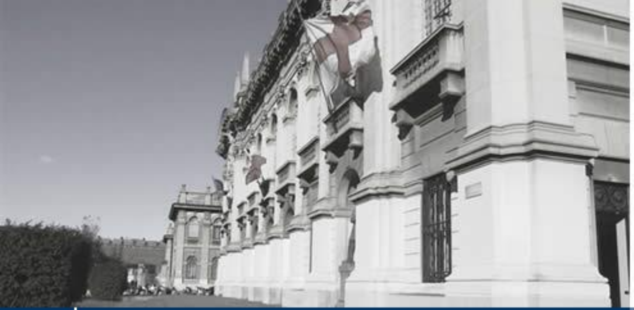

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Hybrid Reduced-Order Modeling and Particle-Kalman Filtering for the Health Monitoring of Flexible Structures

Giovanni Capellari[1], Saeed Eftekhar Azam[2], Stefano Mariani[1]

[1] Politecnico di Milano, Dipartimento di Ingegneria Civile e Ambientale
[2] University of Thessaly, Department of Mechanical Engineering

Singapore: reduce risk related to damage assessment after natural events

Göta Bridge (Sweden): safely extend the lifetime of the ageing bridge

Halifax Metro Center (Canada): Making use of existing structural reserves to allow increased snow and equipment loads on the roof

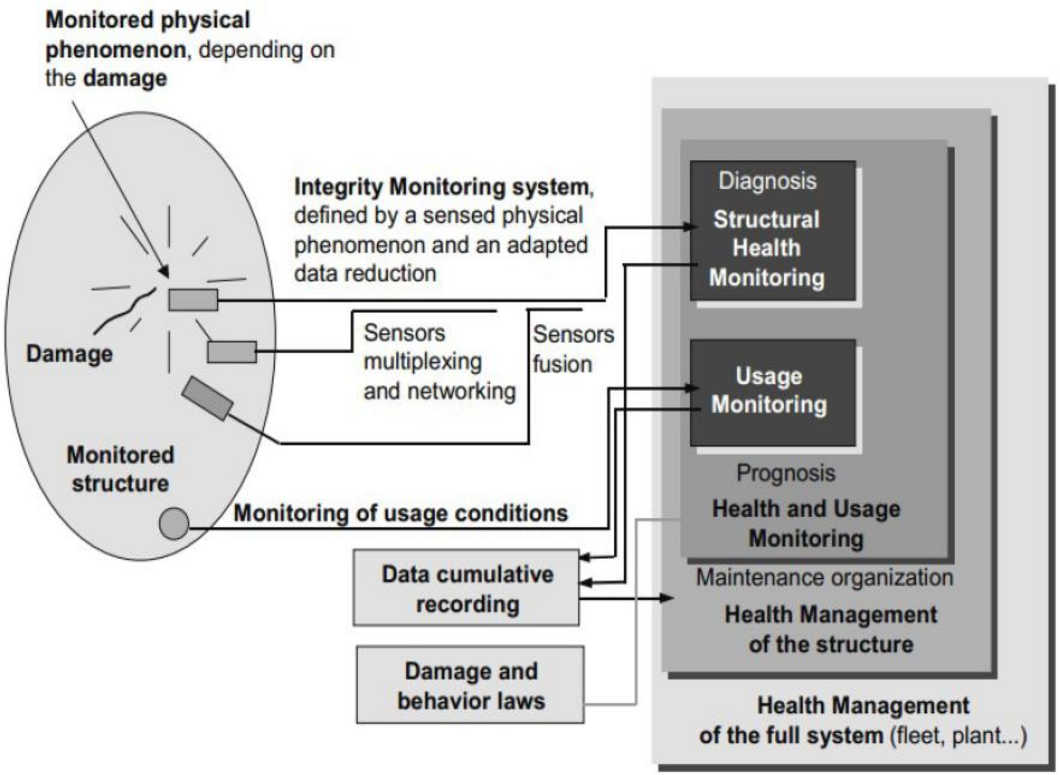
I35W Bridge (USA): reassure public on the safety of the new bridge, support the rapid construction schedule, provide data to local researchers

Damage Identification

1. Observation of the system through periodically spaced measurements

2. Selection of a certain number of features and indexes in order to identify the damage

3. Estimation of the aforementioned indexes using an inverse identification method based on the observations



Balageas et al. 2006

Objective and motivations

Damage identification and localization

Observations $\mathbf{y}(\mathrm{t})$	$\Rightarrow$	Damage indexes $d_1, \ldots, d_{N_p}$

Requirements

- Reduced computational cost $\Rightarrow$ Model order reduction; Hybrid Extended Kalman Particle Filter
- On-line tracking $\Rightarrow$ Dual estimation
- Coupling with FE commercial code $\Rightarrow$ Use of reference substructures

Linear dynamic equation:

Full order *n* model

$$\mathbf{M}\ddot{\mathbf{u}}(t)+\mathbf{D}\dot{\mathbf{u}}(t)+\mathbf{K}\mathbf{u}(t)=\mathbf{F}(t)$$

$$\mathbf{u}\in\mathbb{R}^n,\qquad \mathbf{M},\mathbf{D},\mathbf{K}\in\mathbb{R}^{n\times n}$$

$\Rightarrow$

Reduced order *l*<*n* model

$$\mathbf{M}_r\ddot{\boldsymbol{\alpha}}(t)+\mathbf{D}_r\dot{\boldsymbol{\alpha}}(t)+\mathbf{K}_r\boldsymbol{\alpha}(t)=\mathbf{F}_r(t)$$

$$\boldsymbol{\alpha}\in\mathbb{R}^l,\qquad \mathbf{M}_r,\mathbf{D}_r,\mathbf{K}_r\in\mathbb{R}^{l\times l}$$

- Proper orthogonal decomposition based methods (POD)

$\Downarrow$

Optimization statement

find the projection $\boldsymbol{\Pi}_r=\begin{bmatrix}\boldsymbol{\phi}_1 & \cdots & \boldsymbol{\phi}_n\end{bmatrix}\in\mathbb{R}^{n\times n}$

such that $\int_0^T ||\mathbf{u}(t)-\boldsymbol{\Pi}_r\mathbf{u}(t)||_2^2 dt$ is minimized

ϕ_i **:Proper Orthogonal Modes** (POM)

Calculation of POMs: **Singular Value Decomposition**

Any given matrix U can be decomposed by:

$$\boxed{\mathbf{U} = \mathbf{V\Sigma W}}$$

$\mathbf{U} = (\mathbf{u}(t_i), ..., \mathbf{u}(t_{N_{snap}})) \in \mathbb{R}^{n \times N_{snap}}$: Snapshot matrix

σ_i : Singular values

$\boldsymbol{v}_i$: Left singular vectors

The *i-th* POM can be calculated through:

$$\boxed{\boldsymbol{\phi}_i = \frac{1}{\sigma_i}\mathbf{U}\boldsymbol{v}_i}$$

Level of information:

$$I(l) = 1 - \epsilon_r(l) = \frac{||\mathbf{U} - \Pi_l \mathbf{U}||_F^2}{||\mathbf{U}||_F^2} = \frac{\sum_{i=1}^{l} \sigma_i^2}{\sum_{i=1}^{n} \sigma_i^2}$$

Galerkin-based Projection

The vector $\mathbf{u}(t)$ can be expressed as a linear combination of ϕ_i :

$$\mathbf{u}(t) = \sum_{i=1}^{n} \phi_i y_i(t) = \mathbf{\Phi y}(t)$$

$$\mathbf{u}(t) \approx \sum_{i=1}^{l} \phi_i \alpha_{l_i}(t) = \mathbf{\Phi}_l \boldsymbol{\alpha}(t) \qquad l \ll n$$

$$\mathbf{\Phi}_l = [\phi_1 \cdots \phi_l]$$

$$\boxed{\mathbf{M\ddot{u}}(t) + \mathbf{D\dot{u}}(t) + \mathbf{Ku}(t) = \mathbf{F}(t)}$$

$$\downarrow$$

$$\boxed{\mathbf{M\Phi}_l\ddot{\boldsymbol{\alpha}}(t) + \mathbf{D\Phi}_l\dot{\boldsymbol{\alpha}}(t) + \mathbf{K\Phi}_l\boldsymbol{\alpha}(t) - \mathbf{F}(t) = \mathbf{r}(t)}$$

