_{oc}}{aV_T}-\frac{2V_{mp}-V_{oc}}{aV_T}+\frac{\left(V_{mp}I_{sc}-V_{oc}I_{mp}\right)}{\left(V_{mp}I_{sc}+V_{oc}\left(I_{mp}-I_{sc}\right)\right)}\right)\right)$$

→ a, R_s

3

$$R_{sh} = \frac{\left(V_{mp}-I_{mp}R_s\right)\left(V_{mp}-R_s\left(I_{sc}-I_{mp}\right)-aV_T\right)}{\left(V_{mp}-I_{mp}R_s\right)\left(I_{sc}-I_{mp}\right)-aV_T I_{mp}}$$

→ a, R_s, R_{sh}

4

$$I_0 = \frac{\left(R_{sh}+R_s\right)I_{sc}-V_{oc}}{R_{sh}\exp\left(\frac{V_{oc}}{aV_T}\right)}$$

5

$$I_{pv} = \frac{R_{sh}+R_s}{R_{sh}}I_{sc}$$

Parameters

$a, R_s, R_{sh}, I_0, I_{pv}$

MSP290AS-36.EU (multicrystalline)

MSP290AS-36.EU (multicrystalline)	
n	72
P_{mp} (W)	290
I_{mp} (A)	7.82
V_{mp} (V)	37.08
I_{sc} (A)	8.37
V_{oc} (V)	44.32

PM

EQUIVALENT CIRCUIT PARAMETERS	
a	1.10
$I_{pv,Tr}$	8.37 A
$R_{s,Tr}$	0.162 Ω
$R_{sh,Tr}$	331 Ω
$I_{0,Tr}$	2.86×10^{-9}A

solar panels equivalent circuits at STC (1000W/m²
irradiance, 25˚C cell temperature, AM1.5g spectrum

MSMD290AS-36.EU (monocrystaline)

MSMD290AS-36.EU (monocrystaline)	
n	72
P_{mp} (W)	290
I_{mp} (A)	7.70
V_{mp} (V)	37.66
I_{sc} (A)	8.24
V_{oc} (V)	44.68

PM

EQUIVALENT CIRCUIT PARAMETERS	
a	1.10
$I_{pv.Tr}$	8.24 A
$R_{s,Tr}$	0.130 Ω
$R_{sh,Tr}$	316 Ω
$I_{0,Tr}$	2.36×10^{-9}A

solar panels equivalent circuits at STC (1000W/m^2 irradiance, 25˚C cell temperature, AM1.5g spectrum

IV Dependence on temperature

- *I-V* behaviour of the solar cell depends on temperature.
- Thus parameters of equivalent circuit depends on temperature.
- There are methods that relates parameters with temperature, but…
- We are going to take advantage of the ease of the method to directly calculate the variation of the parameters from manufacturer datasheet

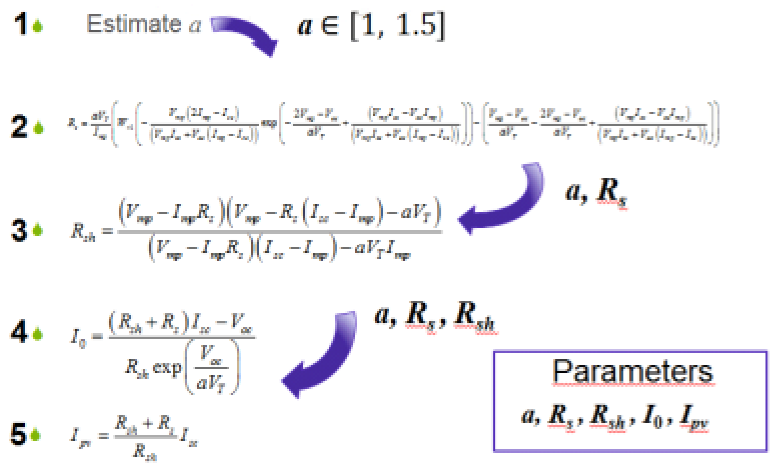
Characteristic points T dependence

- Recalculate characteristic points for temperature T according to manufacturer data.

