



**The 4th World Sustainability
Forum**

1 - 30 November 2014

<http://www.sciforum.net/conference/wsf-4>

World Sustainability Forum 2014 – Conference Proceedings Paper

Effects of Velocity and Thermal Boundary Layer with Sustainable Thermal Control Across Flat Plates

Pius Okpara¹, Emmanuel O.B. Ogedengbe^{1,*} and Marc A. Rosen²

¹ Energhx Research Group, Department of Mechanical Engineering, 353 Faculty of Engineering, University of Lagos, Akoka-Yaba, Lagos, 101017, Nigeria

² Faculty of Engineering and Applied Science, University of Ontario Institute of Technology, 2000 Simcoe Street North, Oshawa, ON, L1H 7K4, Canada

E-Mails: ogedengbe@energhx.com (E.O.B.O.); gmt4popat2012@gmail.com (P. O.); marc.rosen@uoit.ca (M.A. R.)

* Author to whom correspondence should be addressed; Tel.: +234-703-668-9827

ABSTRACT: Numerical simulations of boundary layers play a significant role in the study and interpretation of physical experiments for theoretical explanations of boundary layer disturbances. The influence of thermal boundary layer on the control of heat transport across flat plates is particularly examined. The Crank-Nicolson differential method, which is widely favoured for finite-difference modelling of boundary layer equations, is reviewed. The stability of this method is compared with other numerical approaches in order to establish the appropriate scheme for sustainable applications, involving the design of any conjugate system with heat transfer between the solid and fluid interface. Specific applications to the analysis of cabin comfort in automobiles are anticipated.

Keywords: Thermal boundary layer, Velocity boundary layer, Conjugate heat transfer and Crank-Nicolson method

1. Introduction

A fluid is generally defined as any substance or matter in a readily distorted form such that it deforms continuously when subjected to a shear stress or an unbalanced external force no matter how small [1]. When real fluid motions are observed, two basic types of motion are seen. The

first is a smooth motion in which fluid elements or particles appear to slide over each other in layers, or laminar flow. The second is characterized by random or chaotic motion of individual particles, often with eddies of different sizes, or turbulent flow [2]. The influence of viscosity is dominant in the boundary layer region, especially with increasing Reynolds number. When a fluid moves over a solid surface, the impact of viscosity (and thermal conductivity) within the velocity and thermal boundary layer is significant. This phenomenon is particularly evident in a conjugate heat transfer system of a solid (e.g., flat plate) and a fluid. Flow over a surface is divided into two regions: a region far from the surface of the body in which the effects of such fluid properties as viscosity and thermal conductivity is negligible and a thin region close to the surface where these properties are not negligible. This thin layer of fluid in which the effects of viscosity and thermal conductivity are important is called a boundary layer [3]. If one is interested in fluid momentum, the boundary layer can be described as a region where fluid particles' local velocity, u_0 , is 99% of the free stream velocity of the ambient fluid, u_∞ [4]. Studies on boundary layer flows have significantly increased our understanding of effective velocity and temperature within the zone of the boundary layer.

The effects of velocity and thermal boundary layer flow over a moving surface with temperature dependent viscosity is considered in the present study. In many practical fields, there are significant temperature differences between the surface of a hot body and the free stream. These temperature differences cause density gradients in the fluid medium. This means that the boundary layer flow should not be confined to fluid with uniform viscosity. It is known that this physical property may change significantly with temperature. Conjugate heat transfer (CHT) is the interaction between the heat conduction inside the solid body and the heat transfer in the surrounding fluid. Boundary layer flows are an important consideration in modern computational fluid dynamics [5]. The fundamental governing equations of fluid dynamics are the basis of computational fluid dynamics, and include the continuity equation: $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ [6], momentum equations: $U \frac{\partial u}{\partial x} + V \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$ and $U \frac{\partial v}{\partial x} + V \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right)$ [7][8], and energy equations: $U \frac{\partial T}{\partial x} + V \frac{\partial T}{\partial y} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial x^2} + \frac{\mu(T)}{\rho C_p} \frac{\partial^2 T}{\partial y^2}$ [9]. Note that the examples in the previous sentence are for two-dimensional flows. These equations are statements of the conservation laws of physics upon which all fluid dynamics is based, including the first law of thermodynamics [10].

In recent years studies on boundary layer theory have increased due to their wide range of applications in engineering and industry, e.g., in high speed flows (fighter aircrafts) and in industrial flows such as conveyors [11-12]. Flows over moving surfaces are also observed in high speed flows in nuclear reactors, pollutants emission in refineries, and materials handling in industries [13]. An understanding of boundary layer flow over a moving surface with temperature dependent viscosity is important to improving these and similar applications. This

study investigates boundary layer flow over moving flat surface with temperature dependent viscosity with the objective of solving the velocity and thermal boundary layer equation in laminar flow by discretizing the equation using the Finite-Volume Method and then does the simulation using Visual studio. Beyond improving present understanding of the effects of velocity and thermal boundary layer on the control of temperature gradient across a flat plate, this study also forms the basis for further studies on the application of boundary layer flows to sustainable development of automobiles with efficient control of cabin comfort.

