POLITECNICO DI MILANO

Dipartimento di
Ingegneria Civile e Ambientale

Stochastic Effects on the Dynamics of a Resonant MEMS Magnetometer: a Monte Carlo Investigation

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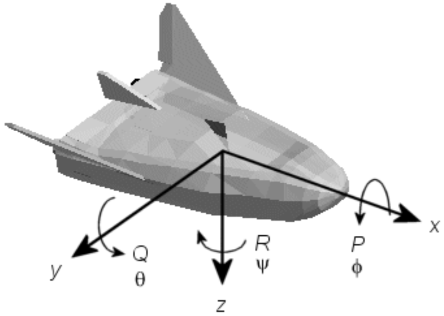
Magnetometers: engineering motivation

The earth magnetic field as a vector quantity

X,Y, Z components

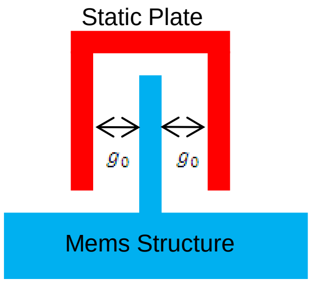
Orientation determination

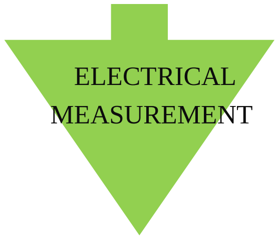


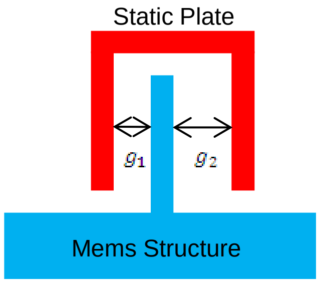
Working principle

The external magnetic fields

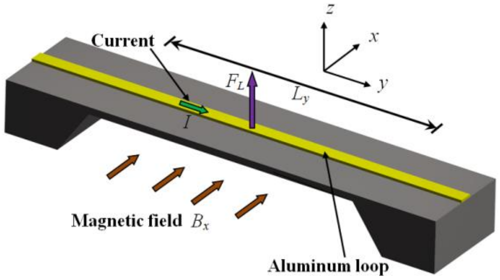
Change of the resonating system configuration (displacement, eigen-frequency)



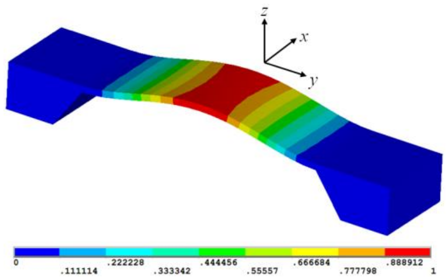




The magnetic field component is defined as a function of the configuration change



