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Reshaping the Science of Reliability with the Entropy Function

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Abstract: The present paper revolves around two argument points. As first, we have observed a certain parallel between the reliability of systems and the progressive disorder of thermodynamical systems; and we import the notion of reversibility/ irreversibility into the reliability domain. As second, we note that the reliability theory is a very active area of research which although has not yet become a mature discipline. Theoretical researchers should continue along the way opened by Gnedenko, and we use the Boltzmann-like entropy to pursue this objective. This paper shows how the results comply with the deductive logic which is typical of mature science.

Keywords: Stochastic Systems; Reliability Theory; Reversibility; Boltzmann-like Entropy; Deductive Logic; Mature Discipline.

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1. Introduction

The reliability of machineries and the mortality of individuals are topics of great interest for the scientific community and common people as well. The *reliability theory* is an abstract approach aimed to gain theoretical insights into engineering and biology. Presently, the vast majority of researchers make conclusions about population based on information extracted from random samples; in short authors follow the inductive logic.

A mature discipline instead complies with the *deductive logic*, that is to say theorists derive the results from principles and axioms using theorems. First Gnedenko takes this course in the reliability domain

[1]. He assumes that the system S is a Markov chain and from this assumption concludes that the probability of good functioning without failure is the *general exponential function*

$$P(t) = e^{-\int_0^t \lambda(t) dt} \quad (1)$$

Where the hazard function $\lambda(t)$ determines the reliability of the system in each instant

$$\lambda(t) = -P'(t)/P(t). \quad (2)$$

Gnedenko demonstrates that probability distribution (1) comes from the conditional probability typical of Markov chains. Eqn. (1) originates from the operations that a systems executes one after the other and Gnedenko's inference can be summarized as follows

Chained Units \Rightarrow General Exponential Function

Several authors believe that the function $\lambda(t)$ follows the so-called *bath tube curve*. They hold that a new system has the decreasing hazard rate in the early part of lifetime where is undergoing burn-in and debugging of machines, and biological systems are growing and bulking. This period is followed by an interval when failures are due to causes resulting in a constant failure rate. The last period of life is one in which the system is experiencing the most severe wear out and thus has an increasing failure rate.

We can make three remarks:

- 1) The bath tube curve is not established on the basis of the deductive logic so far.
- 2) Significant evidence disproves the bath tube curve [2].
- 3) Experimentalists have found that $\lambda(t)$ exhibits very different trends in equipment and living beings.

The methods followed so far to determine the hazard rate prove to be insufficient and in our opinion we should proceed with the deductive logic inaugurated by Gnedenko in the reliability domain.

This is the objective of the present mathematical work.

2. A Lesson from Thermodynamics

The second law of thermodynamics claims that the entropy of an isolated system will increase as the system goes forward in time. This entails – in a way – that physical objects have an inherent tendency towards disorder, and a general predisposition towards decay. Such a wide-spreading process of annihilation hints an intriguing parallel with the decadence of biological and artificial systems to us. The issues of reliability theory are not far away from some issues inquired by thermodynamics and this closeness suggests us to introduce the entropy function for the study of reliable/reparable systems.

We mean to detail the Markovian model and assume that the continuous stochastic system S has m states which are mutually exclusive

$$S = (A_1 \text{ OR } A_2 \text{ OR } \dots \text{ OR } A_m), \quad m > 0. \quad (3)$$

Each state is equipped with a set of *sub-states* or *components* or *parts* which work together toward the same purpose. Formally, the generic state A_i ($i=1, 2, \dots, m$) is equipped with n *sub-states*

$$A_i = (A_{i1} \text{ AND } A_{i2} \text{ AND } \dots \text{ AND } A_{in}), \quad n > 0. \quad (4)$$

We consider that the states of the stochastic system S can be more or less reversible [3], and mean to calculate the reversibility property using the *Boltzmann-like entropy* H_i where P_i is the probability of A_i

$$H_i = H(A_i) = \ln(P_i).$$

We confine our attention to:

- The *functioning state* A_f and the *reliability entropy* H_f ;
- The *recovery state* A_r and the *recovery entropy* H_r .

The meanings of H_f and H_r can be described as follows.

When the functioning state is irreversible, the system S works steadily. In particular, the more A_f is irreversible, the more H_f is high and S is *reliable*. On the other hand, when H_f is low, S often abandons A_f in the physical reality. The system switches to A_r since S fails and is *unreliable*. The recovery entropy calculates the irreversibility of the recovery state, this implies that the more H_r is high, the more A_r is stable and in practice S is *hard to be repaired* and/or cured in the world. In sum H_r expresses the aptitude of S to work or to live without failures; the entropy H_r illustrates the disposition of S toward reparation or restoration to health.