From the orthogonality condition $\mathbf{\Phi}_l^T\mathbf{r}(t) = \mathbf{0}$, we get:

$$\boxed{\mathbf{\Phi}_l^T\mathbf{M\Phi}_l\ddot{\boldsymbol{\alpha}}(t) + \mathbf{\Phi}_l^T\mathbf{D\Phi}_l\dot{\boldsymbol{\alpha}}(t) + \mathbf{\Phi}_l^T\mathbf{K\Phi}_l\boldsymbol{\alpha}(t) - \mathbf{\Phi}_l^T\mathbf{F}(t) = \mathbf{0}}$$

$$\downarrow$$

$$\boxed{\mathbf{M}_l\ddot{\boldsymbol{\alpha}}(t) + \mathbf{D}_l\dot{\boldsymbol{\alpha}}(t) + \mathbf{K}_l\boldsymbol{\alpha}(t) = \mathbf{F}_l(t)}$$

Dual Estimation Problem

Discrete-time state space equations:

State vector

Process noise

Process model $\mathbf{x}_k = \mathbf{f}_k(\mathbf{x}_{k-1}) + \mathbf{w}_k$

Measurement model $\mathbf{y}_k = \mathbf{h}_k(\mathbf{x}_k) + \mathbf{v}_k$

Observations

Measurement noise

Dual estimation:

Dynamic process

$$\mathbf{x}_k = \begin{bmatrix} \mathbf{u}_k \\ \boldsymbol{\theta}_k \end{bmatrix}$$

Parameters

Hypothesis:

$$\mathbf{w}_k = \mathrm{WN}(0, \mathbf{W}) \qquad \mathbf{v}_k = \mathrm{WN}(0, \mathbf{V}) \qquad \mathbf{f}_k, \mathbf{h}_k \text{ linear}$$

1. Initialization $(t_k = t_0)$

$$\hat{\mathbf{x}}_0 = E[\mathbf{x}_0]$$

$$\mathbf{P}_0 = E[(\mathbf{x}_0 - E[\mathbf{x}_0])(\mathbf{x}_0 - E[\mathbf{x}_0])^T]$$

2. Recursive computation $(t_k = t_1, ..., t_N)$

   (a) Prediction stage

$$\hat{\mathbf{x}}_k^- = \mathbf{F}_k \hat{\mathbf{x}}_{k-1}^-$$

$$\mathbf{P}_k^- = \mathbf{F}_k \mathbf{P}_{k-1} \mathbf{F}_k^T + \mathbf{W}_k$$

   (b) Updating stage

$$\mathbf{G}_k = \mathbf{P}_k^- \mathbf{H}_k^T (\mathbf{H}_k \mathbf{P}_k^- \mathbf{H}_k^T + \mathbf{V}_k)^{-1}$$

$$\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_k^- + \mathbf{G}_k(\mathbf{y}_k - \mathbf{H}_k \hat{\mathbf{x}}_k^-)$$

$$\mathbf{P}_k = \mathbf{P}_k^- - \mathbf{G}_k \mathbf{H}_k \mathbf{P}_k^-$$

If $\mathbf{f}_k, \mathbf{h}_k$ non-linear $\rightarrow$

$$\mathbf{F}_k = \nabla_{\mathbf{x}} \mathbf{f}_k(\mathbf{x})|_{\mathbf{x}=\mathbf{x}_{k-1}}$$

$$\mathbf{H}_k = \nabla_{\mathbf{x}} \mathbf{h}_k(\mathbf{x})|_{\mathbf{x}=\mathbf{x}_{k-1}}$$

Extended Kalman Filter vs. Particle Filter

Drawbacks of the Extended Kalman Filter

- linearization error
- computational cost of the Jacobian matrix
- non-holonomic systems

Particle Filter

- no assumptions on the probability distribution function are required
- generation of samples and relative weights from $p(\mathbf{x}_{0:k}|\mathbf{y}_{1:k})$

Drawbacks of the Particle Filter

- number of samples
- degeneracy of the weights

Solutions

- sub-optimal importance function $p(\mathbf{x}_{0:k}|\mathbf{x}^i_{1:k})$
- re-sampling

1. Initialization ($t_k = t_0$)

$$\hat{\mathbf{x}}_0 = E[\mathbf{x}_0] \qquad \mathbf{x}_0^i = \hat{\mathbf{x}}_0 \qquad \omega_0^i = p(\mathbf{y}_0|\mathbf{x}_0)$$

2. Recursive computation ($t_k = t_1, ..., t_N$)

   (a) Prediction stage ($i = 1, ..., N_s$)

$$\mathbf{x}_k^i \sim p(\mathbf{x}_k|\mathbf{x}_{k-1}^i)$$

$$\omega_k^i = \omega_{k-1}^i p(\mathbf{y}_k|\mathbf{x}_k^i)$$

   (b) Resampling stage ($i = 1, ..., N_s$)

$$u_i \sim U\,[0,1]$$

$$\text{find} \quad m \quad s.t. \sum_{i=1}^{m-1} \omega_k^i < u_i \leq \sum_{i=1}^{m} \omega_k^i$$

$$\bar{\mathbf{x}}_k^i = \mathbf{x}_k^m$$

$$\bar{\omega}_k^{*i} = \frac{1}{N_s}$$

   (c) Updating stage

$$\hat{\mathbf{x}}_k = \sum_{i=1}^{N_s} \bar{\omega}_k^i \bar{\mathbf{x}}_k^i$$

State vector:

$$\mathbf{x} = \left[\boldsymbol{\alpha}\ \dot{\boldsymbol{\alpha}}\ \ddot{\boldsymbol{\alpha}}\ \mathbf{d}\right]^T$$

Coordinates of the reduced system

Damage indexes:

$$E_i = (1 - d_i)E$$

Stiffness reduction

Newmark explicit integration method

Process model:

$$\mathbf{x}_k = \begin{bmatrix} \bar{\boldsymbol{\alpha}}_k \\ \mathbf{d}_k \end{bmatrix} = \begin{bmatrix} \mathbf{f}_k^{\boldsymbol{\alpha}}(\bar{\boldsymbol{\alpha}}_{k-1}, \mathbf{d}_{k-1}) \\ \mathbf{d}_{k-1} \end{bmatrix} = \mathbf{f}_k(\mathbf{x}_{k-1}) + \mathbf{w}$$

$$\mathbf{F}_k = \nabla_{\mathbf{x}} \mathbf{f}_k(\mathbf{x})|_{\mathbf{x}=\mathbf{x}_{k-1}}$$

$$\mathbf{f}_k^{\boldsymbol{\alpha}} = \begin{bmatrix} \mathbf{I} - \beta\Delta t^2 \mathbf{M}_{k-1}^{-1}\mathbf{K}_{k-1} & \Delta t\mathbf{I} - \beta\Delta t^3 \mathbf{M}_{k-1}^{-1}\mathbf{K}_{k-1} & \Delta t^2(1/2-\beta)[\mathbf{I} - \mathbf{M}_{k-1}^{-1}\mathbf{K}_{k-1}\Delta t^2\beta] \\ -\Delta t\gamma \mathbf{M}_{k-1}^{-1}\mathbf{K}_{k-1} & \mathbf{I} - \Delta t^2\gamma \mathbf{M}_{k-1}^{-1}\mathbf{K}_{k-1} & (1-\gamma)dt\mathbf{I} - \Delta t^3\gamma(1/2-\beta)\mathbf{M}_{k-1}^{-1}\mathbf{K}_{k-1} \\ -\mathbf{M}_{k-1}^{-1}\mathbf{K}_{k-1} & -\Delta t\mathbf{M}_{k-1}^{-1}\mathbf{K}_{k-1} & -\Delta t^2(1/2-\beta)\mathbf{M}_{k-1}^{-1}\mathbf{K}_{k-1} \end{bmatrix} \begin{bmatrix} \boldsymbol{\alpha}_{k-1} \\ \dot{\boldsymbol{\alpha}}_{k-1} \\ \ddot{\boldsymbol{\alpha}}_{k-1} \end{bmatrix} + \begin{bmatrix} \Delta t^2\beta \mathbf{M}_{k-1}^{-1}\mathbf{F}_k \\ \Delta t\gamma \mathbf{M}_{k-1}^{-1}\mathbf{F}_k \\ \mathbf{M}_{k-1}^{-1}\mathbf{F}_k \end{bmatrix}$$