$$F_l = IB_x l_y$$



Design requirements

Design Demands

1 High sensitivity

+

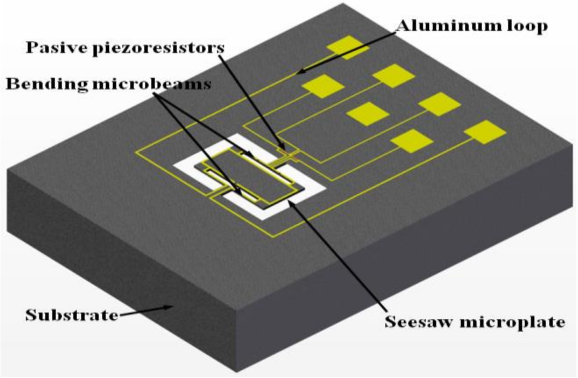
2 Process limitations

+

3 Mechanical acceleration filtering

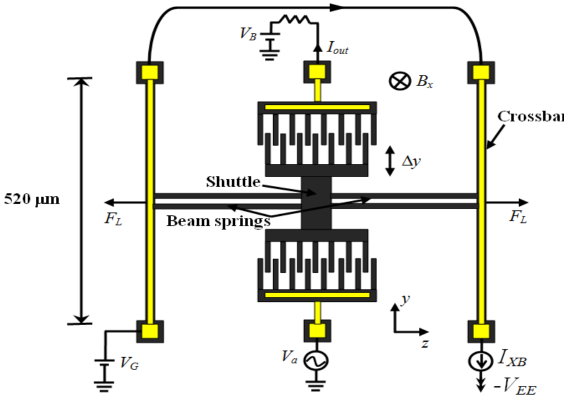
The goal of our designs

Some Designs In The Literature



Herrera-May et al

1 2 3



Behraad Bahreyni

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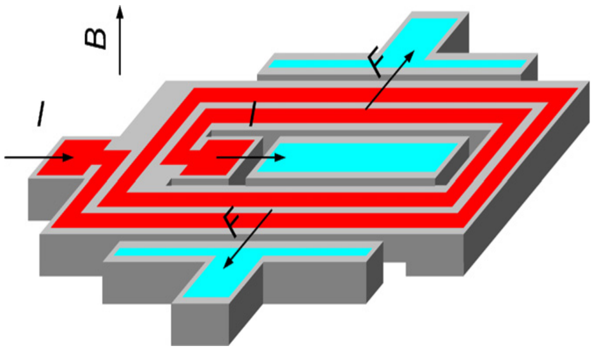
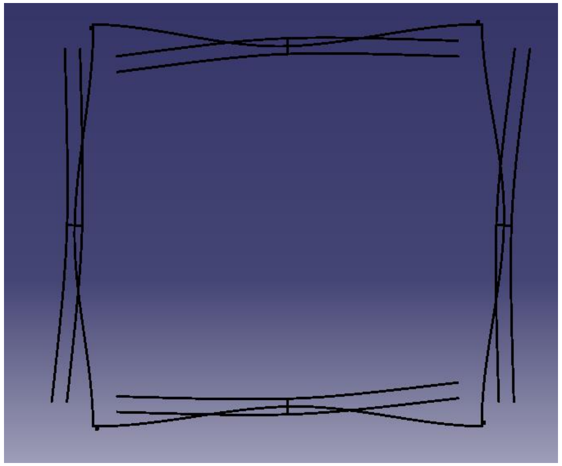
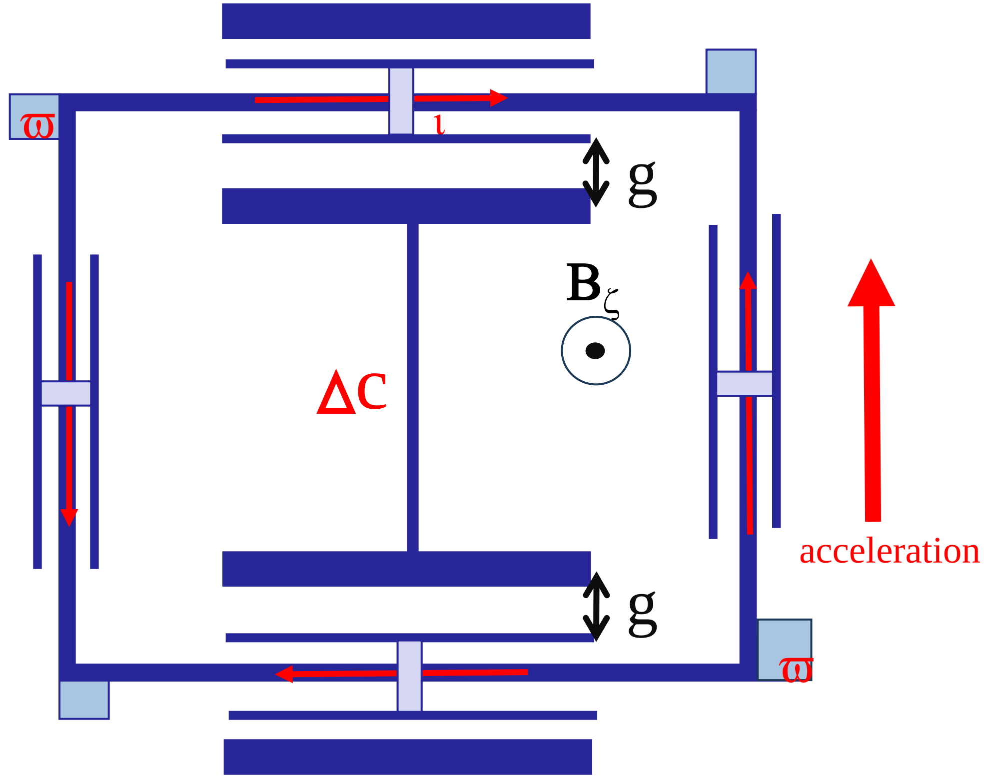


VTT technical research center

1 2 3

New design ideas (Capacitive parallel plates)





d_m Displacement due to magnetic field

d_a Displacement due to acceleration

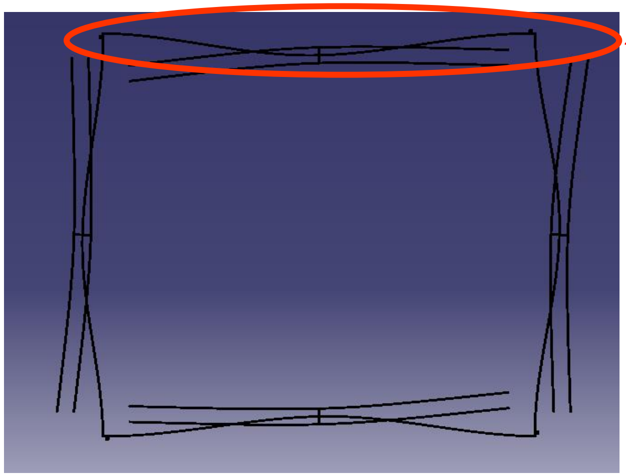
Absence of acceleration $\Delta c = f(+d_m + d_m)$