$$I_{sc,T} = I_{sc,T_r}\left(1 + \frac{\alpha I_{sc}(T - T_r)}{100}\right) \qquad V_{oc,T} = V_{oc,T_r}\left(1 + \frac{\beta V_{oc}(T - T_r)}{100}\right)$$

$$I_{mp,T} = I_{mp,T_r}\left(1 + \frac{\alpha I_{mp}(T - T_r)}{100}\right) \qquad V_{mp,T} = V_{mp,T_r}\left(1 + \frac{\beta V_{mp}(T - T_r)}{100}\right)$$

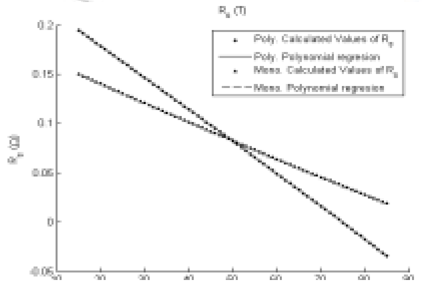
- For the new characteristic points repeat solving sequence.



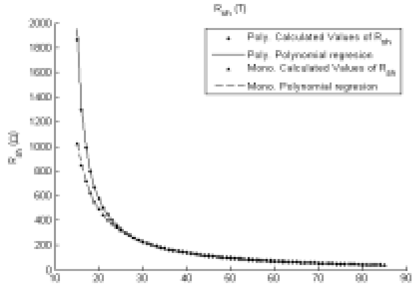
- Do for the entire interval of T.

Characteristic points *T* dependence

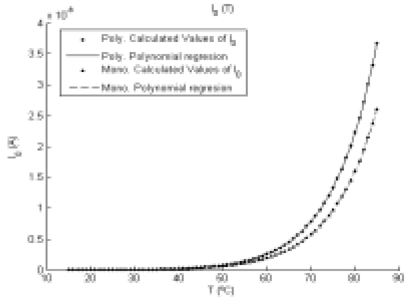
- R_s



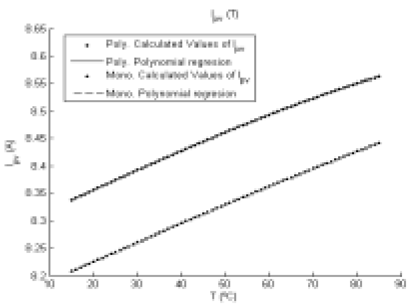
- R_{sh}



- I_0



- I_{pv}



MSP290AS-36.EU
(multycrystaline)

$$I_{pv}(T) = 8.37 + 3.62\cdot 10^{-3}\Delta T - 3.38\cdot 10^{-6}\Delta T^2 - 7.58\cdot 10^{-8}\Delta T^3,$$

$$R_s(T) = 1.62\cdot 10^{-1} - 3.21\cdot 10^{-3}\Delta T + 7.05\cdot 10^{-7}\Delta T^2 - 3.01\cdot 10^{-8}\Delta T^3,$$

$$R_{sh}(T) = 1/(3.03\cdot 10^{-3} + 2.65\cdot 10^{-4}\Delta T + 1.50\cdot 10^{-6}\Delta T^2 + 1.56\cdot 10^{-8}\Delta T^3),$$

$$I_0(T) = \exp(-1.97\cdot 10^{1} + 1.44\cdot 10^{-1}\Delta T - 4.80\cdot 10^{-4}\Delta T^2 + 1.15\cdot 10^{-6}\Delta T^3).$$

MSMD290AS-36.EU
(monocrystaline)

$$I_{pv}(T) = 8.24 + 3.49\cdot 10^{-3}\Delta T - 1.68\cdot 10^{-6}\Delta T^2 - 2.41\cdot 10^{-8}\Delta T^3,$$

$$R_s(T) = 1.30\cdot 10^{-1} - 1.97\cdot 10^{-3}\Delta T + 2.53\cdot 10^{-6}\Delta T^2 - 1.07\cdot 10^{-8}\Delta T^3,$$

$$R_{sh}(T) = 1/(3.18\cdot 10^{-3} + 2.33\cdot 10^{-4}\Delta T + 1.27\cdot 10^{-6}\Delta T^2 + 1.33\cdot 10^{-8}\Delta T^3),$$

$$I_0(T) = \exp(-1.98\cdot 10^{1} + 1.41\cdot 10^{-1}\Delta T - 4.69\cdot 10^{-4}\Delta T^2 + 1.13\cdot 10^{-6}\Delta T^3).$$