Stiffness matrix:

$$\mathbf{K}(d_1, \dots, d_{N_p}) = \sum_{i=1}^{N_p} E_i \frac{\mathbf{K}_{und} - \mathbf{K}_i}{E - \bar{\kappa} E} = \sum_{i=1}^{N_p} \frac{1 - d_i}{1 - \bar{\kappa}} (\mathbf{K}_{und} - \mathbf{K}_i)$$

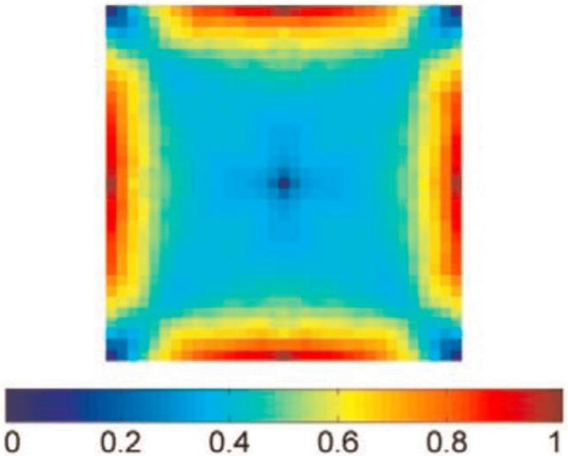
- coupling with any FE commercial code $\Longrightarrow$ Abaqus: use of keywords ELEMENT MATRIX OUTPUT applied to a fictiotious substructure
- the parametric formulation of the stiffness matrix is not required

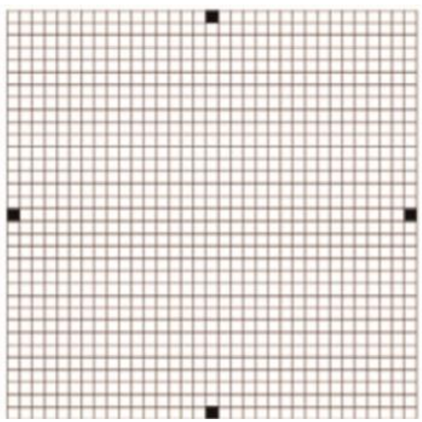
Measurement model: $\mathbf{y}_k = \mathbf{h}_k(\mathbf{x}_k) + \mathbf{v} = \mathbf{H}\mathbf{L}_{k-1}\mathbf{x}_k + \mathbf{v}$

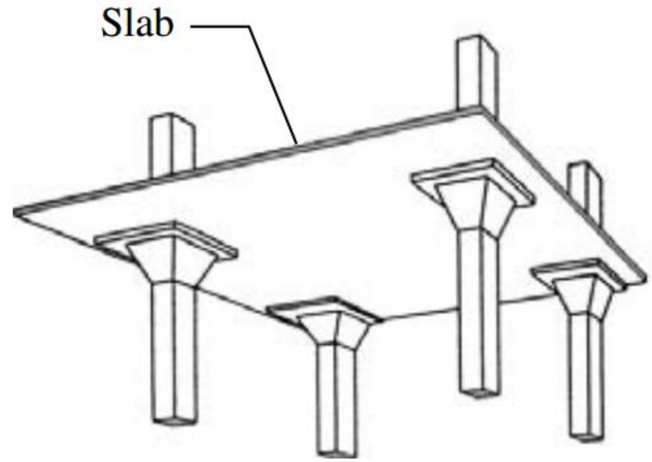
$$\mathbf{L}_k = \begin{bmatrix} \mathbf{\Phi}_{l,k} & & & \\ & \mathbf{\Phi}_{l,k} & & \\ & & \mathbf{\Phi}_{l,k} & \\ & & & \mathbf{0} \end{bmatrix}$$

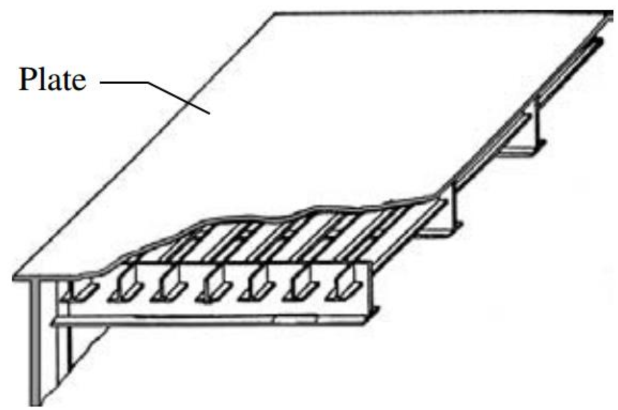
POMs

Application to structural dynamics





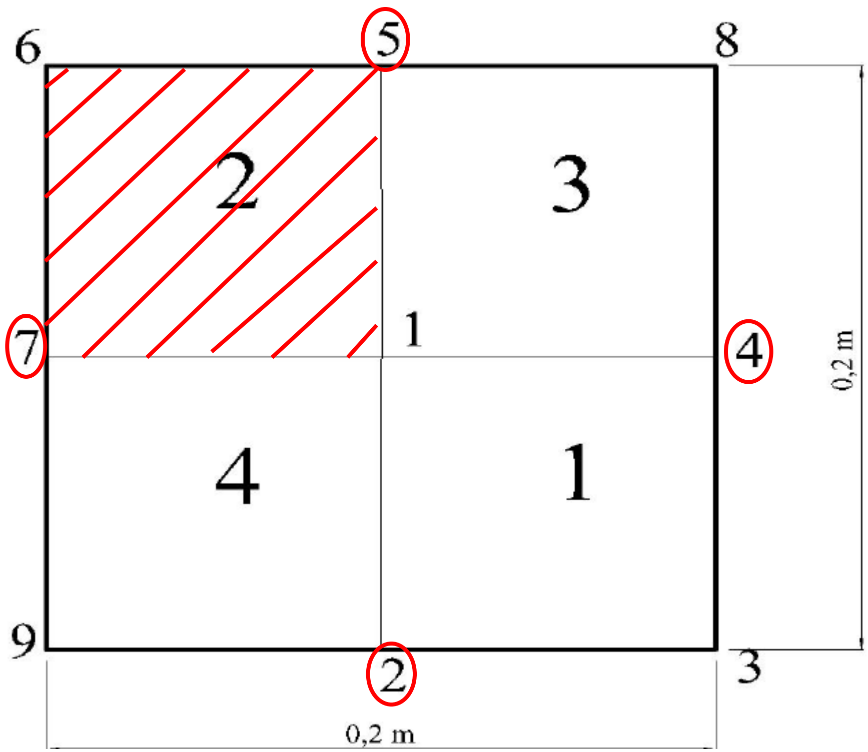




Previous works:

- **Bruggi**, **Mariani**, Optimization of sensor placement to detect damage in flexible plates (Engineering Optimization, 2012)
- **Mariani**, **Bruggi**, **Caimmi**, **Bendiscioli**, Optimal placement of MEMS sensors for damage detection in flexible plates (Structural Longevity, 2014)