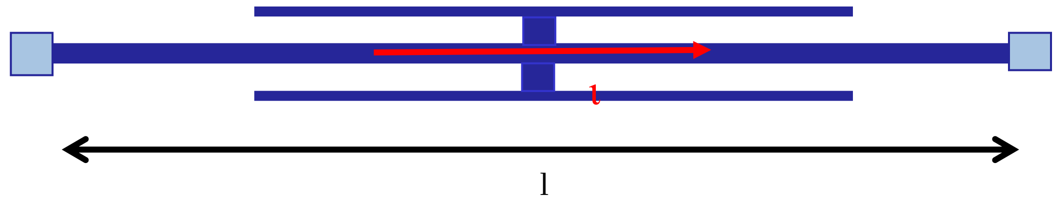
Presence of acceleration $\Delta c = f(+d_m + \not{d}_a - \not{d}_a + d_m)$

$\Rightarrow \Delta c = f(2d_m)$

Mechanical acceleration filtered out

Multi-physics modeling





Hamilton's principle

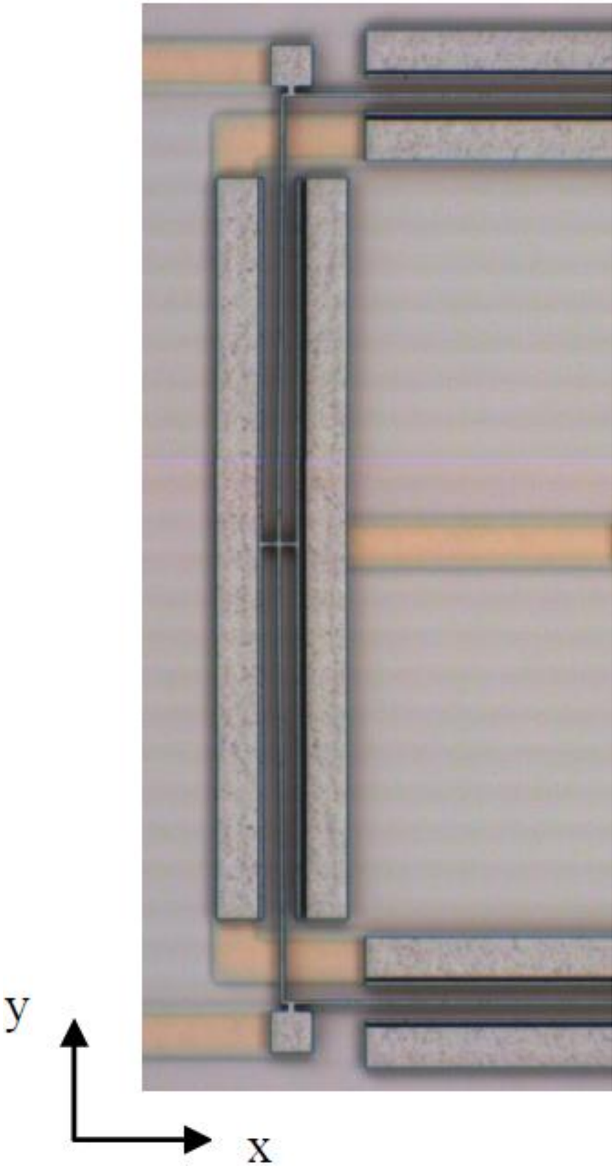
$$m\frac{\partial^2 v}{\partial t^2} + c_d\frac{\partial v}{\partial t} + EI\frac{\partial^4 v}{\partial x^4} + p\frac{\partial^2 v}{\partial x^2} - \frac{EA}{2L}\frac{\partial^2 v}{\partial x^2}\int_0^L \left(\frac{\partial v}{\partial x}\right)^2 dx - F = 0$$

Thermo electro magneto mechanical problem

Joule effect

Lorentz force and eddy current

Multi-physics modeling



Applying Galerkin method to Hamilton's principle

First Eigen mode

$$\frac{d^2q}{dt^2} + 2\mu\frac{dq}{dt} + {\omega_0}^2 q + Kq^3 = F_0 \cos \omega t$$