V Dependence on irradiation

- *I-V* behaviour of the solar cell depends on irradiation
- Manufacturer data for this solar cell is referred to AM1,5g (G_r = 1000 W/m^2)
- Experimental behaviour with irradiation *G*
  - I_{sc} varies lineally with *G*
  - V_{oc} varies logarithmic with *G*
  - R_s is constant with *G*
- Parameter behaviour
  - $I_{pv,G} = I_{pv,Gr}\ G/G_r$
  - I_0, R_s, R_{sh} and a non dependent of *G*

Summarizing MSP290AS-36.EU

MSP290AS-36.EU			
n	72	T_r (ºC)	25
P_{mp} (W)	290	γ (%/ºC)	-0.45
I_{mp} (A)	7.82	αI_{mp} (%/ºC)	-
V_{mp} (V)	37.08	βV_{mp} (%/ºC)	-0.35
I_{sc} (A)	8.37	αI_{sc} (%/ºC)	+0.04
V_{oc} (V)	44.32	βV_{oc} (mV/ºC)	-0.33

- Eq. circuit parameters expresions have been calculated taking in account
  - Manufacturer experimental data for temperature dependance
  - Dependance with irradiation G

- a: $a = 1.1$
- R_s: $R_s(T) = 1.62\cdot10^{-1} - 3.21\cdot10^{-3}\Delta T + 7.05\cdot10^{-7}\Delta T^2 - 3.01\cdot10^{-8}\Delta T^3,$
- R_{sh}: $R_{sh}(T) = 1/(3.03\cdot10^{-3} + 2.65\cdot10^{-4}\Delta T + 1.50\cdot10^{-6}\Delta T^2 + 1.56\cdot10^{-8}\Delta T^3),$
- I_0: $I_0(T) = \exp(-1.97\cdot10^{1} + 1.44\cdot10^{-1}\Delta T - 4.80\cdot10^{-4}\Delta T^2 + 1.15\cdot10^{-6}\Delta T^3),$
- I_{pv}: $I_{pv}(T) = \left(8.37 + 3.62\cdot10^{-3}\Delta T - 3.38\cdot10^{-6}\Delta T^2 - 7.58\cdot10^{-8}\Delta T^3\right)\dfrac{G}{G_r}.$

Summarizing MSMD290AS-36.EU

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V_{mp} (V)	37.66	βV_{mp} (%/ºC)	-0.35
I_{sc} (A)	8.24	αI_{sc} (%/ºC)	+0.04
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- Eq. circuit parameters expresions have been calculated taking in account
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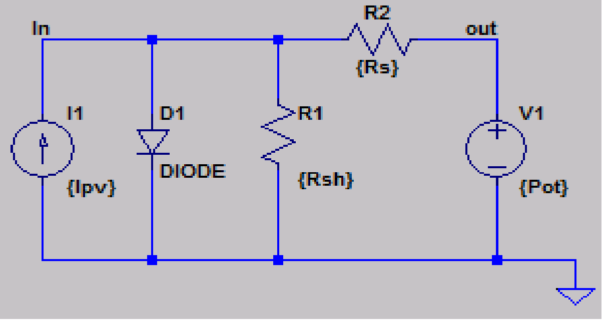
  - Dependance with irradiation *G*