Results



Elements S4R (Mindlin-Reissner)

$$\mathbf{u} = \begin{bmatrix} u_x & u_y & u_z & \varphi_x & \varphi_y & \varphi_z \end{bmatrix}^T$$

$$E = 68.9 \text{ MPa}$$

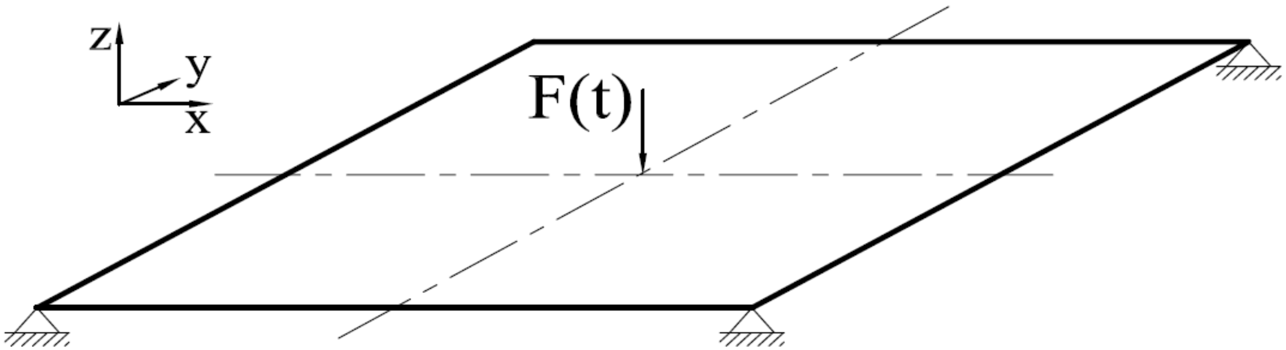
$$\rho = 2.5 \cdot 10^3 \text{ kg/m}^3$$

$$F_z^1(t) = A\sin(\omega t)$$

$$A = 100 \text{ N}$$

$$\omega = 500 \text{ rad/s}$$

$$t = 5\ mm$$

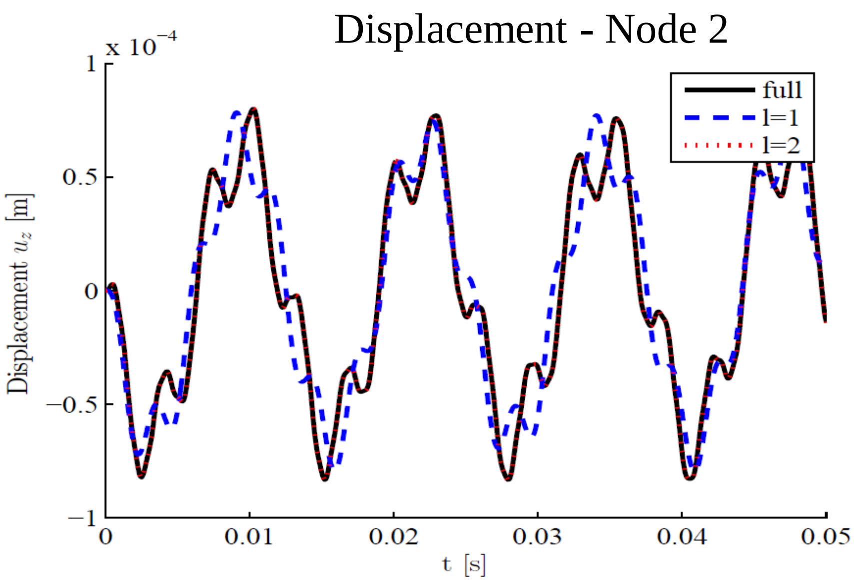


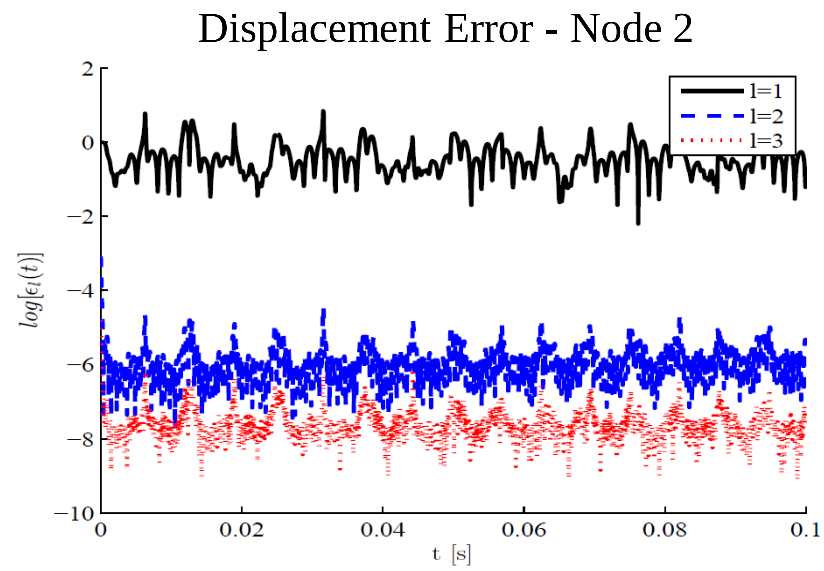
Results: Benchmark Analysis

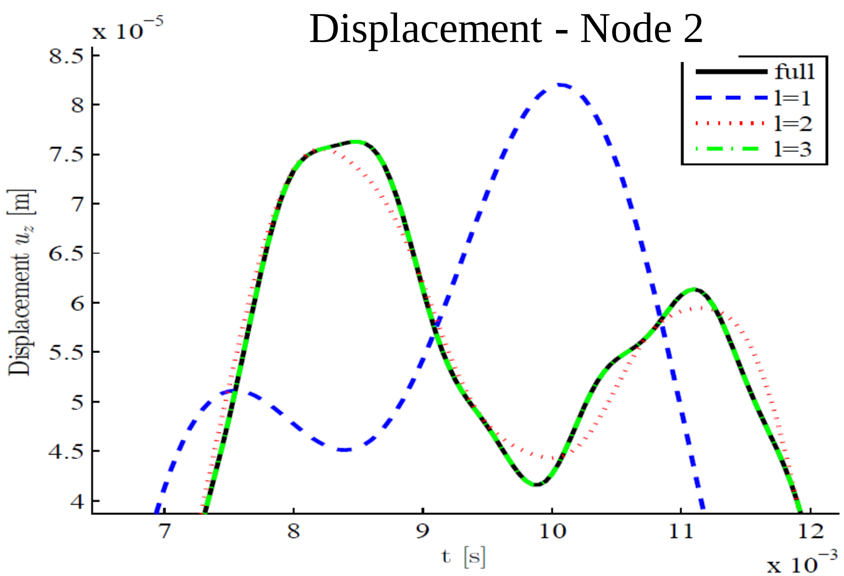
The damage identification method is evaluated in function of the following features:

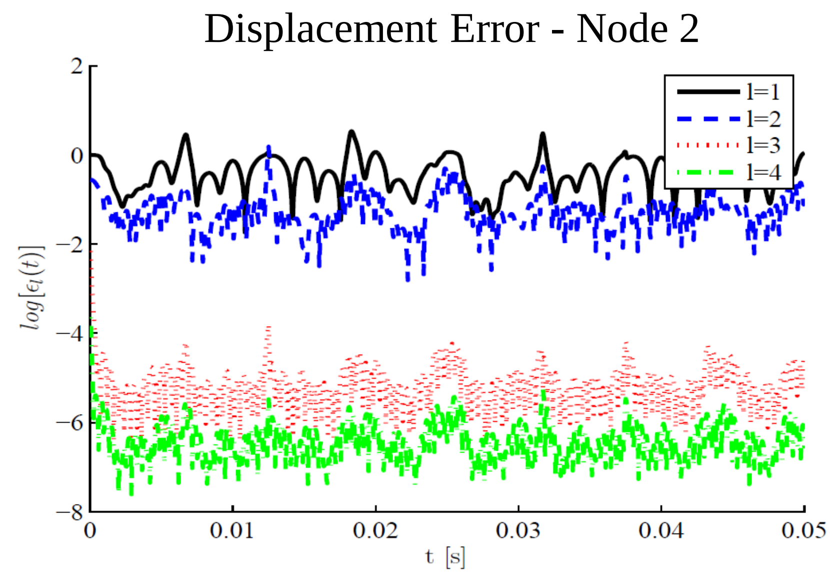
- order of the reduced system
- initial conditions
- measurement noise
- process noise
- number of observations
- mesh refinement
- POMs convergence

