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Are the Flocks Critical Phenomena?

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Abstract: Almost all flock models are constructed using a self-propelled particle system (SPPs). In an SPP method, each individual will interact with neighbors found within a certain radius. Recent investigations are forcing us to reconsider the notion of the neighborhood in flocks. Cavagna et al. found a scale-free correlation in which the sub-flocks use the same information and where their size is proportional to the flock size. This finding indicates that the flock neighborhood dynamically changes the shape and formation of the flock. They defined this state of the flock as the "noise critical phenomenon". However, is it a sufficient interpretation of the scale-free correlation? The agent of the metric-topological interaction model, which we proposed, changes its neighborhood by adjusting between the individual and classed collection cognitions. These differences in the neighborhood of each agent enables their flock to rapidly change direction without external noise and shows a scale-free correlation that is supported with empirical research. The metric-topological interaction model suggests that the flock emerges as a scale-free correlation without considering noise critical phenomena.

Keywords: Collective Behavior; Metric Distance; Topological Distance; Scale-Free Correlation

1. Introduction

Almost all models of collective behavior were constructed using the self-propelled particle system (SPP) proposed by Vicsek et al. (Vicsek et al., 1995). Each agent in an SPPs model interacts with its neighbors within a pre-defined radius that is provided by the modeler, and this interaction determines the agent's direction. Despite the simplicity of SPPs, it shows a phase transition from disorder to order that corresponds to either the density or external noise environment for each individual. The SPPs appears to be a reasonable model for understanding collective animal behavior (e.g., swarms, fish schools, and bird flocks) because several empirical studies confirm the results of the density dependent phase transition. For instance, locusts will align their direction when the density of individuals is above 73.8 locusts/m^2; a model exists to explain this phenomenon in the context of an SPPs (Buhl et al., 2006; Parrich, 1999). Therefore, many researchers tried to understand collective behavior by adding additional, plausible properties to SPPs models (Couzin et al., 2002; Goldstone and Gureckis, 2009; Strefler et al., 2008; Giardina, 2008).This methodology produced useful results for explaining the formations of fish schooling or birds flocking. For example, Couzin et al. attempted to understand collective behavior by adding additional, realistic properties (Couzin et al., 2002). They gave each individual a blind zone (the range outside of the agent's perception), a repulsion zone (the zone in which the agent avoids other agents), and an attractive zone (the zone in which the agent is attracted to other agents). They showed that this model could produce various formations of schooling, such as swarming, marching, and torus, by tuning the attractive or repulsive parameters. From their simulation, they suggested the idea of a collective memory, which influenced the flocking formation based on past formations (Couzin et al., 2002; Couzin, 2007).

Recently, Ballerini et al. showed that, using empirical evidence, a bird never interacts with his or her neighbors using the metric distance (SPPs), but rather uses the topological distance (Ballerini et al., 2008a,b). The topological distance is when the bird interacts with its nearest 7 neighbors, no matter how far away they are. Moreover, they also indicated that flock models of the topological distance were more robust in their simulations to a predator's attack than if the metric distance was used because a flock based on the metric distance could not interact with other agents when they separated beyond a given distance. The important part of the topological distance is not the limit of a bird's perception (how many neighbors a bird can count), but that the bird can adapt to changing relationships with their neighbors. In other words, each bird can keep a constant coherence with neighbors regardless of if the density of his neighborhood is high or low. We previously pointed out that, if we used this interpretation, the notion of the topological distance implicitly hypothesized the metric distance because the notion of density is needed to consider the neighborhood (Niizato and Gunji, 2010b). Almost all researchers, however, neglect this dynamic aspect of the topological distance and emphasized the static aspect of the topological distance (Giardina, 2008). Thus, we consider that this uncertainly neighborhood can be interpreted as dynamic neighborhood by switching between the metric and topological distances. We propose a metric-topological interaction model to unite these ideas. The metric-topological interaction model showed a medium property between the metric distance and the topological distance (Niizato and Gunji, 2010b). One wonders, however, if a sufficient model for understanding the topological distance can be constructed similar to a SPPs model, without needing to account for the neighborhood of uncertainty.