2. Methodology

2.1 Model development

We consider here a model formulation for steady, two-dimensional flow in the (x, y) -plane in Cartesian coordinates. Consider the fluid flow geometry for a boundary layer flow over a moving flat surface (e.g., a car roof as in Fig. 1), over which flows a fluid having a temperature-dependent viscosity (moist air or water).

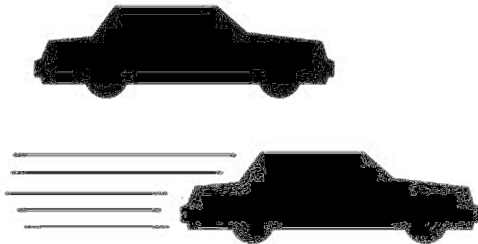


Figure 1: Schematic view of a car in motion.

The boundary layer thickness, $\sigma(x)$ is shown in Fig. 2.

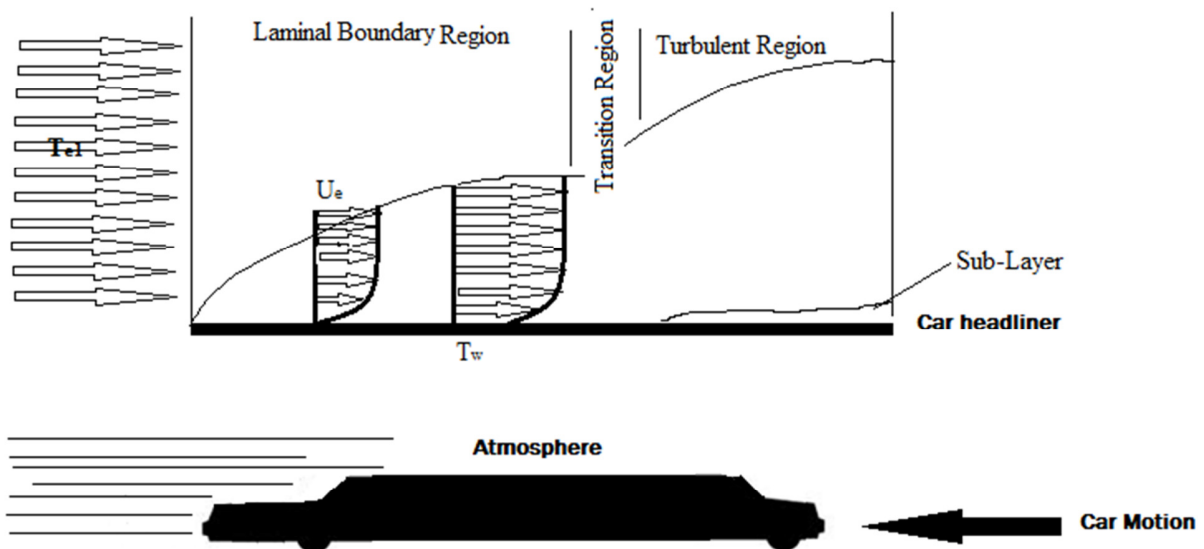


Figure 2: Velocity and thermal boundary layer flow geometry with conjugate heat transfer on a moving car model.

2.2 Governing Equation

The governing boundary layer equations of the flow are transformed into a dimensionless system of equations using a similarity variable (x, y). The resulting sets of coupled non-linear ordinary differential equations are solved numerically by applying a shooting iteration technique with the Crank-Nicolson approach.

We consider the following conditions:

- ✓ Steady flow.
- ✓ Two-dimensional flow.
- ✓ Laminar boundary layer flow.
- ✓ Viscous incompressible Newtonian fluid.
- ✓ Moving flat surface with variable plate velocity (car roof), $U_p(x)$ and streaming free stream edge velocity $U_e(x)$ parallel to the surface.
- ✓ No suction and injection at the solid surface, so $v(x; 0) = V(x; 1) = 0$.
- ✓ Uniform temperature of the flat surface, T_w .
- ✓ For boundary layer flow over a moving flat surface with temperature dependent viscosity, a temperature dependent viscous term, μ , expressed as $\mu(T)$.