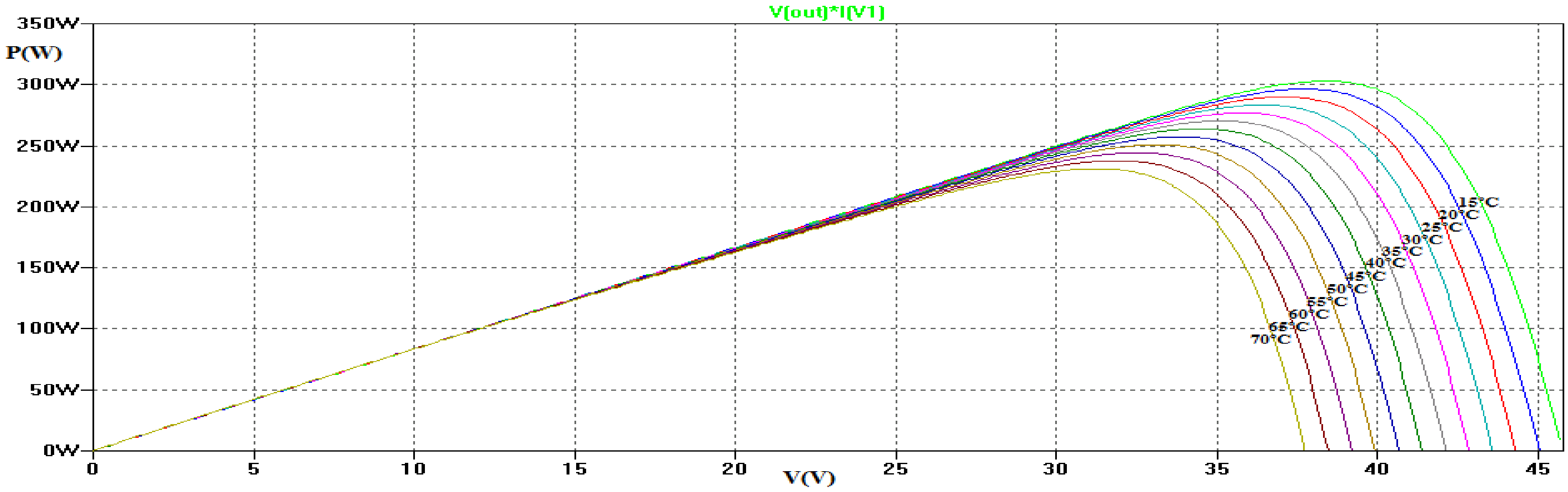
- ***a*:** $a = 1.1$
- **R_s:** $R_s(T) = 1.30 \cdot 10^{-1} - 1.97 \cdot 10^{-3} \Delta T + 2.53 \cdot 10^{-6} \Delta T^2 - 1.07 \cdot 10^{-8} \Delta T^3,$
- **R_{sh}:** $R_{sh}(T) = 1/(3.18 \cdot 10^{-3} + 2.33 \cdot 10^{-4} \Delta T + 1.27 \cdot 10^{-6} \Delta T^2 + 1.33 \cdot 10^{-8} \Delta T^3),$
- **I_0:** $I_0(T) = \exp(-1.98 \cdot 10^{1} + 1.41 \cdot 10^{-1} \Delta T - 4.69 \cdot 10^{-4} \Delta T^2 + 1.13 \cdot 10^{-6} \Delta T^3),$
- **I_{pv}:** $I_{pv}(T) = \left(8.24 + 3.49 \cdot 10^{-3} \Delta T - 1.68 \cdot 10^{-6} \Delta T^2 - 2.41 \cdot 10^{-8} \Delta T^3\right) \frac{G}{G_r}.$

LTSpice Examples

- **LTSpice model. MSP290AS-36.EU Temperature variation (15:5:70) :**

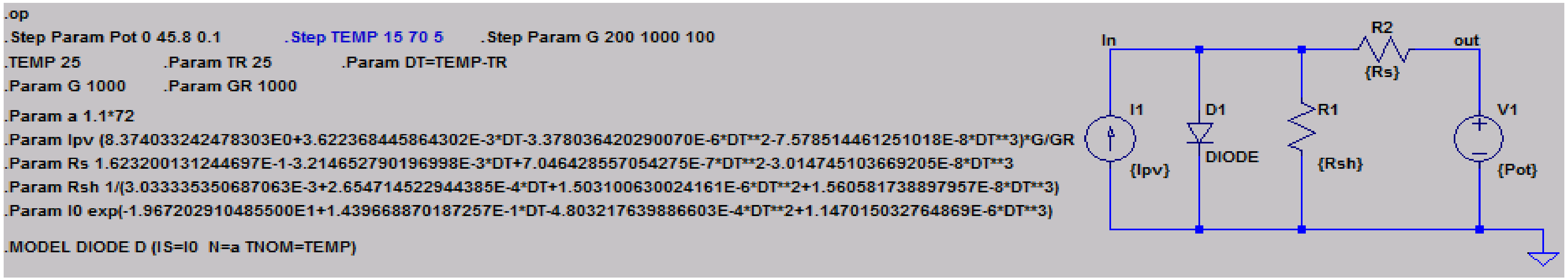
```
.op
.Step Param Pot 0 45.8 0.1        .Step TEMP 15 70 5        .Step Param G 200 1000 100
.TEMP 25          .Param TR 25          .Param DT=TEMP-TR
.Param G 1000     .Param GR 1000
.Param a 1.1*72
.Param Ipv (8.374033242478303E0+3.622368445864302E-3*DT-3.378036420290070E-6*DT**2-7.578514461251018E-8*DT**3)*G/GR
.Param Rs 1.623200131244697E-1-3.214652790196998E-3*DT+7.046428557054275E-7*DT**2-3.014745103669205E-8*DT**3
.Param Rsh 1/(3.033335350687063E-3+2.654714522944385E-4*DT+1.503100630024161E-6*DT**2+1.560581738897957E-8*DT**3)
.Param I0 exp(-1.967202910485500E1+1.439668870187257E-1*DT-4.803217639886603E-4*DT**2+1.147015032764869E-6*DT**3)
.MODEL DIODE D (IS=I0  N=a TNOM=TEMP)
```