Results
Model Order Reduction – Undamaged vs Damaged Case

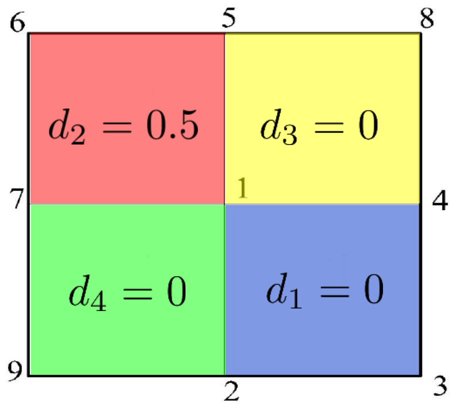


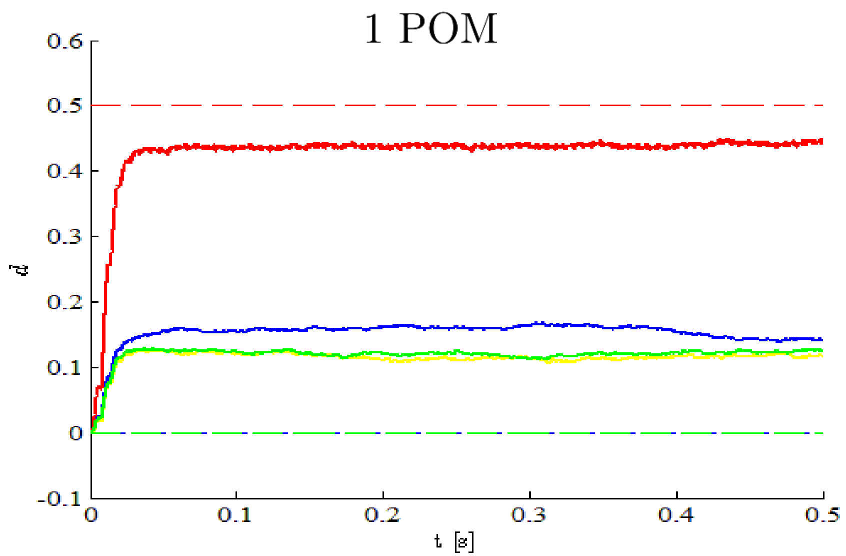


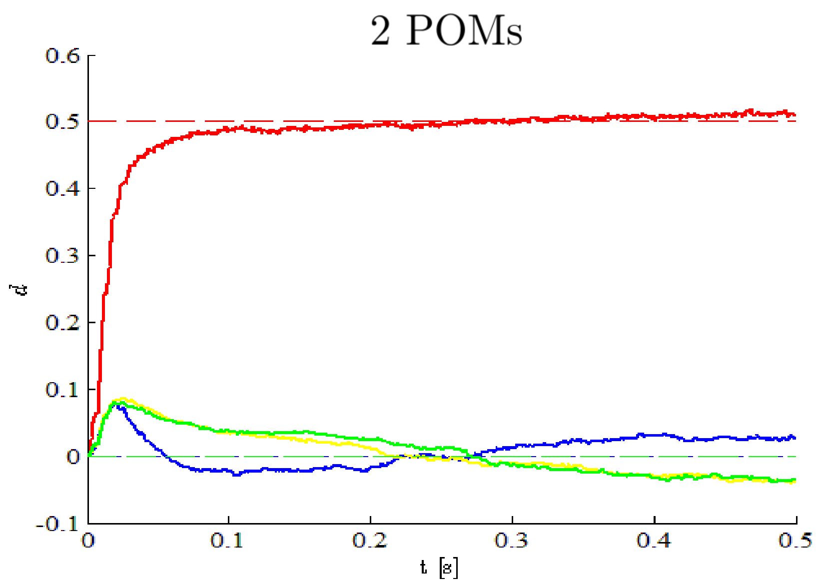


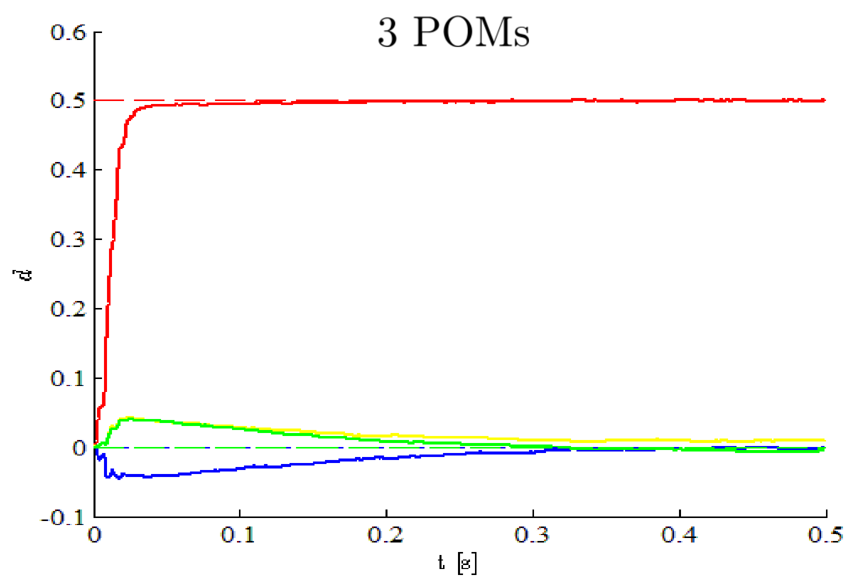


Number of POMs retained



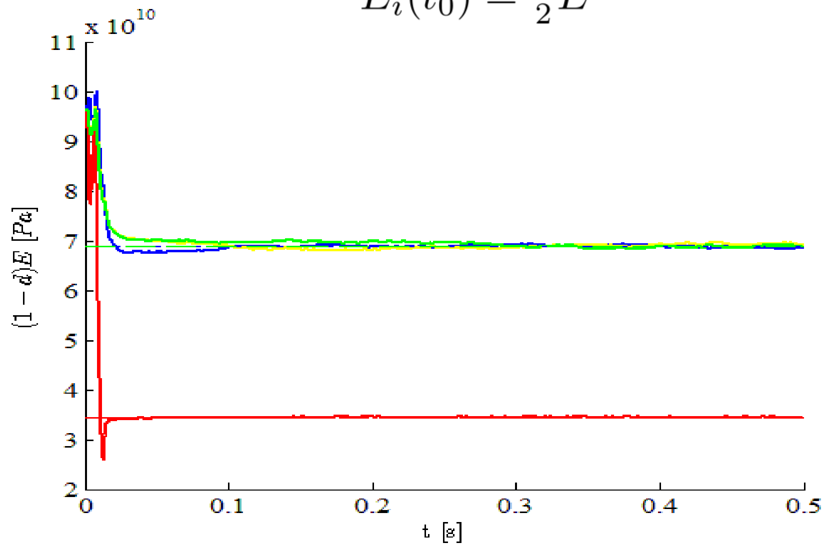




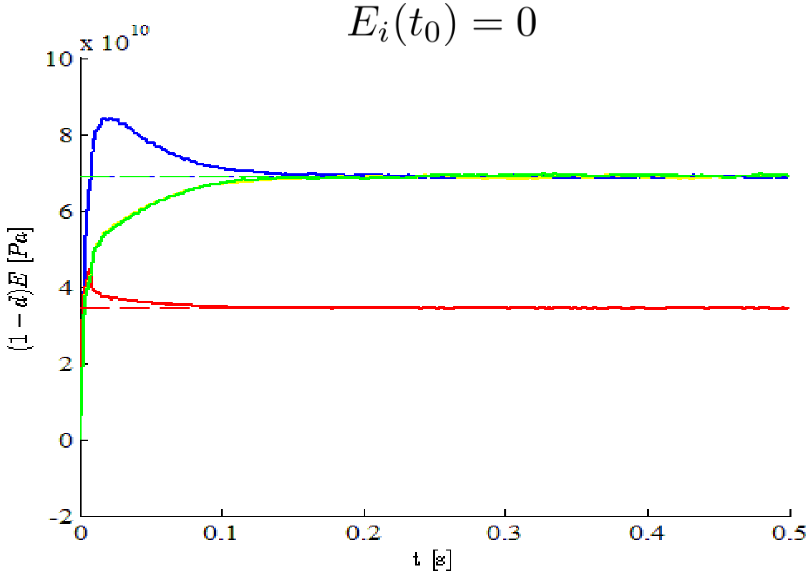


Initial conditions

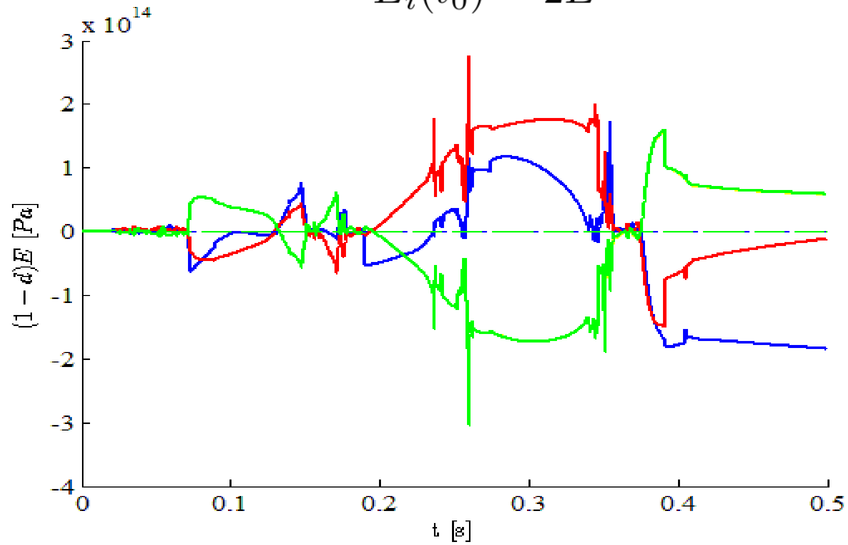