Clamped - Clamped

$$2\mu = \frac{c_d}{\rho A} \quad {\omega_0}^2 = \frac{16EI\pi^4}{3\rho A l^4} - \frac{El\alpha_s\rho_e i^2\pi^2}{9\rho A^2 k_H} \quad K = \frac{E\pi^4}{3\rho l^4} \quad F_0 = \frac{4iB_z}{3\rho A}$$

One degree of freedom equivalent system (Duffing nonlinear equation)

Multi-physics modeling

$$\frac{d^2q}{dt^2} + 2\eta\frac{dq}{dt} + \omega_0^2 q + Kq^3 = F_0 \cos\omega t$$

Maximum amplitude of oscillation is given by the solution of

$$\left(\frac{F_0}{\omega_0{}^2}\right)^2 = \left(2\left(1-\frac{\omega}{\omega_0}\right)Z_{MAX} + \frac{3}{4}\frac{K}{\omega_0{}^2}Z_{MAX}{}^3\right)^2 + \left(\frac{2\mu}{\omega_0}Z_{MAX}\right)^2$$

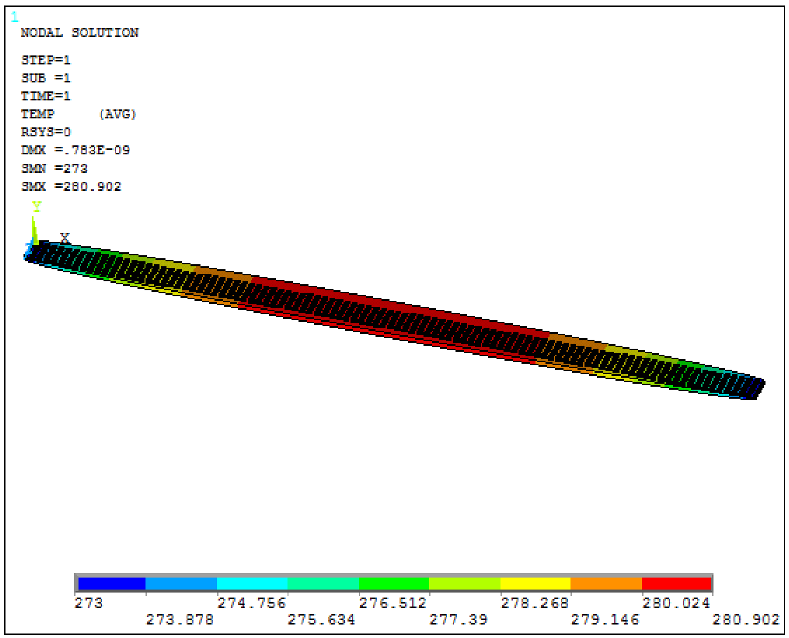
Current frequency $\omega = \omega_0$

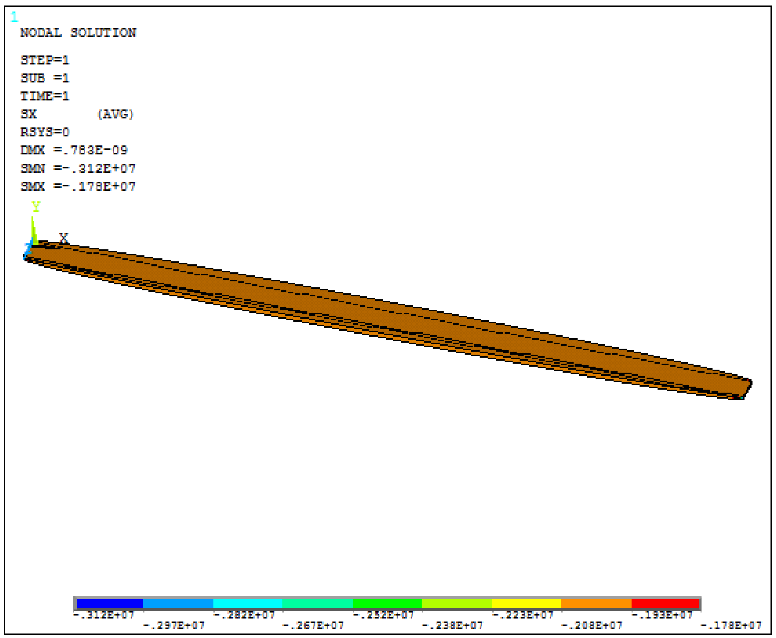
$$Z_{Max} = \left[\left(\left(\frac{64F_0{}^4}{81K^4} + \frac{262144\mu^6\omega_0{}^6}{19683K^6}\right)^{\frac{1}{2}} + \frac{8F_0{}^2}{9K^2}\right)^{\frac{1}{3}} - \frac{64\mu^2\omega_0{}^2}{27K^2\left(\left(\frac{64F_0{}^4}{81K^4} + \frac{262144\mu^6\omega_0{}^6}{19683K^6}\right)^{\frac{1}{2}} + \frac{8F_0{}^2}{9K^2}\right)^{\frac{1}{3}}}\right]^{\frac{1}{2}}$$