A recent study by Cavagna et al. found that there was more to dynamic neighborhoods than the metric distance and the topological distance (Cavagna et al., 2009). They defined fluctuation vectors that could be obtained by subtracting the average of each velocity vector from each velocity vector and examined the distribution of these correlations. They investigated each bird's fluctuation vectors and found that the flock had domains of correlating fluctuation vectors. Furthermore, they found that the domains of the correlation of fluctuating vectors were proportional to the flock size. This scale-free correlation is clearly distinct from the topological distance and the metric distance because both of these concepts of neighborhood are static. On the other hand, the idea of a scale-free correlation suggests a dynamic neighborhood for the flock. They compared the scale-free correlation and SPPs models in the context of the noise critical phenomenon and admitted that these notions never conflicted, with the reservation that the correlation function was given using unbound flocks (Czirok and Vicsek, 2006). We considered that this conclusion is not an appropriate interpretation of the scale-free correlation. The noise critical phenomenon becomes a problem only when we consider that the individuals (or particles) never have a relationship with the whole, and it can never explain the proportional relationship between flock size and the correlated domains. The scale-free correlation arises from the result of adjustments between each individual and its flock. To understand the scale-free correlation, we have to create an alternative model, such as the SPPs model. In this paper, we review the metric-topological interaction model as a combination of both the metric distance and the topological distance properties and show that these two notions can replace the way of cognition, that is, the classed collection cognition and the individual cognition. The metric-topological interaction model shows that collective behaviors can cause changes of direction without external noise. This model has a correlation domain that closely resembles those seen in empirical research and shows a scale-free correlation. The results show that the metric-topological interaction model can explain these three notions of a "neighborhood", that is, the metric distance, the topological distance and scale-free correlation, without contradictions.

2. Result and Analysis

2.1. Metric and Topological Interacion Model

First, we show the outline of the metric-topological interaction model. We insisted that the notion of a topological distance was an implicit assumption of the metric distance because the agent must use the notion of metric distance to recognize its nearest neighbors (i.e., how far apart are the individuals?). On the other hand, the notion of a metric distance is also an implicit assumption of the topological distance. The radius of each neighborhood has to be adjusted depending on the number of agents employed in the neighborhood when a flock is maintained despite changes in density. We reinterpreted these two neighborhoods using the notion of classed collection cognition (e.g., kinds, classes, roles, variables) and individual cognition (e.g., individuals, instances, filters, values) (Niizato and Gunji, 2010a,b). While a set of individuals is grasped and averaged by agents, these individuals are selected based on distance, which is evaluated by roles. Thus, it can be interpreted that the classed collection cognition corresponds to the metric distance and the individual cognition corresponds to the topological distance. Our previous study showed that the metric-topological interaction model has an intermediate property between the metric distance and the topological distance (Niizato and Gunji, 2010b).

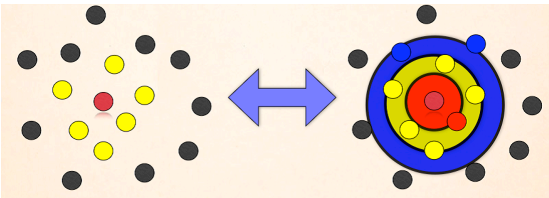

Figure 1: The image of the metric-topological interaction model. The left figure corresponds to the individual cognition (that is, the topological distance) and the right figure corresponds to the classed collection cognition (that is, the metric distance).

The algorithm of the metric-topological interaction model is listed under **Algorithm**. We did not add other properties, such as the blind zone for each agent. The main focus of the metric-topological interaction model is switching between the classed collection and individual cognitions. Fig.1 shows an image of the metric-topological interaction model. The left figure corresponds to the topological distance (that is, the individual cognition) and the right figure corresponds to the metric distance (the classed collection cognition). Each agent on the left side of the figure uses the topological distance and aligns his or her direction using each fixed set of six neighbors. The agent on the right side of the figure uses the metric distance; the range of the metric distance is based on the previous topological distances used by that agent. In this study, we added two areas: a repulsion zone (red area), which is the area in which other agents are avoided, and an attraction zone (blue area), which is the area in which other agents are attracted. These three areas represent the expansion and contraction of the metric domain. The agent of the metric-topological interaction model uses these two methods of cognition and chooses his direction by switching between them. Note that any external noise is added in the metric-topological interaction model. The randomness only emerges in this model when each agent is either correct or incorrect when checking his neighborhood. The role of noise arises due to switching between the classed collection and individual cognitions. The parameters of the metric-topological interaction model are listed in Table 1. In this paper, we set the velocity to be constant for all steps.