$E_i(t_0) = \frac{3}{2}E$



$E_i(t_0) = 0$



$E_i(t_0) = 2E$



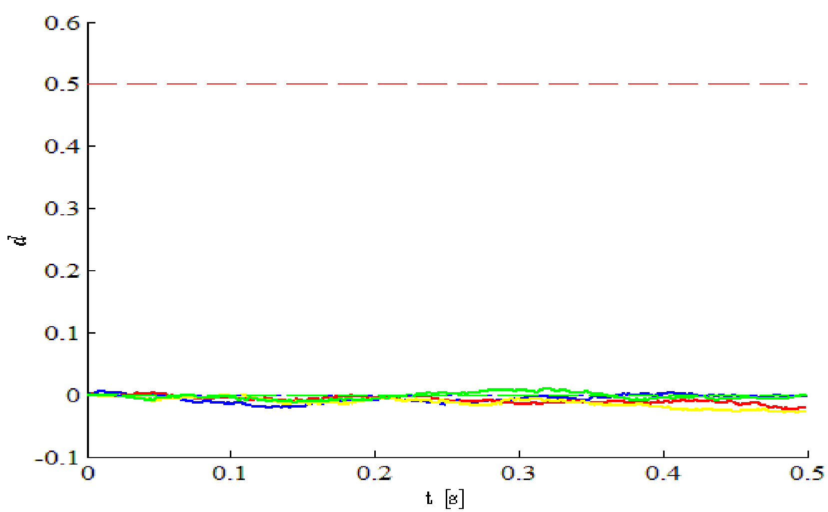
Measurement noise

$$\mathbf{y}_k = \mathbf{h}_k(\mathbf{x}_k) + \mathbf{v}$$

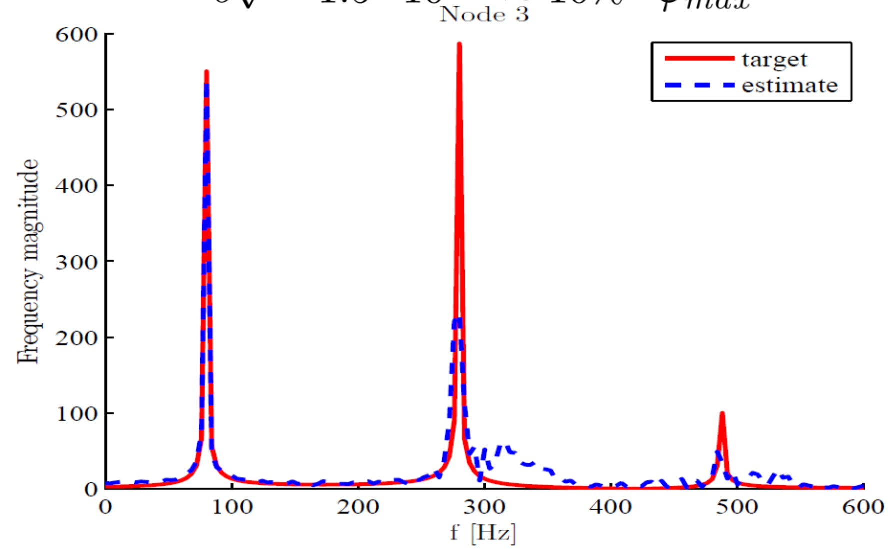
$$\mathbf{V} = \sigma_{\mathbf{v}}^2 \mathbf{I}$$

Maximum amplitude

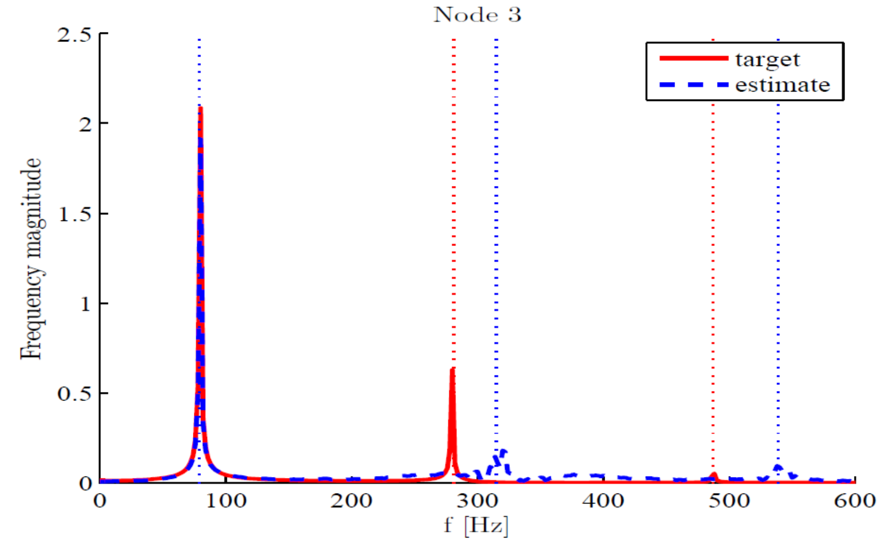
$$\sigma_{\mathbf{v}} = 1.5 \cdot 10^{-3} \approx 100\% \cdot \varphi_{max}$$