Following the Ling and Dybbs model [14] this temperature-dependent dynamic viscosity can be expressed as:

$$\mu(T) = \frac{\mu_\infty}{1 + \tau(T - T_\infty)} \quad (1)$$

Two dimensional boundary layer equations for boundary layer flow over a moving surface with temperature dependent viscosity are given by:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad [\text{Continuity Equation}] \quad (2)$$

$$U \frac{\partial u}{\partial x} + V \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} \quad [\text{x-Momentum Equation}] \quad (3)$$

$$0 = -\frac{1}{\rho} \frac{\partial p}{\partial y} \quad [\text{y-Momentum Equation}] \quad (4)$$

$$U \frac{\partial T}{\partial x} + V \frac{\partial T}{\partial y} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial x^2} + \frac{\mu(T)}{\rho C_p} \frac{\partial^2 T}{\partial y^2} \quad [\text{Energy equation}] \quad (5)$$

The following boundary conditions apply:

1. At the wall: $u(x; 0) = U_p(x)$; $v(x; 0) = 0$; $T(x; 0) = T_w$
 2. At the edge: as $y \rightarrow \infty$; $u(x; \infty) = U_e(x)$; $v(x; \infty) = 0$; $T(x; \infty) = T_\infty$
- For the y-momentum equation analysis, the pressure across the boundary layer and edge are constant, i.e.,

$$\frac{\partial p}{\partial y} = \frac{\partial T}{\partial y} = 0$$

This implies $p = p(x)$. Therefore:

$$-\frac{1}{\rho} \frac{dp}{dx} = Ue \frac{\partial Ue}{\partial x} \quad (6)$$

The x-direction boundary layer equation becomes:

$$U \frac{\partial u}{\partial x} + V \frac{\partial u}{\partial y} = Ue \frac{\partial Ue}{\partial x} + \frac{1}{\rho} \frac{\partial}{\partial y} \left(\mu(T) \frac{\partial u}{\partial y} \right) \quad (7)$$

We assume power law variations in $U_p(x)$ and $U_e(x)$, i.e.

$$U_p(x) = X^n U_w \quad (8)$$

$$U_e(x) = X^n U_\infty \quad (9)$$

where U_∞ and U_w are constant reference velocities.

The pressure gradient is given as:

$$n = \frac{x}{U(x)} \frac{dU(x)}{dx} \quad (10)$$

where:

$$U_x = (U_\infty + U_w) X^n = B_0 X^n; \quad (11)$$

$$B_0 = U_w + U_\infty \quad (12)$$

Since both the fluid and the surface are moving, we consider the boundary layer flow at a specific local point. Therefore, we deal with local variables and non-dimensional terms, and define the Reynolds number based on distance x along the wall. Hence this local Reynolds number can be written as:

$$Re_x = \frac{U(x).x}{\nu} = \frac{\rho U(x).x}{\mu_\infty} \quad (13)$$

2.3 Resolving the Velocity Boundary Layer using Finite Volume Method

The solution to Eqns. 1 – 13 can be obtained using the finite-difference based Crank-Nicolson method. However, special numerical technique is required in order to super-impose the obtained velocity and temperature distribution on the staggered grid of finite volume method for solving the distribution within the solid domain (as represented by the car grid in Fig. 3).

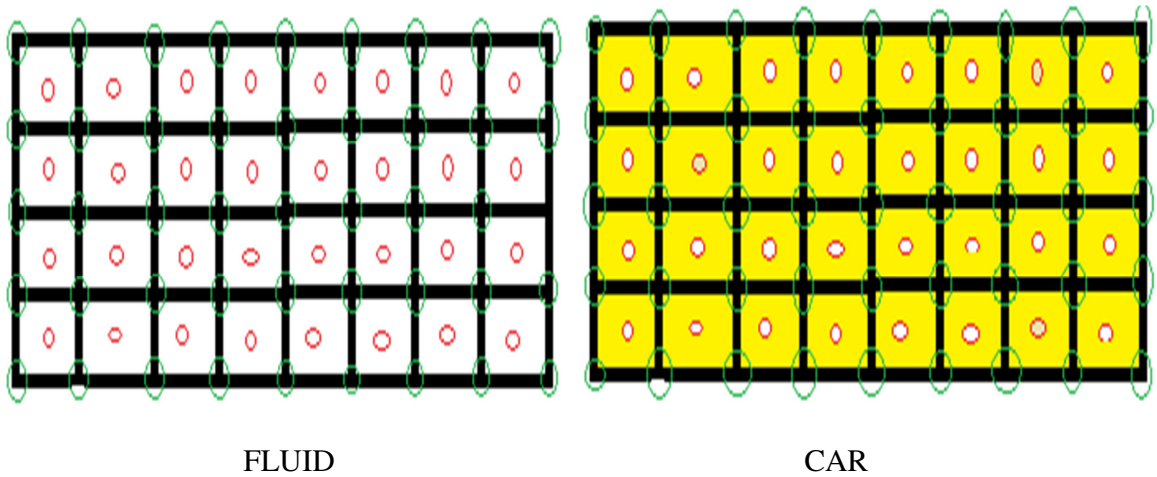




Figure 3: Discretization of the flowing fluid and overhead roof computational domain.