Multi-physics simulation

To check the model, Ansys multi-physics simulations were performed

Static thermo-electro-structural analysis





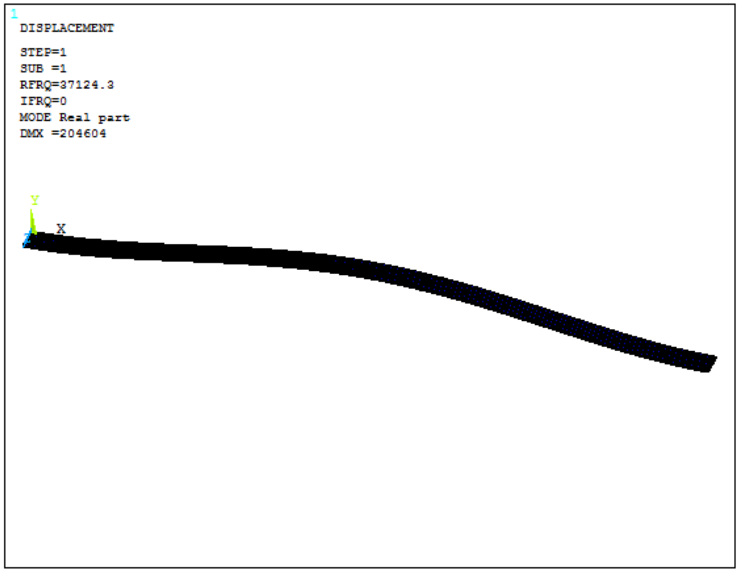
***Temperature field*:**

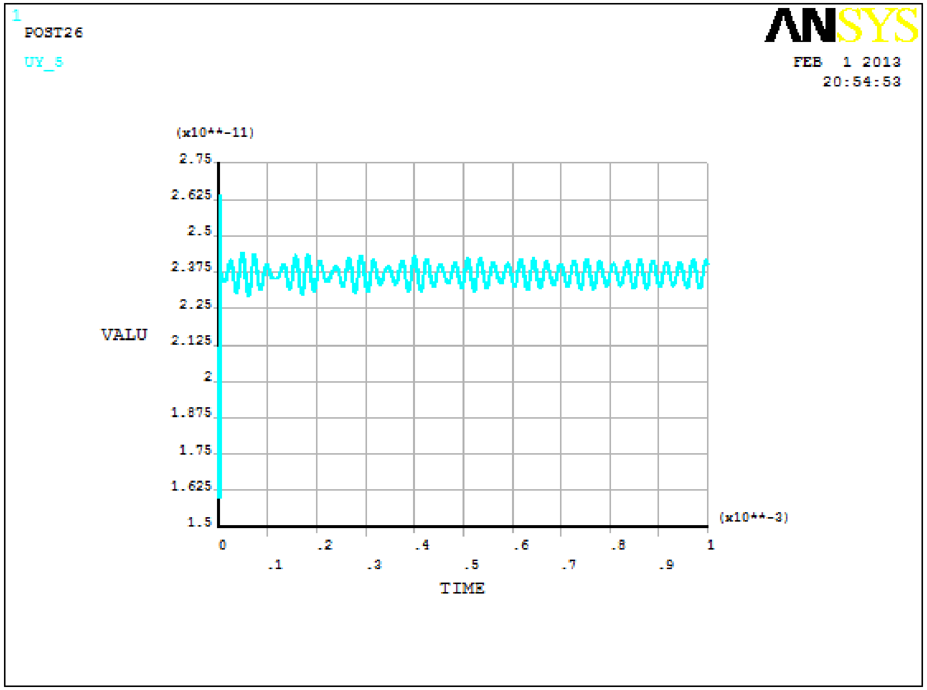
$$T_{Max} = 280.90 \text{ (Ansys)}$$
$$280.90 \text{ (Analytical)}$$

***Stress field*:**

$$\sigma_{th} = -1.9756E+06\ Pa \text{ (Ansys)}$$
$$-1.9757E+06\ Pa \text{ (Analytical)}$$

Multi-physics simulation





Prestressed modal analysis:

$$\boldsymbol{f = 37124\ Hz}\text{(Ansys)}$$
$$\boldsymbol{37334\ Hz}\ \text{(Analytical)}$$