Name	Symbol	Value
Speed	v	4.0
Threshold Value of The Individual Cognition	a	0.2
Threshold Value of The Clas. Colle. Cognition	b	0.2
Minimum Replutive Zone	mR_1	40
Minimum Alignment Zone	mR_2	60
Increasing Rate of Replutive Zone	Rp	1.5
Increasing Rate of Attractive Zone	At	2.5
Increasing Rate of Alignment Zone	Al	5.0

Table 1: The parameters of the metric-topological interaction model. Each symbol corresponds to the **Algorithm**.

2.2. The Behavior of the Metric-Topological Interaction Model

The behavior of the metric-topological interaction model appears to be fairly different from the SPPs model. The SPPs model never shows spontaneous, rapid changes of directions in the flock (Vicsek et al., 1995; Couzin et al., 2002). The flock in an SPPs can change their direction only due to the noise for each agent because the SPPs model sets some degree of noise for each agent in advance. In the SPPs model, the noise for the each agent is usually interpreted as the freedom of the individual from the rule of the social interaction.

Algorithm

1. Each agent is allocated space and given a direction at random. Then, go to 2.
2. Individual cognition

$\mathbf{v}_k{}^{t+1} = \mathbf{v}_k{}^{t} + <\mathbf{v}>_{N-IND}$

$\cdot N-IND \equiv \{l \in N | rank(l) = 6\}$

$\cdot rank(l)$; the order of the distance between $\mathbf{x}_k$ and $\mathbf{x}_l$.

$\cdot <>_S$ means the mean value for elements of a set S.

if $(\exists i \in$ N-IND$, |\mathbf{v}_i{}^t - <\mathbf{v}>_{N-IND}| \geq a)\{$

The agent k uses individual cognition for the next step.

$\}$

else $\{$

$R_2 = |\mathbf{x}_k^t - \mathbf{x}_s^t|$ where $rank(s) = 6$

$R_1 = mR_1 + c \cdot Rp$

$R_3 = mR_2 + c \cdot At$

$\cdot c \in N \cup \{0\}$ and $R_2 = Al \cdot c$

The agent k uses classed collection cognition for the next step.

$\}$

3. Classed collection cognition

· for repulsion zone

$\mathbf{v}_k^{rep} = -\sum_{l \in N-CLAS_{rep}} \frac{\mathbf{x}_l^t - \mathbf{x}_k^t}{|\mathbf{x}_l^t - \mathbf{x}_k^t|}$

· N-CLAS$_{rep} \equiv \{l \in N | 0 < |\mathbf{x}_k^t - \mathbf{x}_l^t| \leq R_1\}$

· for alignment zone

$\mathbf{v}_k^{align} = \sum_{l \in N-CLAS_{align}} \frac{\mathbf{v}_l^t}{|\mathbf{v}_l^t|}$

· N-CLAS$_{align} \equiv \{l \in N | R_1 \leq |\mathbf{x}_k^t - \mathbf{x}_l^t| \leq R_2\}$

· for attractive zone

$\mathbf{v}_k^{attr} = \sum_{l \in N-CLAS_{attr}} \frac{\mathbf{x}_l^t - \mathbf{x}_k^t}{|\mathbf{x}_l^t - \mathbf{x}_k^t|}$

· N-CLAS$_{attr} \equiv \{l \in N | R_2 \leq |\mathbf{x}_k^t - \mathbf{x}_l^t| \leq R_3\}$

Then

$\mathbf{v}_k^{t+1} = v \cdot \frac{\mathbf{v}_k^t + \mathbf{v}_k^{rep} + \mathbf{v}_k^{align} + \mathbf{v}_k^{attr}}{|\mathbf{v}_k^t + \mathbf{v}_k^{rep} + \mathbf{v}_k^{align} + \mathbf{v}_k^{attr}|}$

$\cdot v = |\mathbf{v}_k^t|$: v is constant for each individual.

if $(\forall i, j \in$ N-CLAS$, |\mathbf{v}_i{}^t - \mathbf{v}_j{}^t| \leq b)\{$

The agent k uses classed collection cognition for the next step.

$\}$

else $\{$

The agent k uses individual cognition for the next step.

$\}$

4. All agents simultaneously move in the decided-upon direction with velocity v and go to 2 or 3, as determined by the previous step.