Legend:

 and  are control volumes

 are grid nodes (boundary nodes)

 are finite volume computational nodes

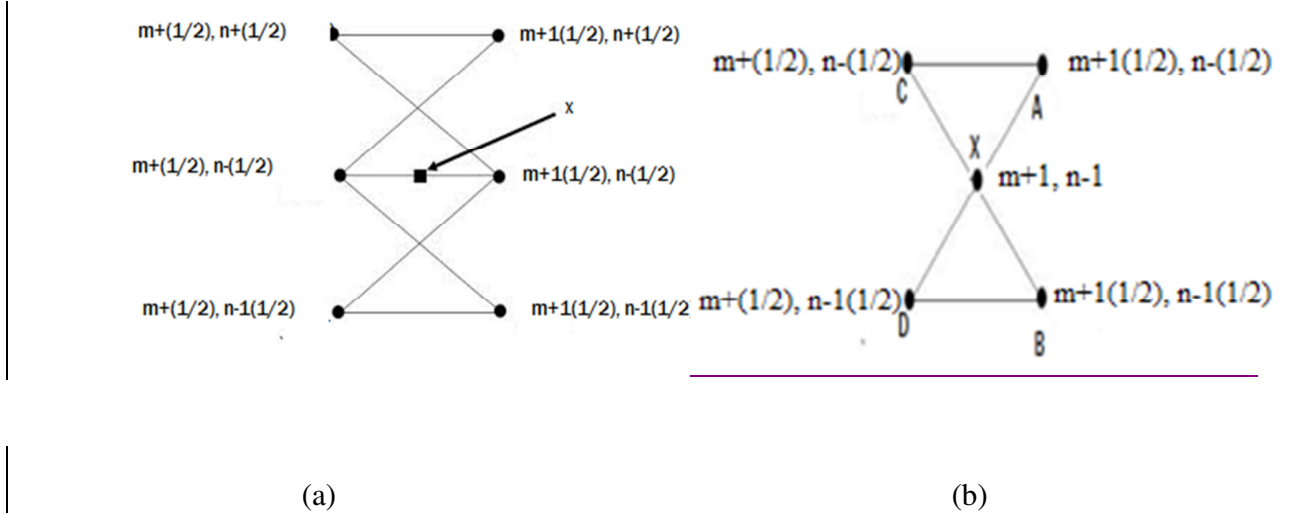


Figure 4: (a) Momentum equation molecule and (b) continuity equation molecule.

STEP 1. DISCRETIZING MOMENTUM DIMENSIONLESS EQUATION

$$f'_i \xi_i \frac{f'_i}{\xi_i} + \bar{V}_i \frac{\partial f'_i}{\partial \eta} = (1 - f'^2_i) \beta_i + \frac{\partial^2 f'_i}{\partial \eta^2}$$

1st term
2nd term
3rd term
4th term.

1ST TERM: $f'_i \xi_i \frac{f'_i}{\xi_i}$

$$\xi f' \frac{\partial f'}{\partial \xi} \approx \frac{\xi_{m+1} f'_{m+1(\frac{1}{2}),n-(\frac{1}{2})} \left[f'_{m+1(\frac{1}{2}),n-(1/2)} - f'_{m+(\frac{1}{2}),n-(\frac{1}{2})} \right]}{\Delta \xi} \quad (14)$$

2ND TERM: $\bar{V}_i \frac{\partial f'_i}{\partial \eta}$

$$\bar{V} \frac{\partial f'}{\partial \eta} \approx \frac{1}{2} \bar{V}_{m+(\frac{1}{2}),n-(\frac{1}{2})} \left(\frac{f'_{m+(\frac{1}{2}),n+(\frac{1}{2})} - f'_{m+(\frac{1}{2}),n-1(\frac{1}{2})}}{2\Delta \eta} + \frac{f'_{m+(\frac{1}{2}),n+(\frac{1}{2})} - f'_{m+(\frac{1}{2}),n-1(\frac{1}{2})}}{2\Delta \eta} \right) \quad (15)$$