3. Basic Assumption

Real events are multi-fold. Mechanical, electrical, thermal, chemical and other material effects interfere in the physical reality. The generic component A_{ig} ($g=1,2,\dots,n$) involves a series of collateral physical mechanisms that run in parallel A_{ig} . Universal experience brings evidence how side effects change A_{ig} . Parallel interferences work by time passing and at last impede the correct functioning to A_{ig} . Thus we can establish a general property for the system components

$$\text{The part } A_{ig} \text{ degenerates as time goes by.} \quad (5)$$

For example, Carnot defines an abstract model for the heat engine that includes two bodies at temperature T_1 and T_2 ($T_1 \neq T_2$), the gas A_{ig} does the mechanical work via cycles of contractions and expansions. The mounting disorder of the molecules results in the decreasing performances of A_{ig}

which is qualified by the thermodynamic entropy. In other words, a side effect progressively harms the gas of the heat engine.

4. Simple Degeneration of Systems

We detail (5) and establish *the regular degeneration of components* . The reliability entropy of A_{ig} decreases linearly as time goes by

$$H_{fg} = H_{fg}(t) = -c_g t. \quad c_g > 0 \quad (6)$$

From hypothesis (6) one can prove that the probability of good functioning P_f follows *the exponential law* with constant hazard rate [4]

$$P_f = P_f(t) = e^{-c t}, \quad c > 0. \quad (7)$$

$$\lambda(t) = c. \quad (8)$$

5. Complex Degeneration of Systems

When assumption (6) comes true over a certain period of time, the components $A_{f1}, A_{f2}, \dots, A_{fn}$ worsen to the extent that they set up a *cascade effect* [4]. The cascade effect consists of the generic part A_{ig} that spoils one or more close components while the system proceeds to run. A cascade effect can be linear or otherwise compound.

In the first stage we assume the component A_{ig} harms the close part A_{ik} and this in turn damages another one and so on.

The cascade effect is linear. (9)

Suppose the linear cascade effect occurs while principle (6) is still true of necessity, one can prove that the probability of good functioning is the *exponential-power function*

$$P_f = P_f(t) = b e^{-a t^n}, \quad a, b > 1. \quad (10)$$

The hazard function is a *power of time*

$$\lambda(t) = a t^{n-1} \quad (11)$$

In the second stage we suppose that the component A_{ig} damages the components all around

The cascade effect is compound. (12)

This hypothesis – alternative to linear waterfall effect – yields that the probability of functioning is *the exponential-exponential function* and the hazard rate is *exponential of time*

$$P_f = P_f(t) = g e^{-de^t}, \quad g, d > 1. \quad (13)$$

$$\lambda(t) = de^t$$

6. Conclusive Remarks

A) Chaining implies a true dependency between chained operations, and Gnedenko derives the general exponential function from this dependency property. The present work follows a different course using the Boltzmann-like entropy and develops the ensuing inferences:

$$\begin{aligned} \text{Regular degeneration of system's components} &\Rightarrow \text{Exponential Function} \\ \text{Regular degeneration + linear cascade effect} &\Rightarrow \text{Exponential-Power Function} \\ \text{Regular degeneration + composite cascade effect} &\Rightarrow \text{Exponential-Exponential Function} \end{aligned} \quad (15)$$

Both the assumptions and the conclusions of (15) fit with Gnedenko's work. In particular:

- *Assumptions* of (15): We model S by mean of (3) and (4) that are Markovian chains. The regular degeneration of A_{ig} and the cascade effects make explicit some special behaviors of chained operations.
- *Conclusions* of (15): Mathematical results (7), (10) and (13) are special cases of function (1).

The present theoretical frame proves to be consistent with Gnedenko's work.

B) During the middle age the organs of appliances and biological beings often degenerate at constant rate in accordance with (8). Machines and equipment have frequently a linear shape and the probability of good functioning follows the Weibull distribution during ageing that corresponds to (10). The body of animals and humans appear rather intricate; during ageing compound cascades effects result in the Gompertz distribution that conforms to (13).

In short, the theoretical results obtained in this work match with empirical investigations. Differently from the statistical modes, the present approach adopts the deductive logic and relates the system decay to precise causes.

C) Each result in (15) has been obtained from precise hypotheses, and those hypotheses may come true during the system juvenile period, the maturity and the senescence alike. For example the hazard rate of Hydra magnipapilata is constant throughout the entire lifetime [5]. This empirical data means that the organs of Hydra magnipapilata are subjected to regular degeneration from the birth to the death and do not undergo any special collateral effect during the lifetime.

Therefore, the present frame does not hold that the bath tube curve is systematically true in the world in accordance with statistical enquiries.

In conclusion, remarks A, B and C show how the Boltzmann-like entropy sustains a promising approach for developing a deductive theory of aging integrating mathematical methods with engineering notions and specific biological knowledge.

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