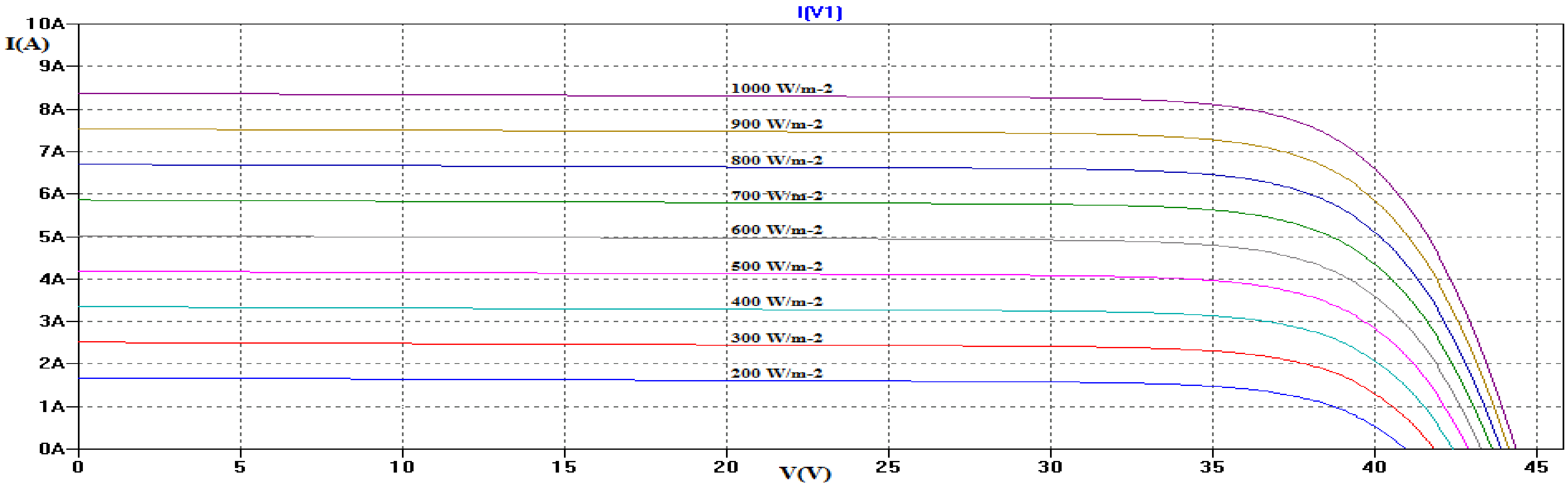




LTSpice Examples

- **LTSpice model. MSP290AS-36.EU Irradiance variation (200:100:1000) :**

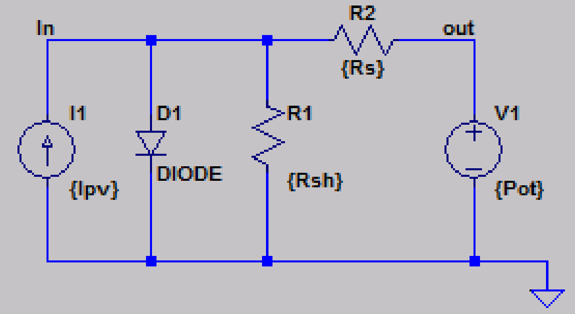


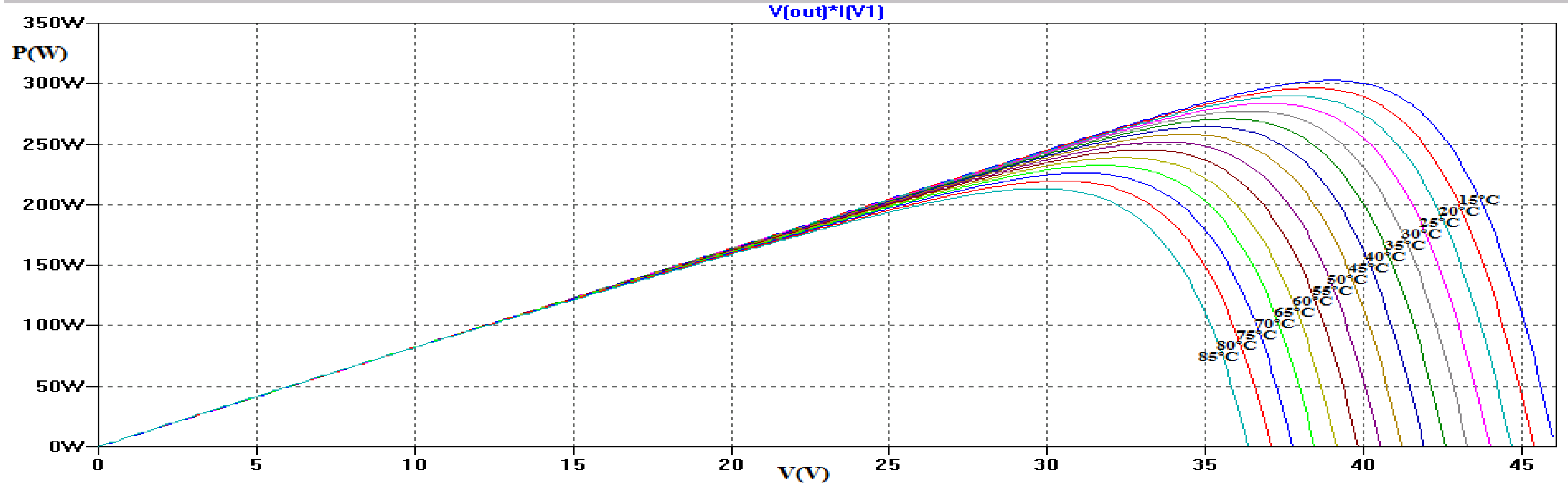


LTSpice Examples

- **LTSpice model. MSMD290AS-36.EU Temperature variation (15:5:85) :**

```
.op
.Step Param Pot 0 46.1 0.1        .Step TEMP 15 85 5     .Step Param G 200 1000 100
.TEMP 25             .Param TR 25          .Param DT=TEMP-TR
.Param G 1000     .Param GR 1000

.Param a 1.1*72
.Param Ipv (8.243373734964720+3.493669589695205E-3*DT-1.682220698209894E-6*DT**2-2.411861663329314E-8*DT**3)*G/GR
.Param Rs 1.301691164106565E-1-1.969177512608361E-3*DT+2.530824231968095E-6*DT**2-1.068189326561281E-8*DT**3
.Param Rsh 1/(3.175975114505289E-3+2.332234796430174E-4*DT+1.266815894525395E-6*DT**2+1.331661146468357E-8*DT**3)
.Param I0 exp(-1.986584201191201E1+1.408868299494937E-1*DT-4.690212078627850E-4*DT**2+1.130804108563038E-6*DT**3)

.MODEL DIODE D (IS=I0  N=a TNOM=TEMP)
```

