RESHAPING THE SCIENCE OF RELIABILITY WITH THE ENTROPY FUNCTION

Paolo Rocchi

IBM
via Shangai 53, 00144 Roma

LUISS UNIVERSITY.
via Salvini 2, 00197 Roma,
procchi@luiss.it

Giulia Capacci

UNIVERSITY of PERUGIA
Piazzale Menghini 1, Perugia
giu.capacci@gmail.com

The current state of the Reliability Theory

The Reliability Theory has become a noteworthy collection of models, methods and tools

The current state of the Reliability Theory

"**On fait la science avec des faits, comme on fait une maison avec des pierres: *mais une accumulation de faits n'est pas plus une science* qu'un tas de pierres n'est une maison** (Science is built up with facts, as a house is made of stones. But a collection of facts is no more a science than a heap of stones is a house)"

Henri Poincaré

A large amount of formulas does not make a consistent sector of studies

The reliability theory has not yet matured as a science

The current state of the Reliability Theory

An exact science is grounded on **the deductive logic**
Theorists deduce the results from axioms or precise assumptions

Gnédenko, Soloviev and other eminent authors from the Russian school made the first attempt to lay the logical foundations of the reliability science
Following a deductive approach

Gnedenko's Cornerstone

Gnedenko assumes that the system S is a Markov chain and concludes that the probability of good functioning without failure is the *general exponential function*

$$P(t) = e^{-\int_0^t \lambda(t)\,dt} \qquad (1)$$

Where the hazard function $\lambda(t)$ determines the reliability of the system in each instant

$$\lambda(t) = -P'(t)/P(t). \qquad (2)$$

Gnedenko demonstrates that (1) originates from the operations that a systems executes one after the other. The deductive inference is the following

Chained Units ⇒ General Exponential Function

The present theoretical research is an attempt to continue

Gnedenko's seminal work adopting a different method

From Thermodynamics

The second law of thermodynamics claims that the entropy of an isolated system will increase as the system goes forward in time.

Any physical objects have an inherent tendency towards disorder and a general predisposition towards decay.

Such a wide-spreading process of annihilation hints an intriguing parallel between thermodynamics and the reliability theory.

The failures of biological and artificial systems are not far away from the issues inquired by thermodynamics.

The Entropy Function

We import the concept of ***reversibility and irreversibility*** from thermodynamics.

We express the ability of the stochastic system S to evolve from the state A_i using the **Boltzmann-like entropy** $H(A_i)$:

$$H = H(A_i) = \ln(P_i)$$

- When the state A_i is irreversible, $H(A_i)$ is “high”
- When the state A_i is reversible, $H(A_i)$ is “low”

Physical Meaning of The Boltzmann-Like Entropy

1) - If the entropy $H_f = H(A_f)$ of the *functioning state* A_f is high, S is stable in A_f, namely ***S* is *reliable.***

2) - If $H(A_f)$ is low, the system ***S* is *unreliable*.**

3) - If the entropy $H_r = H(A_r)$ of the *failure state* A_f is high, S is stable in A_r, namely ***S* is *hard to repair*.**

4) - If $H(A_r)$ is low, A_r is reversible, the system ***S* is easily *repairable.***

$H(P_f)$ and $H(P_r)$ express the attitude of S to work and to be repaired respectively.

The Basic Assumption and
The Simple Degeneration of Systems

Assumption: Every active component A_{fg} of A_f decays at constant speed, that is the entropy of A_{ig} decreases linearly with time

$$H_{fg} = H_{fg}(t) = - c_g\, t, \qquad c_g > 0. \qquad (6)$$

From (6) one obtains the probability of good functioning until the first failure is the exponential function and the hazard rate is constant

$$P_f = P_f(t) = e^{-c\, t}, \qquad c > 0. \qquad (7)$$

$$\lambda(t) = c\,.$$

Complex Degeneration of Systems

Linear cascade effect

The component A_{ig} harms the close part A_{ik} and this in turn damages another one and so on; the probability of good functioning is the *exponential-power function*

$$P_f = P_f(t) = b \bullet e^{-at^n}, \qquad a, b > 1. \tag{10}$$

The hazard function is a *power of time*

$$\lambda(t) = at^{n-1}. \tag{11}$$

Compound cascade effect

The component A_{ig} damages the components all around and the probability of functioning is *the exponential-exponential function*

$$P_f = P_f(t) = \mathsf{g} \cdot e^{-\mathsf{d} e^t}, \qquad \mathsf{g}, \mathsf{d} > 1. \tag{13}$$

And the hazard rate is *exponential of time*

$$\lambda(t) = \mathsf{d} \cdot e^t. \tag{14}$$

Conclusion

A) Gnedenko proves:

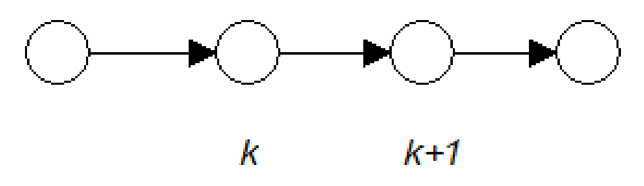
$$\mathfrak{Chained\ Units} \Rightarrow \mathfrak{General\ Exponential\ Function} \tag{15}$$

The present calculations are special cases of (15) in that
Each assumption depicts a special Markov chain:

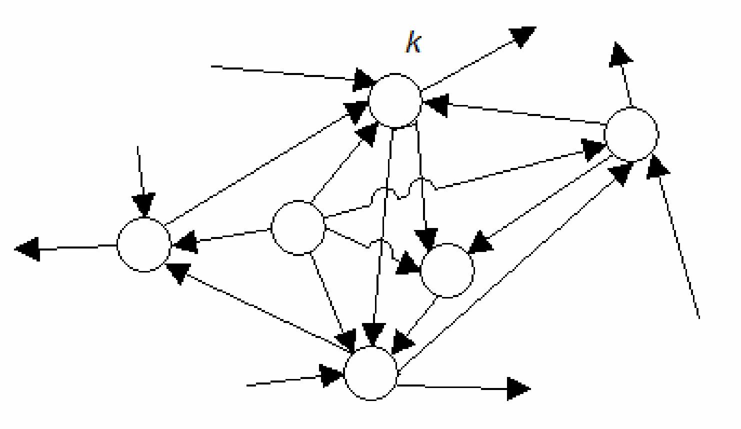
$$\mathfrak{Regular\ degeneration\ of\ system's\ components} \Rightarrow \mathfrak{Exp.\ Function}$$
$$\mathfrak{Regular\ degeneration} + \mathfrak{linear\ cascade\ effect} \Rightarrow \mathfrak{Exp.}-\mathfrak{Power\ Function}$$
$$\mathfrak{Regular\ degeneration} + \mathfrak{composite\ cascade\ effect} \Rightarrow \mathfrak{Exp.}-\mathfrak{Exp.\ Function}$$

The present theory is consistent with Gnedenko's result

B) Artificial systems have linear structures and empirical data complies with the Weibull distribution



Biological system exhibits the mesh pattern and empirical data complies with the Gompertz distribution



The present mathematical frame fits with experience

C) Frequently the hazard rate does not conform with the bath tube shape but has an irregular trend

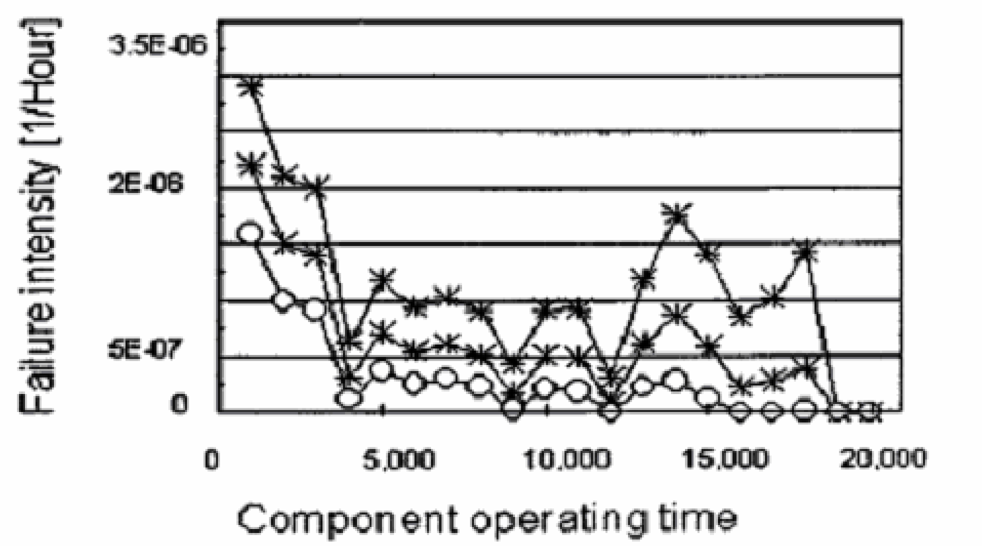


Figure: The roller-coaster hazard rate curve of electronic equipment

The assumptions of the present theory can occur in any period of system lifetime and can justify a variety of hazard rate curves.

In conclusion, the Boltzmann-like entropy supports a promising approach for developing a deductive theory of aging integrating mathematical methods with engineering notions and specific biological knowledge.

THANKS FOR YOUR ATTENTION !