3RD TERM: $(1 - f'^2_i) \beta_i$

$$\beta(1 - f'^2) = \beta_{m+1} \left(1 - f'_{m+1(\frac{1}{2}),n-(\frac{1}{2})} \cdot f'_{m+(\frac{1}{2}),n-(\frac{1}{2})} \right) \quad (16)$$

$$\begin{aligned}
\mathbf{4^{TH} TERM:} \quad & \frac{\partial^2 f'_i}{\partial \eta^2} \\
& \frac{\partial^2 f'}{\partial \eta^2} \approx \\
& \frac{1}{2} \left[\frac{f'_{m+(\frac{1}{2}),n+(\frac{1}{2})} - 2f'_{m+(\frac{1}{2}),n-(\frac{1}{2})} + f'_{m+(\frac{1}{2}),n-1(\frac{1}{2})}}{(\Delta \eta)^2} + \right. \\
& \left. \frac{f'_{m+1(\frac{1}{2}),n+(1/2)} - 2f'_{m+1(\frac{1}{2}),n-(\frac{1}{2})} + f'_{m+1(\frac{1}{2}),n-(\frac{1}{2})}}{(\Delta \eta)^2} \right] \tag{17}
\end{aligned}$$

Notes:

- ❖ The essence of averaging $\frac{\partial f}{\partial \eta}$ at point m and m+1 is the essential characteristic of the Crank-Nicolson method.
- ❖ Discretization is used to linearize continuity and momentum equations.
- ❖ Linear functions f' at m+1 are known.
- ❖ The only known value of transformed momentum equation is $\bar{V}_{m,n}$.
- ❖ The known value of $\bar{V}_{m,n}$ allows us to solve for f' in the transformed momentum equation; then the value is used to evaluate the transformed continuity equation to obtain \bar{V} .
- ❖ The Crank-Nicolson method is implicit because we solve for all values of f' at all times.
- ❖ Implicit methods are known to be very stable and to allow large step sizes in the ξ_i direction with good accuracy.

Combining the terms, we have:

$$\begin{aligned}
& \frac{\xi_{m+1} f'_{m+1(\frac{1}{2}),n-(\frac{1}{2})} \left[f'_{m+1(\frac{1}{2}),n-(1/2)} - f'_{m+(\frac{1}{2}),n-(\frac{1}{2})} \right]}{\Delta \xi} + \\
& \frac{1}{2} \bar{V}_{m+(\frac{1}{2}),n-(\frac{1}{2})} \left(\frac{f'_{m+(\frac{1}{2}),n+(\frac{1}{2})} - f'_{m+(\frac{1}{2}),n-1(\frac{1}{2})}}{2\Delta \eta} + \frac{f'_{m+(\frac{1}{2}),n+(\frac{1}{2})} - f'_{m+1(\frac{1}{2}),n-1(\frac{1}{2})}}{2\Delta \eta} \right) = \beta_{m+1} \left(1 - \right. \\
& \left. f'_{m+1(\frac{1}{2}),n-(\frac{1}{2})} \cdot f'_{m+(\frac{1}{2}),n-(\frac{1}{2})} \right) + \frac{1}{2} \left[\frac{f'_{m+(\frac{1}{2}),n+(\frac{1}{2})} - 2f'_{m+(\frac{1}{2}),n-(\frac{1}{2})} + f'_{m+(\frac{1}{2}),n-1(\frac{1}{2})}}{(\Delta \eta)^2} + \right. \\
& \left. \frac{f'_{m+1(\frac{1}{2}),n+(1/2)} - 2f'_{m+1(\frac{1}{2}),n-(\frac{1}{2})} + f'_{m+1(\frac{1}{2}),n-(\frac{1}{2})}}{(\Delta \eta)^2} \right] \tag{18}
\end{aligned}$$

Equation 18 is the finite volume momentum equation and it contains unknowns $f'_{m+(\frac{1}{2}),n-(\frac{1}{2})}$,

$f'_{m+1(\frac{1}{2}),n+(\frac{1}{2})}$, and $f'_{m+1(\frac{1}{2}),n-(\frac{1}{2})}$, which have the form:

$$\begin{aligned}
& A_{m+(\frac{1}{2}),n-(\frac{1}{2})} f'_{m+1(\frac{1}{2}),n+(\frac{1}{2})} + B_{m+(\frac{1}{2}),n-(\frac{1}{2})} f'_{m+1(\frac{1}{2}),n-(\frac{1}{2})} + C_{m+(\frac{1}{2}),n-(\frac{1}{2})} f'_{m+1(\frac{1}{2}),n-1(\frac{1}{2})} = \\
& D_{m+(\frac{1}{2}),n-(\frac{1}{2})} \tag{19}
\end{aligned}$$