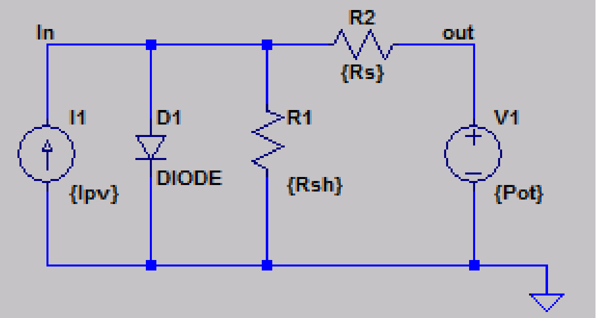


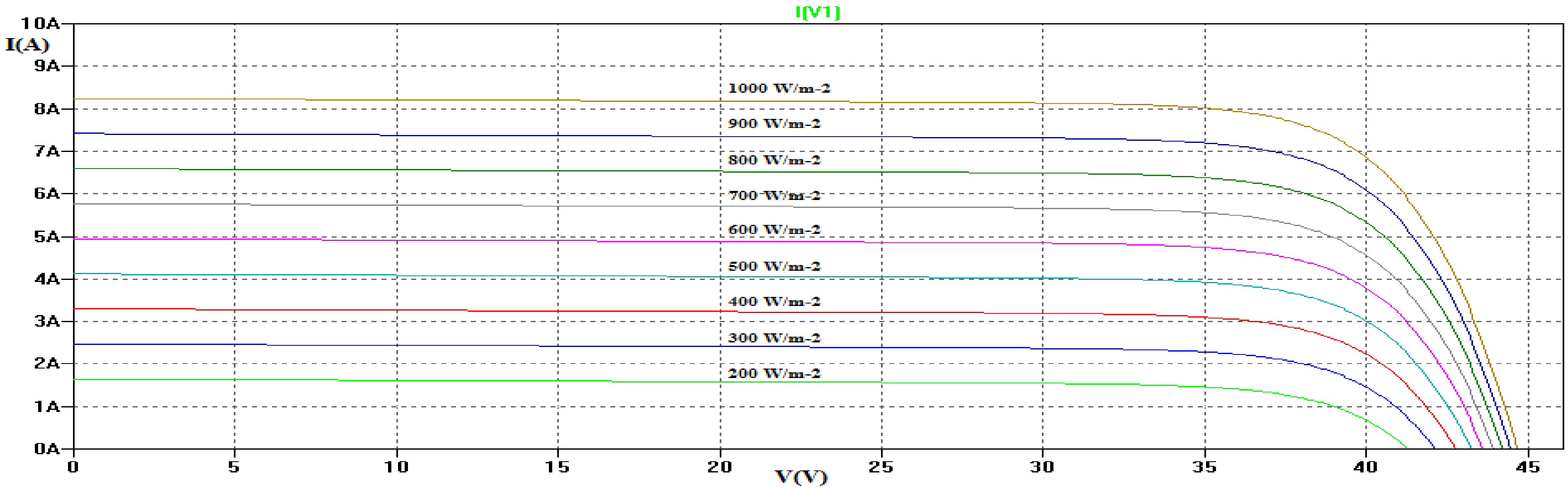


LTSpice Examples

- **LTSpice model. MSMD290AS-36 Irradiance variation (200:100:1000) :**

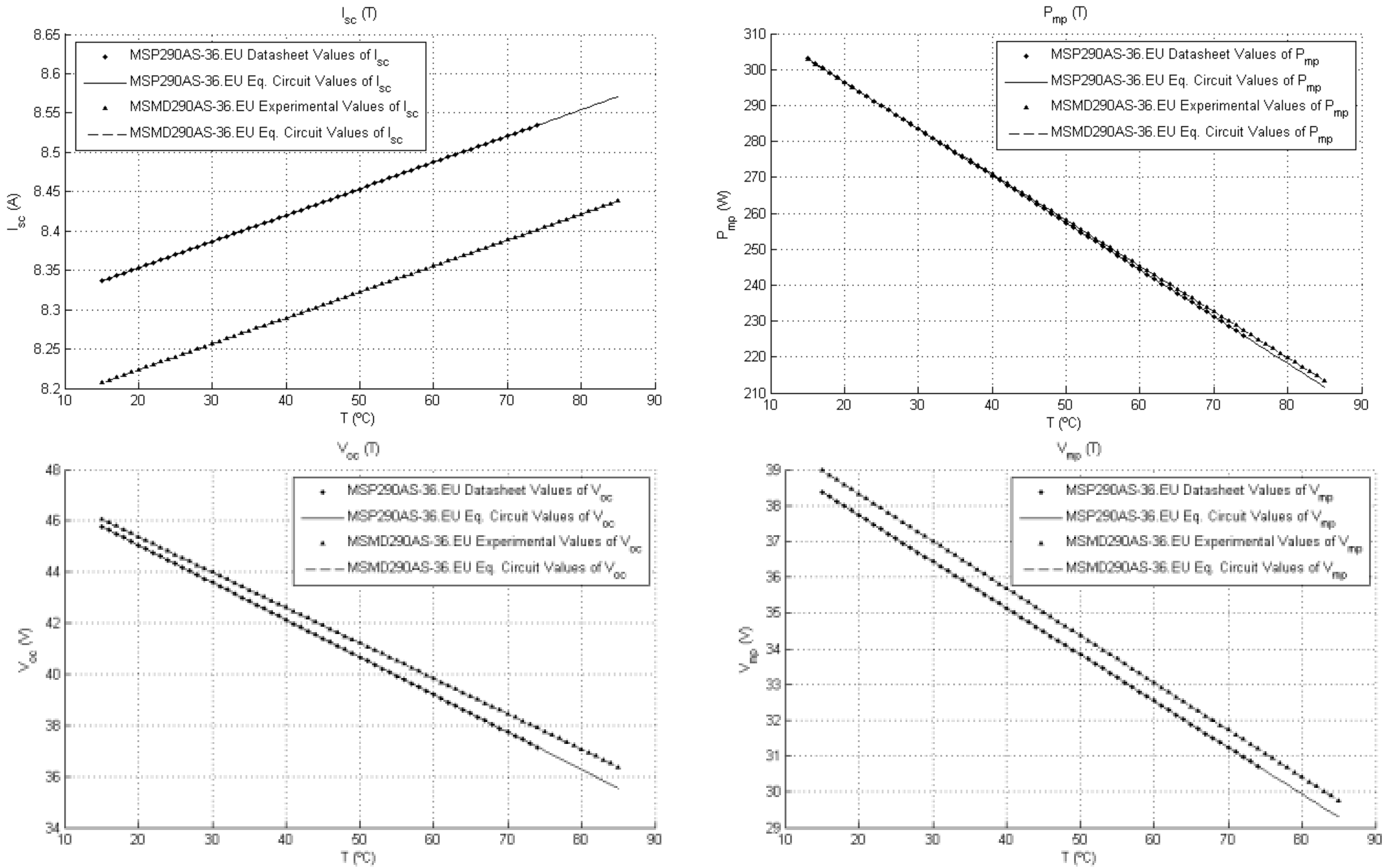
```
.op
.Step Param Pot 0 46.1 0.1        .Step TEMP 15 85 5     .Step Param G 200 1000 100
.TEMP 25             .Param TR 25           .Param DT=TEMP-TR
.Param G 1000      .Param GR 1000

.Param a 1.1*72
.Param Ipv (8.243373734964720+3.493669589695205E-3*DT-1.682220698209894E-6*DT**2-2.411861663329314E-8*DT**3)*G/GR
.Param Rs 1.301691164106565E-1-1.969177512608361E-3*DT+2.530824231968095E-6*DT**2-1.068189326561281E-8*DT**3
.Param Rsh 1/(3.175975114505289E-3+2.332234796430174E-4*DT+1.266815894525395E-6*DT**2+1.331661146468357E-8*DT**3)
.Param I0 exp(-1.986584201191201E1+1.408868299494937E-1*DT-4.690212078627850E-4*DT**2+1.130804108563038E-6*DT**3)

.MODEL DIODE D (IS=I0  N=a TNOM=TEMP)
```





Accuracy

- The results of the simulations performed reproduce with high accuracy the experimental results for the characteristic points, regarding the temperature variations, included in the datasheet.