This type of the interpretation is seen for most models from many researchers (Buhl et al., 2006; Couzin et al., 2002; Strefler et al., 2008; Giardina, 2008). Notice that this interpretation of noise means that the freedom of the individual is not a relationship between as the entirety of the flock. The modeler must prepare a well-defined degree of freedom for the individual. However, this type of noise never can explain the movement of the entire flock, e.g., rapidly changing direction. To change the flock's direction in a short amount of time, the direction of each individual must not vary from his neighbors, but rather all must have a consensus as to where the neighbors will go as a flock.

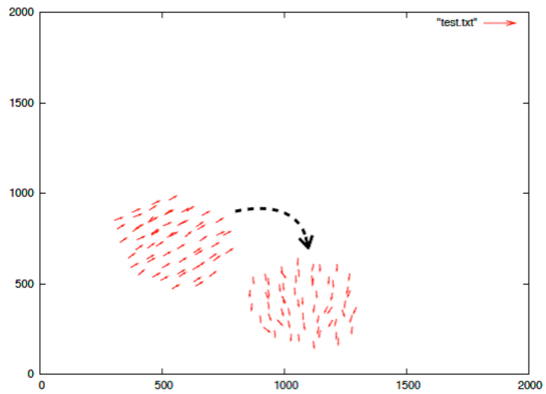

Figure 2: The flock of the metric-topological interaction model rapidly changes their direction (nearly 90°) in 200 steps.

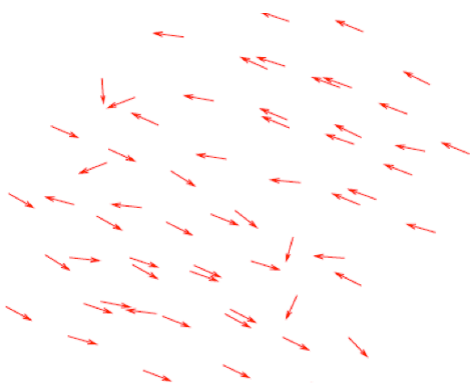

Figure 3: The figure of the flock when represented using the fluctuating vectors. There are two regions that are strongly correlated. The metric-topological interaction model spontaneously makes these domains.

On the other hand, the metric-topological interaction model can show rapid direction as a flock without any external noise. Each agent of the metric-topological interaction model has a different neighborhood or neighbors from the other agents because each agent updates his or her classed collection or individual cognition based on the past state of his or her neighborhood. This difference in the recognition of the surrounding neighborhood results in a noise-like effect for each agent. The neighborhood of each agent, moreover, overlaps with the neighborhoods of other agents and generates a certain direction for groups in the flock. We add the fact that rapid changes of direction never emerge from randomly distributed neighborhoods (the repulsion zone, alignment zone, and attractive zone) for each agent. In this situation, the agents never make a flock and instead behaved like in an SPPs model. This fact suggests that it is important for each agent of the metric-topological interaction model to check his or her neighborhood and to adjust his or her environment based on the movement of his neighbors when we consider agents behaving as a flock.

2.3. Scale-Free Correlation in the Metric-Topological interaction Model

Cavagna et al. originally proposed the notion of scale-free correlation based on their empirical research (Cavagna et al., 2009). They defined the fluctuation vectors that were given by subtracting the average of each velocity vector from each velocity vector. Thus, the fluctuation vector $\mathbf{u}_i$ is given by;

$$\mathbf{u}_i = \mathbf{v}_i - \frac{1}{N}\sum_{k=1}^{N} \mathbf{v}_k, \qquad (1)$$

where N is number of the agents, i is index of each agent, and $\mathbf{v}_i$ is the velocity vector of each agent. To compute each fluctuation vector, we obtain additional information for the interior of the flock. In other words, the fluctuation vector shows the direction of the bias of each agent in the flock. The correlation function $C(r)$ can be computed using the delta function;

$$C(r) = \frac{\sum_{i,j}^{N} \mathbf{u}_i \cdot \mathbf{u}_j \delta(r - r_{ij})}{\sum_{i,j}^{N} \delta(r - r_{ij})}, \qquad (2)$$