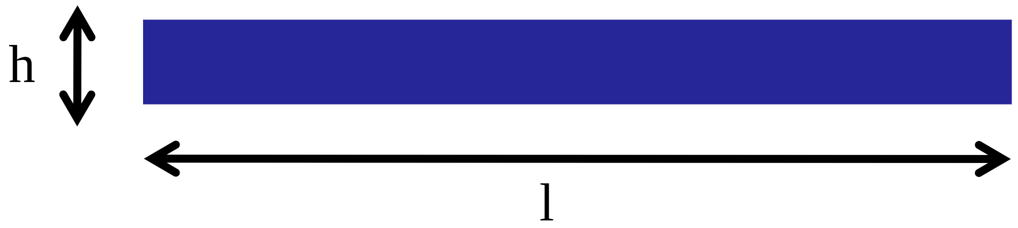
$$\boldsymbol{max_{Dis_{midpoint}} = 2.41e - 11}\text{(Ansys)}$$
$$\boldsymbol{2.36e - 11}\text{(Analytical)}$$

Parameter optimization

Sensor's performance

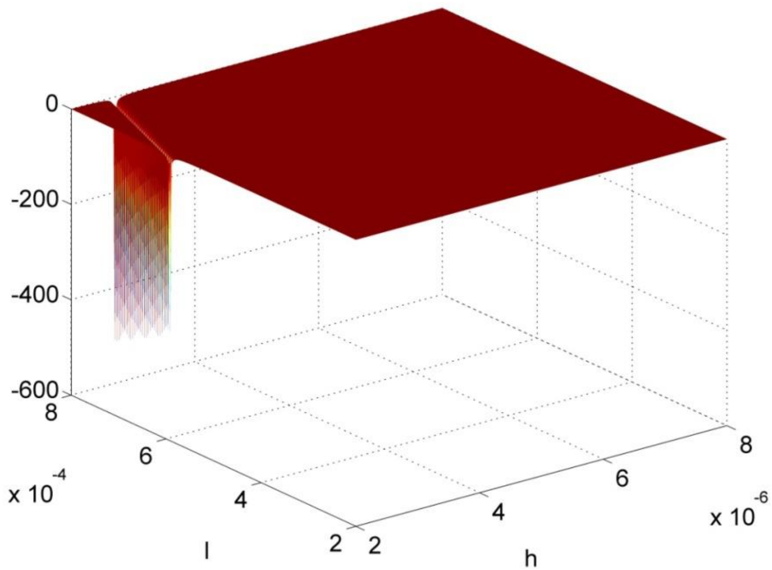
- Sensitivity (Maximum amplitude)
- Power consumption (Minimum electrical resistance)

To have an optimal device, we perform a multi objective optimization procedure, consisting of a structural objective function (Z_{Max}) and an electrical objective function (R_{elec})