$$\sigma_{\mathbf{v}} = 1.5 \cdot 10^{-4} \approx 10\% \cdot \varphi_{max}$$



$$\sigma_{\mathbf{v}} = 1.5 \cdot 10^{-3} \approx 100\% \cdot \varphi_{max}$$

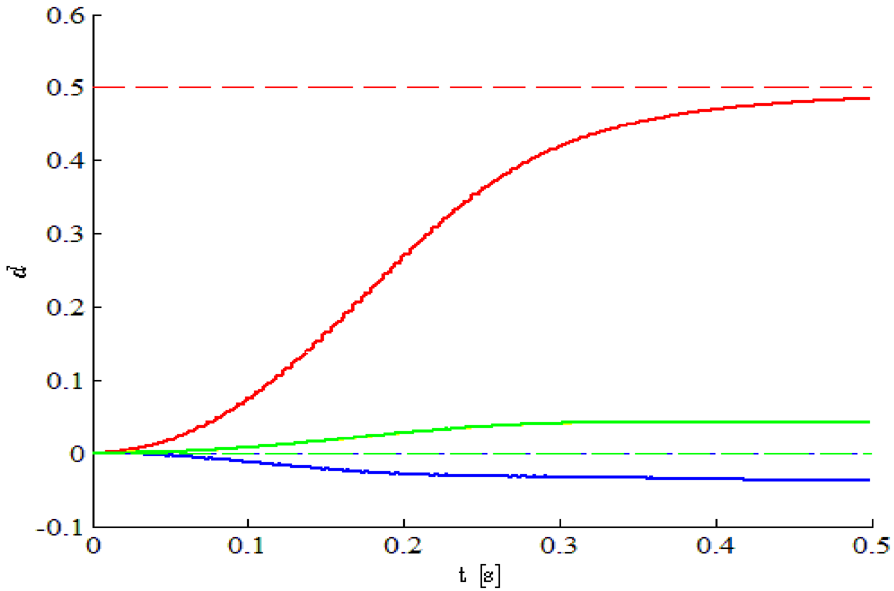


Process noise

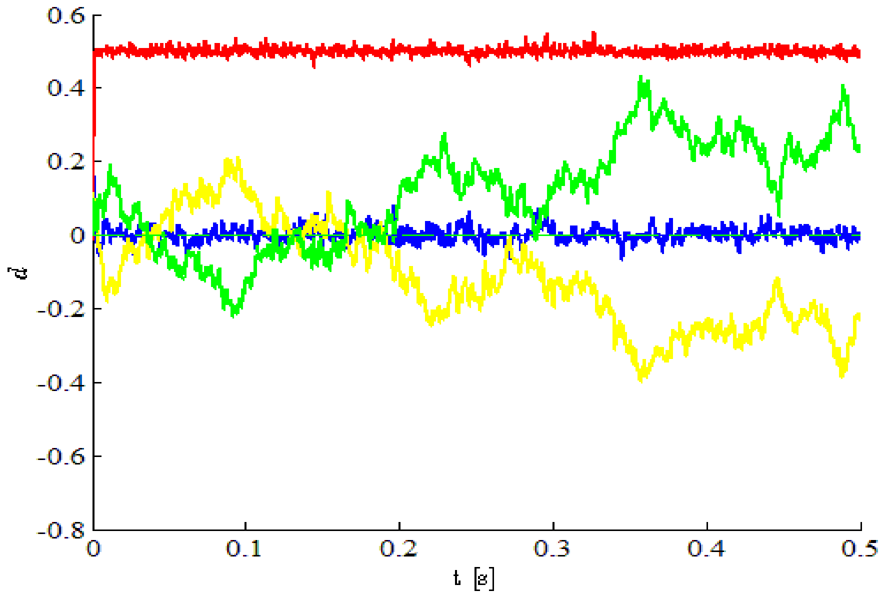
$$\mathbf{x}_k = \mathbf{f}_k(\mathbf{x}_{k-1}) + \mathbf{w}$$

$$\mathbf{W} = \sigma_{\mathbf{w}}^2 \mathbf{I}$$

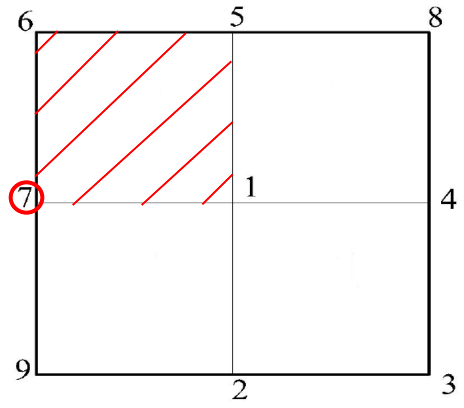
$\sigma_{\mathbf{w}} \approx 10\% \cdot \varphi_{max}$



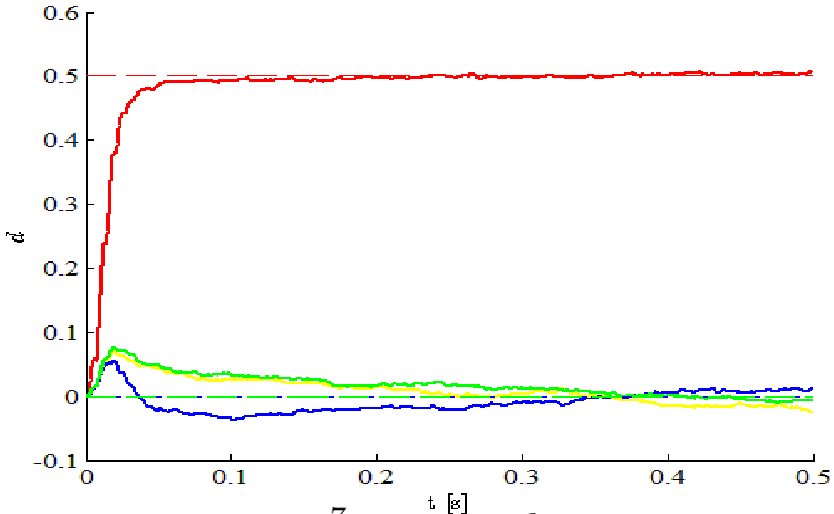
$\sigma_{\mathbf{w}} > 100\% \cdot \varphi_{max}$