On comparing the coefficients, we note the following:

$$\begin{aligned}
A_{m+\frac{1}{2},n-\frac{1}{2}} &= \frac{\bar{V}_{m+\frac{1}{2},n-\frac{1}{2}}}{4\Delta\eta} - \frac{1}{2(\Delta\eta)^2} \\
B_{m+\frac{1}{2},n-\frac{1}{2}} &= \xi_{m+1} f'_{m+\frac{1}{2},n-\frac{1}{2}} \frac{1}{\Delta\xi} + \beta_{m+1} f'_{m+\frac{1}{2},n-\frac{1}{2}} + \frac{1}{(\Delta\eta)^2} \\
C_{m+\frac{1}{2},n-\frac{1}{2}} &= - \left\{ \frac{\bar{V}_{m+\frac{1}{2},n-\frac{1}{2}}}{4\Delta\eta} + \frac{1}{2(\Delta\eta)^2} \right\} \\
D_{m+\frac{1}{2},n-\frac{1}{2}} &= \beta_{m+1} + \xi_{m+1} f'^2_{m+\frac{1}{2},n-\frac{1}{2}} \frac{1}{\Delta\xi} - \bar{V}_{m+\frac{1}{2},n-\frac{1}{2}} \frac{f'_{m+\frac{1}{2},n+\frac{1}{2}} - f'_{m+\frac{1}{2},n-\frac{1}{2}}}{4\Delta\eta} \\
&+ \frac{f'_{m+\frac{1}{2},n+\frac{1}{2}} - 2f'_{m+\frac{1}{2},n-\frac{1}{2}} + f'_{m+\frac{1}{2},n-1\frac{1}{2}}}{4\Delta\eta}
\end{aligned} \tag{20}$$

Since at $m+1(1/2)$ level (A, B and C), f' are unknown and at point $m+(1/2)$, f' are known then, it is possible to drop the $m + \left(\frac{1}{2}\right)$ and $m + 1(1/2)$ notation. Therefore, we have:

$$A_{n-\frac{1}{2}} f'_{n+\frac{1}{2}} + B_{n-\frac{1}{2}} f'_{n-\frac{1}{2}} + C_{n-\frac{1}{2}} f'_{n-1\frac{1}{2}} = D_{n-\frac{1}{2}} \tag{21}$$

STEP 2. DISCRETIZING CONTINUITY DIMENSIONLESS EQUATION

$$\begin{array}{cccc}
\xi \frac{\partial f'}{\partial \xi} & + & \beta f' & + & \frac{\eta}{2} (\beta - 1) \frac{\partial f'}{\partial \eta} & + & \frac{\partial V}{\partial \eta} = 0 \\
\swarrow & & \downarrow & & \downarrow & & \searrow \\
1^{\text{st}} \text{ Term} & & 2^{\text{nd}} \text{ Term} & & 3^{\text{rd}} \text{ Term} & & 4^{\text{th}} \text{ Term}
\end{array}$$

1ST TERM: $\xi \frac{\partial f'}{\partial \xi}$

$$\xi \frac{\partial f'}{\partial \xi} = \frac{1}{2} \xi_{m+1} \left\{ \frac{f'_{m+\frac{1}{2},n-\frac{1}{2}} - f'_{m+\frac{1}{2},n-\frac{1}{2}}}{\Delta\xi} + \frac{f'_{m+\frac{1}{2},n-1\frac{1}{2}} - f'_{m+\frac{1}{2},n-1\frac{1}{2}}}{\Delta\xi} \right\} \tag{22}$$

2ND TERM: $\beta f'$

$$\beta f' = \frac{1}{4} \beta_{m+1} \left\{ f'_{m+\frac{1}{2},n-\frac{1}{2}} + f'_{m+\frac{1}{2},n-1\frac{1}{2}} + f'_{m+\frac{1}{2},n-\frac{1}{2}} + f'_{m+\frac{1}{2},n-1\frac{1}{2}} \right\} \tag{23}$$

3RD TERM: $\frac{1}{2} \eta (\beta - 1) \frac{\partial f'}{\partial \eta}$

$$\frac{1}{2} \eta (\beta - 1) \frac{\partial f'}{\partial \eta} = \frac{1}{4} \eta_{n-1} (\beta_{m+1} - 1) \left\{ \frac{f'_{m+\frac{1}{2},n-\frac{1}{2}} - f'_{m+\frac{1}{2},n-1\frac{1}{2}}}{\Delta\eta} + \frac{f'_{m+\frac{1}{2},n-\frac{1}{2}} - f'_{m+\frac{1}{2},n-1\frac{1}{2}}}{\Delta\eta} \right\} \tag{24}$$

4TH TERM: $\frac{\partial V}{\partial \eta}$

$$\frac{\partial V}{\partial \eta} = \frac{1}{2} \left\{ \frac{V_{m+\frac{1}{2},n-\frac{1}{2}} - V_{m+\frac{1}{2},n-1\frac{1}{2}}}{\Delta \eta} + \frac{V_{m+1\frac{1}{2},n-\frac{1}{2}} - V_{m+1\frac{1}{2},n-1\frac{1}{2}}}{\Delta \eta} \right\} \quad (25)$$