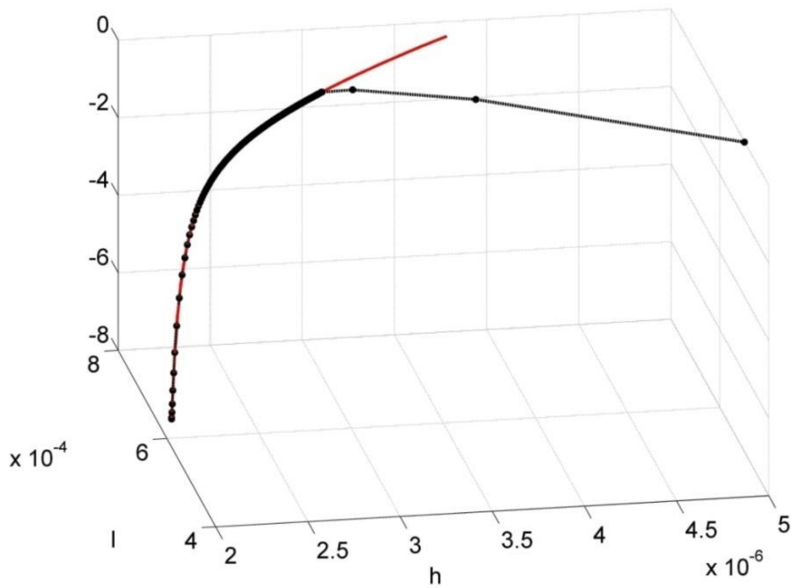


Optimal *h*, *l* of the multi-physics solution for the fundamental component

Optimal design for dynamic compliance



Unconstrained objective function

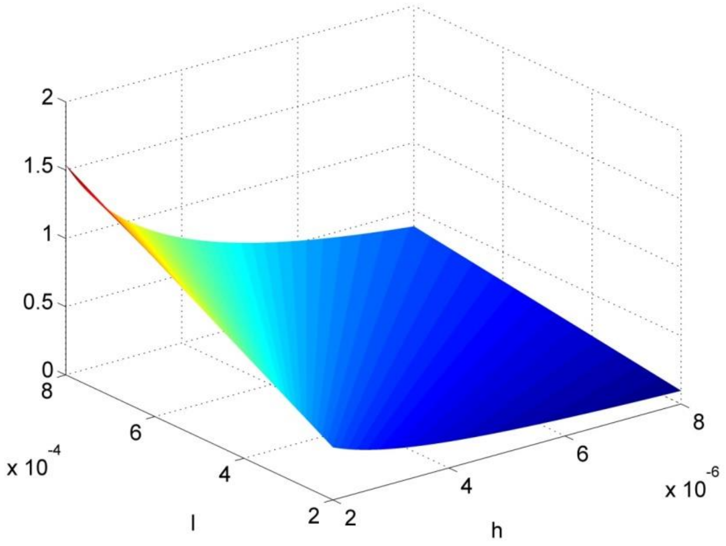


Constrained objective function (red line), and optimal path followed by the minimization algorithm (black dotted line)

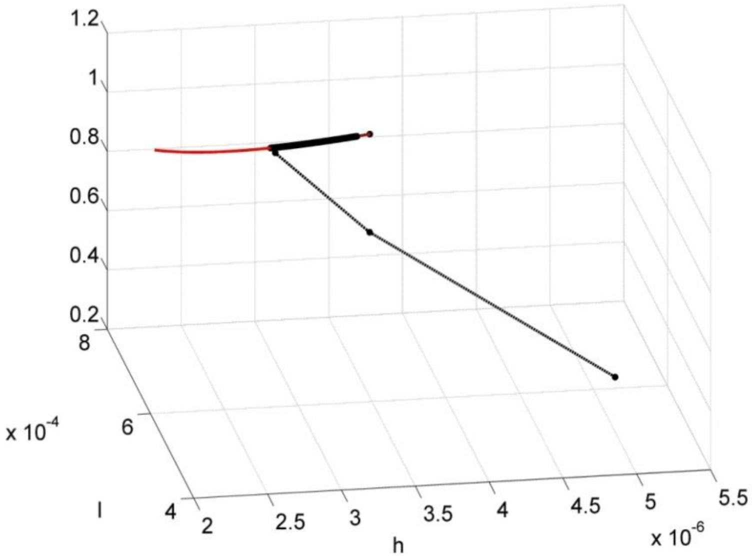
Except for the first iterations that move from a point violating the prescribed equality constraint, the optimizer provides a set of feasible solutions to the arising sub-problems.

It finally ends with the expected global minimum h=2μm, l=580.9 μm

Optimal design for power consumption



Unconstrained objective function



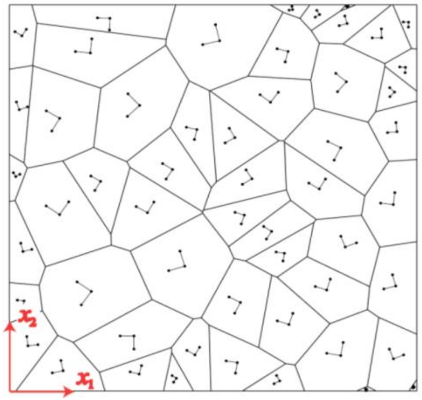
Constrained objective function (red line), and optimal path followed by the minimization algorithm (black dotted line)

Except for the first iterations that move from a point violating the prescribed equality constraint, the optimizer provides a set of feasible solutions to the arising sub-problems.

It finally ends with the expected global minimum h=3.8μm, l=798.1 μm

Sources of uncertainty in the model and probability density functions (PDFs)