VIII Conclusions

- New analytical methodology
  - Explicit
  - Non-Iterative
  - Straight forward
- Parameter identification of the equivalent circuit for a solar cell/panel.
- Meeting manufacturer datasheet for any
  - Temperature
  - Illumination

VIII Conclusions

Possible applications:

- End users with little calculation and testing resources
- Analysis that imply profuse calculations.
- Determination of initial values for numerical methods
- Construct realistic models of solar panels that can be used in simulations of MPPT algorithms


Thank
You
Merci
Gracias
Arigato
Dankscheen
Mehrbani
Ekhmet
bolzïn
Tashakkur
Maketai
hui
Sanco
Shukuria
Paldies
Hatur
Wabeeja
Medawagse
Mersi
unalchéesh
Tingki
Komapsumnida
atu
aniha
Maake
Denkauja
Aguyje
Spassibo
gozaimashita
Fakaaue
Spasibo
Nenachalhya
Baiika
Yuspagarätam
Minmonchar
Atto
Gaejtho
Yaqhanyelay
Efcharisto
Gui
Maiteka
ekoju
Tavtapuch
suksama
Sikomo
Shukria
lah
Merastawhy
Dhanyabaad
Chaltu
nuhun
Biyan
Grazie
Snachalhuya
Juspaxar

Any questions?

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