Substituting the finite difference equations in equation (23) to (26) into the continuity equation, we have:

$$\begin{aligned} & \frac{1}{2} \xi_{m+1} \left\{ \frac{f'_{m+1\frac{1}{2},n-\frac{1}{2}} - f'_{m+\frac{1}{2},n-\frac{1}{2}}}{\Delta \xi} + \frac{f'_{m+1\frac{1}{2},n-1\frac{1}{2}} - f'_{m+\frac{1}{2},n-1\frac{1}{2}}}{\Delta \xi} \right\} + \\ & \frac{1}{4} \beta_{m+1} \left\{ f'_{m+\frac{1}{2},n-\frac{1}{2}} + f'_{m+\frac{1}{2},n-1\frac{1}{2}} + f'_{m+1\frac{1}{2},n-\frac{1}{2}} + f'_{m+1\frac{1}{2},n-1\frac{1}{2}} \right\} + \frac{1}{4} \eta_{n-1} (\beta_{m+1} - \\ & 1) \left\{ \frac{f'_{m+\frac{1}{2},n-\frac{1}{2}} - f'_{m+\frac{1}{2},n-1\frac{1}{2}}}{\Delta \eta} + \frac{f'_{m+1\frac{1}{2},n-\frac{1}{2}} - f'_{m+1\frac{1}{2},n-1\frac{1}{2}}}{\Delta \eta} \right\} + \frac{1}{2} \left\{ \frac{V_{m+\frac{1}{2},n-\frac{1}{2}} - V_{m+\frac{1}{2},n-1\frac{1}{2}}}{\Delta \eta} + \right. \\ & \left. \frac{V_{m+1\frac{1}{2},n-\frac{1}{2}} - V_{m+1\frac{1}{2},n-1\frac{1}{2}}}{\Delta \eta} \right\} = 0 \end{aligned} \quad (26)$$

All variables in equation (26) are known except $V_{m+1\frac{1}{2},n-\frac{1}{2}}$. Collecting like terms, the above equation becomes

$$\begin{aligned} V_{m+1\frac{1}{2},n-\frac{1}{2}} = & V_{m+1\frac{1}{2},n-1\frac{1}{2}} + V_{m+\frac{1}{2},n-1\frac{1}{2}} - V_{m\frac{1}{2},n-\frac{1}{2}} + 2\Delta \eta \left[\left(-\frac{1}{4} \beta_{m+1} - \right. \right. \\ & \left. \frac{1}{2\Delta \xi} \xi_{m+1} - \frac{1}{4\Delta \eta} \eta_{n-1} (\beta_{m+1} - 1) \right) f'_{m+1\frac{1}{2},n-\frac{1}{2}} + \left(-\frac{1}{4} \beta_{m+1} - \frac{1}{2\Delta \xi} \xi_{m+1} + \frac{1}{4\Delta \eta} \eta_{n-1} (\beta_{m+1} - \right. \\ & \left. 1) \right) f'_{m+1\frac{1}{2},n-1\frac{1}{2}} + \left(-\frac{1}{4} \beta_{m+1} + \frac{1}{2\Delta \xi} \xi_{m+1} - \frac{1}{4\Delta \eta} \eta_{n-1} (\beta_{m+1} - 1) \right) f'_{m+\frac{1}{2},n-\frac{1}{2}} + \\ & \left. \left(-\frac{1}{4} \beta_{m+1} + \frac{1}{2\Delta \xi} \xi_{m+1} + \frac{1}{4\Delta \eta} \eta_{n-1} (\beta_{m+1} - 1) \right) f'_{m+\frac{1}{2},n-1\frac{1}{2}} \right] \end{aligned} \quad (27)$$

For simplification, four new variables are introduced, called the continuity equation coefficients. Comparing with

$$\begin{aligned}
V_{m+1(\frac{1}{2}),n-(\frac{1}{2})} &= V_{m+1(\frac{1}{2}),n-1(\frac{1}{2})} + V_{m+(\frac{1}{2}),n-1(\frac{1}{2})} - V_{m(\frac{1}{2}),n-(\frac{1}{2})} \\
&+ 2\Delta\eta \left[(A_n^c) f'_{m+1(\frac{1}{2}),n-(\frac{1}{2})} + (B_n^c) f'_{m+1(\frac{1}{2}),n-1(\frac{1}{2})} \right. \\
&\left. + (C_n^c) f'_{m+(\frac{1}{2}),n-(\frac{1}{2})} + (D_n^c) f'_{m+(\frac{1}{2}),n-1(\frac{1}{2})} \right]
\end{aligned}$$

we have;