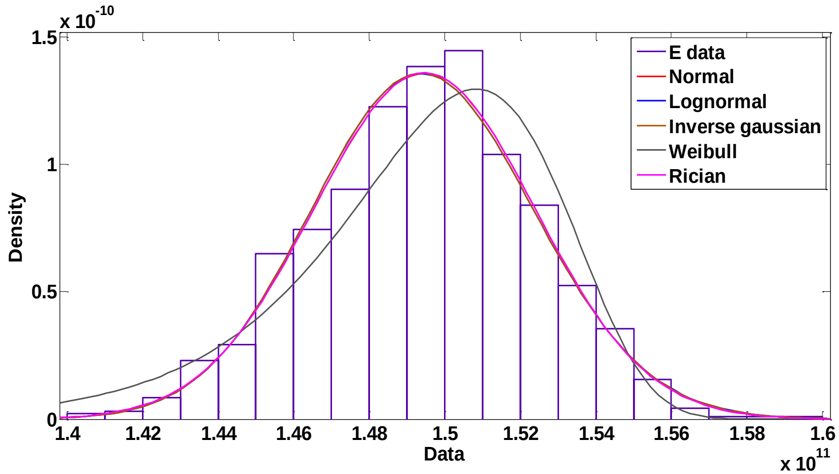
Young's modulus



RVE of polysilicon

E distribution

Maximum likelihood

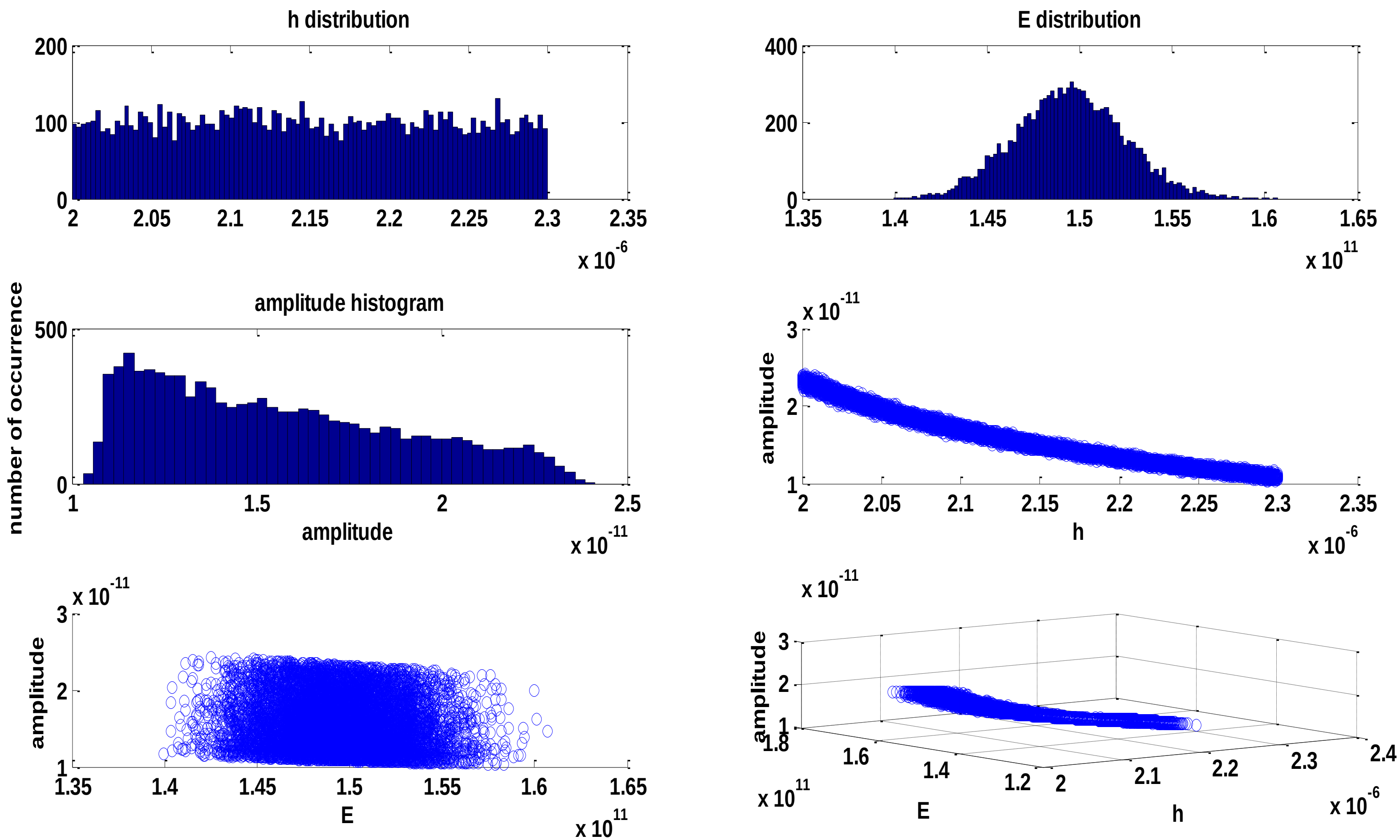


PDF of E

Over etching

uncertainty in beam width due to process (uniform PDF)

Monte Carlo simulation: effect of uncertainties on Z_{Max} around the optimal values



Planned activities

- Validate the multi-physics model by a commercial FEM code (either Ansys or Comsol)

- Adopt a topology optimization approach to find the optimal shape of the base component (to search for the optimal shape, not only beam shaped structures, but also tapered, curved and any other arbitrary shape)

- Performing the experimental tests on the devices