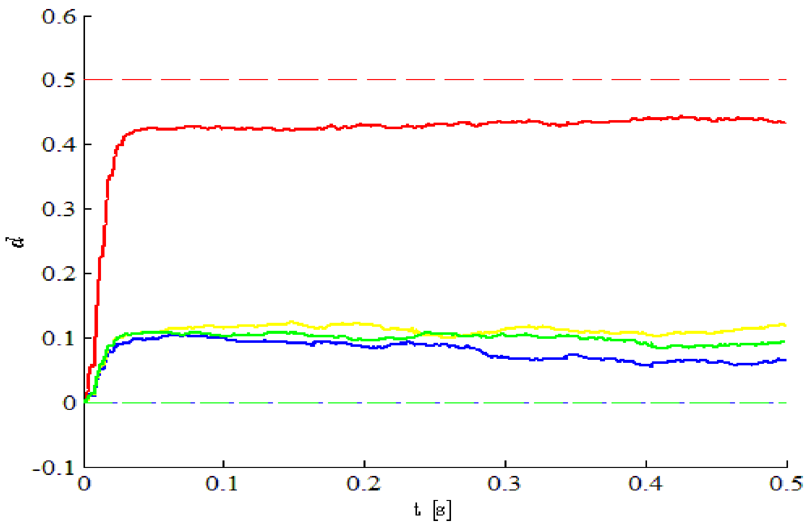
Number of observed degrees of freedom



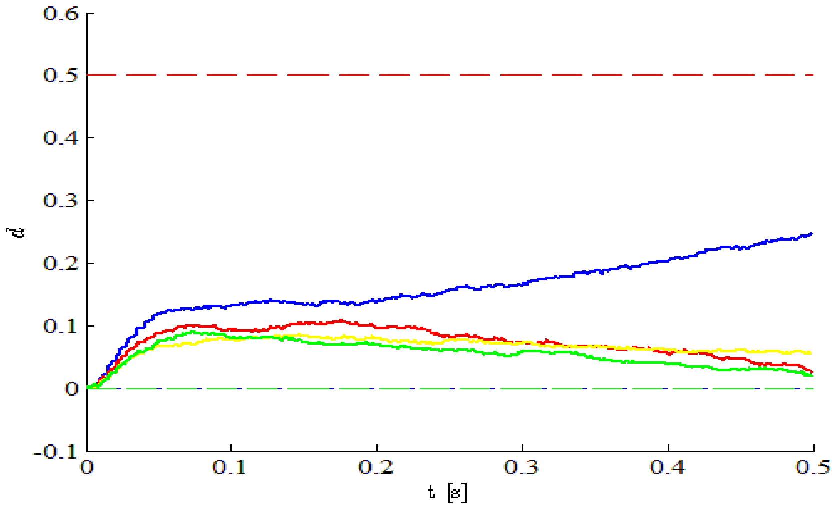
φ_x^7 and φ_y^7 measured



φ_x^7 measured



φ_y^7 measured



Results
Damage parameters estimation

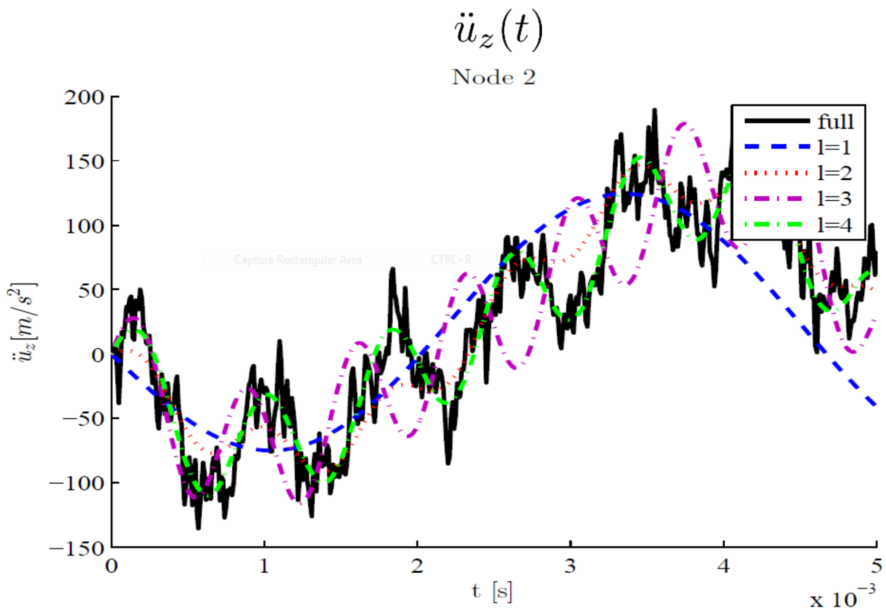
Mesh refinement

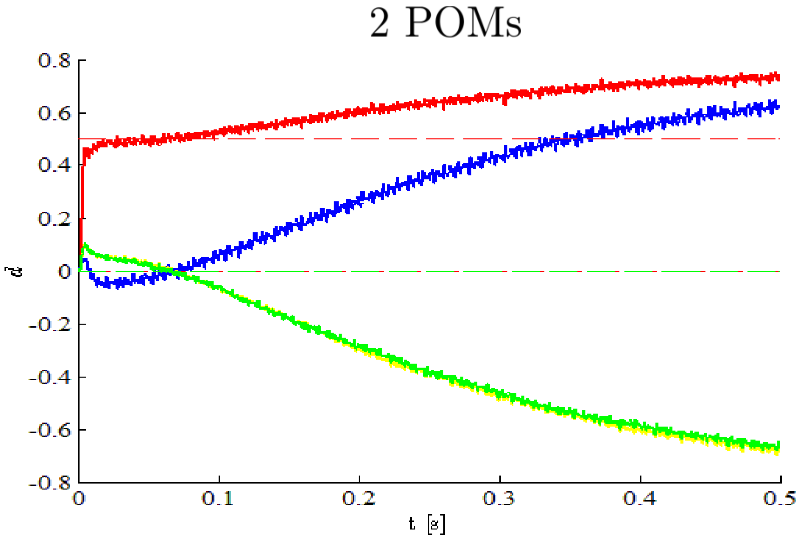
3x3 nodes $\longrightarrow$ 11x11 nodes: 2182 d.o.f.

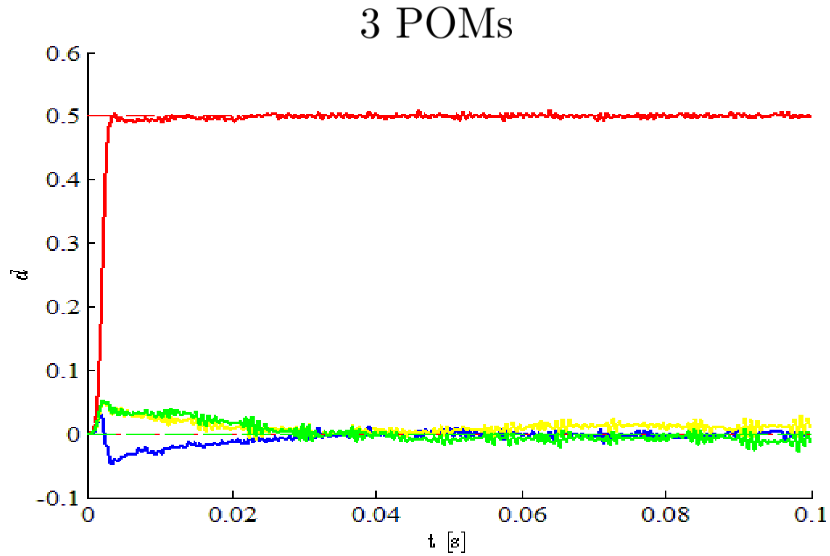
Speed-up

2 POMs: 10 d.o.f. $\longrightarrow \simeq 400$

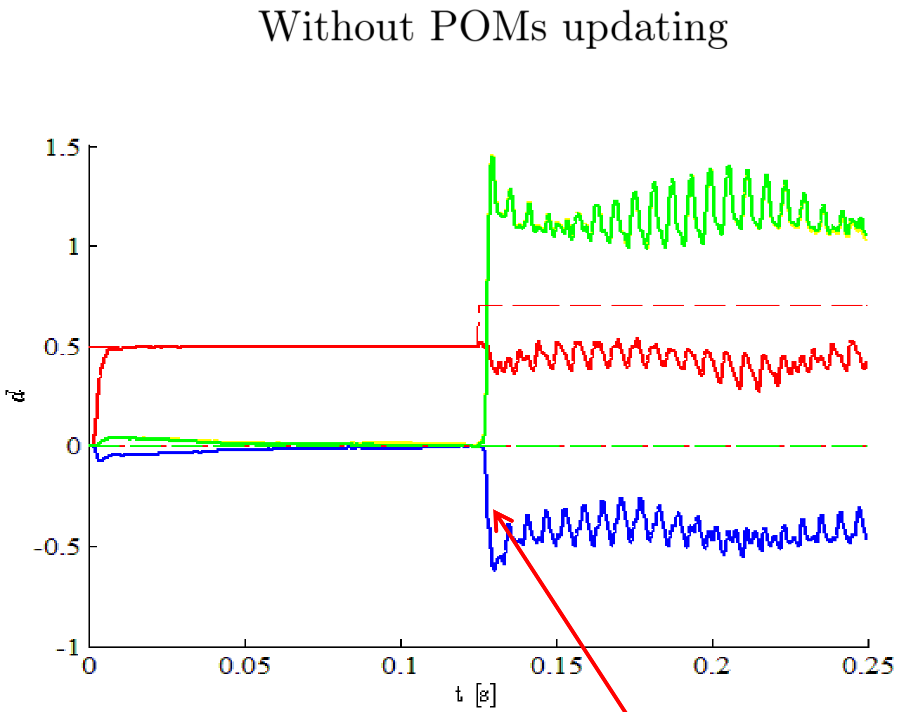
3 POMs: 13 d.o.f. $\longrightarrow \simeq 250$

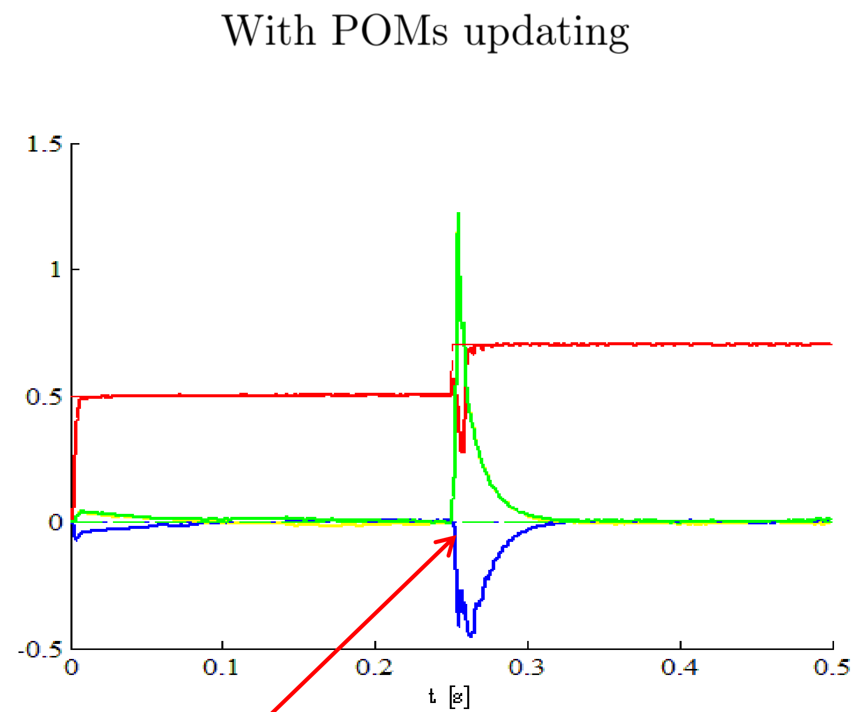






Non-stationary case





variation of stiffness

Conclusions

We have introduced several innovations with respect to previous works:

- Identification and estimation of damage indexes related to the reduction of stiffness
- Localization of damage
- Coupling with commercial FE code

We assessed the effects on the algorithmic performance of:

- Number of POMs retained
- Initial conditions
- Measurement noise
- Process noise
- Number of observations
- Mesh refinement
- On-line variation of the structural health