$$\begin{aligned}
A_n^c &= -\frac{1}{4}\beta_{m+1} - \frac{1}{2\Delta\xi}\xi_{m+1} - \frac{1}{4\Delta\eta}\eta_{n-1}(\beta_{m+1} - 1) \\
B_n^c &= -\frac{1}{4}\beta_{m+1} - \frac{1}{2\Delta\xi}\xi_{m+1} + \frac{1}{4\Delta\eta}\eta_{n-1}(\beta_{m+1} - 1) \\
C_n^c &= -\frac{1}{4}\beta_{m+1} + \frac{1}{2\Delta\xi}\xi_{m+1} - \frac{1}{4\Delta\eta}\eta_{n-1}(\beta_{m+1} - 1) \\
D_n^c &= -\frac{1}{4}\beta_{m+1} + \frac{1}{2\Delta\xi}\xi_{m+1} + \frac{1}{4\Delta\eta}\eta_{n-1}(\beta_{m+1} - 1)
\end{aligned} \tag{28}$$

From the boundary condition equation $V_1 = 0$ when $n = 1$, \bar{V} is seen to be algebraically related to V as follows:

$$\bar{V} = \frac{\eta}{2}(\beta - 1)f' + V \tag{29}$$

Matrix Laboratory (MATLAB) or VISUAL BASIC programs can be used to solve the boundary layer equation in order to obtain the horizontal and vertical velocity distribution. The calculation proceeds iteratively until all values for f' and V are been obtained for the entire computational domain. The effects of various parameters of the flow such as velocity ratio and viscosity variation parameter can then be investigated. Stability analysis can also be considered. Applications of these results can include: high speed flows, pollutants emission flow, conveyors for materials handlings, aerodynamics (airplanes, rockets, projectiles), hydrodynamics (ships, submarines, torpedoes), transportation (automobiles, trucks, cycles), wind engineering (buildings, bridges, water towers), and ocean engineering (buoys, breakwaters, cables).

3. Conclusions

Fundamental theory of velocity and thermal boundary layer over a flat plate has been presented. The methodology for super-imposition of finite-difference based Crank-Nicolson method over a staggered grid finite volume method provides the common stencil for the solution of velocity and temperature distribution across the solid-fluid interface. Iterative computation of these distributions across the entire domain is anticipated to introduce the required control of heat transfer toward the design of automobiles with efficient cabin comfort.

References and Notes

1. McDonough, J. M., (2009), Lectures in Elementary Fluid Dynamics: Physics, Mathematics and Applications, University of Kentucky, Lexington, KY 40506-0503 c 2009.
2. Shivamoggi, B.K., (1997), Theoretical Fluid Dynamics, John Wiley & Sons Inc, New York.
3. Rodgers, D. F, (1992), Laminar Flow Analysis, Cambridge University Press, Cambridge.
4. Cengel, A.Y., Cimbala, J.M., (2006), Fluid Mechanics: Fundamentals and Applications, McGraw-Hill Inc., New York.
5. Schlichting, H., Gersten, K., (2000), Boundary Layer Theory, Springer-Verlag, Berlin.
6. Cebeci, T., Cousteix, J., (2005), Modelling and Computation of Boundary Layer Flows, New Horizons Publishing, California.
7. Anderson, J. D. Jr., (2005), Computational Fluid Dynamics: The Basics with Applications, McGraw-Hill Inc., New York.
8. Versteeg, H.K., Malalasekera, W., (1995), An Introduction to Fluid Dynamics: The Finite Volume Method, Addison Wesley Longman Ltd, London.
9. Chorin, A.J., Marsden, J.E., (1992), A Mathematical Introduction to Fluid Mechanics, Springer Verlag, New York Inc.
10. Pozrikidis, C., (2009), Fluid Dynamics: Theory, Computation and Numerical Simulations, Springer-Verlag, New York.

11. Mureithi, E. W., Mason, D. P., (2010), Local Non-Similarity Solutions For a Forced-Free Boundary Layer Flow With Viscous Dissipation, *Mathematical and Computational Applications*, Vol. 15, No. 4, pp. 558-573.
12. Mureithi, E. W, Mason, D. P, (2001), On the Stability of a Forced-Free Boundary Layer Flow With Viscous Heating, *Fluid Dynamics Research*, Vol. 31, PP. 65-78.
13. Zarrini, M., Pralhad, R. N (2010), Numerical Investigation of two-dimensional Boundary Layer Flow Over a Moving Surface, *International Journal of Mathematical and Computer Sciences*. Vol:4, No:1
14. Hossain, A, Munir, S, (2000), Mixed Convection Flow from a Vertical Flat Plate with Temperature Dependent Viscosity, *International Journal of Thermal Science*, Vol. 39, pp 173-183.
15. Ali, M. E, (2006), The Effects of Variable Viscosity On Mixed Convection Heat Transfer Along a Vertical Moving Surface, *International Journal Of Thermal Science*, vol. 45, pp 60-69.