The distance between each agent is given by r_{ij}. The delta function is defined by $\delta(r - r_{ij}) = 1$ if $r = r_{ij}$; $\delta(r - r_{ij}) = 0$, otherwise. In real flocks that are represented by fluctuation vectors, the correlation value of any bird in the flock is high when r is small, but it gradually decays when r is large and, finally, becomes zero at a certain length. Cavagna et al. defined this length as the "correlation length"; that is, the sub-flock shares the same information or interests. In other words, the correlation length is defined as,

$$C(r = \xi) = 0, \quad (3)$$

By calculating the value of ξ, we can determine the distance over which the fluctuation vectors are correlated. Cavagna et al. showed that this "correlation length" was proportional to the flock size. Flock sizes are calculated by determining the maximum distance between two birds belonging to the flock. If we represent the flock size as L, then the relation of the correlation length and the flock size must be satisfied such that

$$\xi = aL, \quad (4)$$

For a real flock, the value of the proportional constant, a, is 0.35 (Cavagna et al., 2009). This shows that the correlation length changes to correspond to the flock size. Comparing the scale-free correlation with the metric distance and the topological distance, the scale-free correlation dynamically changes its correlation domain based on the flock size. This result seems peculiar because each bird only interacts with seven neighbors in the topological distance. The correlation domain is clearly larger than the range of the topological distance and, supposedly, is also larger than the metric distance. This shows that each bird shares information, which it knows directly from its neighbor or neighborhood. The flock modeled with the metric distance never showed scale-free correlation.

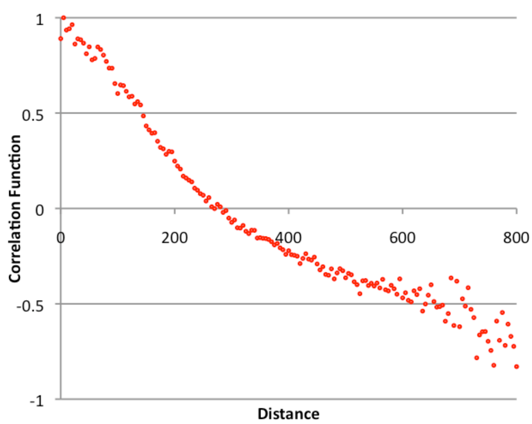

Figure 4: Example of a figure of the correlation function in the flock. The number of agents is 100. The correlation function gradually decreases its value and corresponds to the distance. The correlation length is given when the correlation function crosses 0 on the x-axis. In this case, the correlation length is 270.

We examined the scale-free correlation in a time series of a flock. The number of agents in the flock is 100. The value of the correlation length is obtained from the average of the value of the correlation function for 1,000 steps. The flock lasted for 8,000 steps of the simulation, so we were able to collect 8 data points. One example of the relationship between the correlation function and the distance is shown in 4. It confirms that the value of the correlation function gradually decreases in relation to the distance. The correlation function is obtained by checking where the correlation function first intersects with zero. In this case, the correlation length is 270. We plotted the data in Fig.5 this way. The horizontal axis corresponds to the flock size and the vertical axis corresponds to the correlation length. The correlation length and the flock size are clearly correlated. The value of the proportional constant, a, is 0.35. This value is in good agreement with empirical results. We simulated other cases where we changed the numbers of agents and obtained similar results from 100 times simulations, that is, 0.353±0.022 It is worthwhile to point out that the flock size widely varies from 780 to 900. The agents of the metric-topological interaction model always make their neighborhood along with their environment. It may be that these properties of the metric-topological interaction model are reflected in the scale-free correlation.

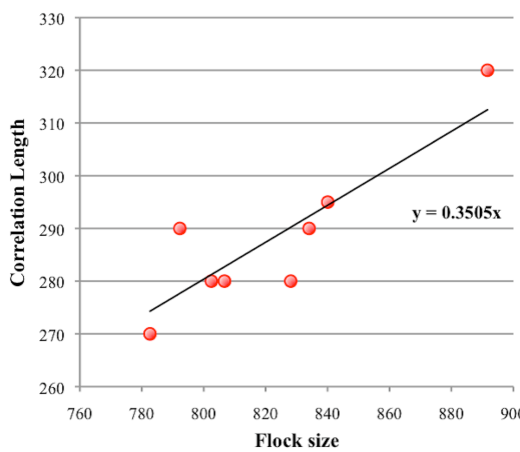

Figure 5: The scale free correlation of the metric-topological interaction model when the number of agents is 100. The horizontal axis corresponds to the flock size and the vertical axis corresponds to the correlation length. The metric-topological interaction model shows a correlation of 0.89 between the correlation length and flock size.

2.4. Role of the Classed Collection and Individual Cognition

To use the classed collection-individual analogy, where the metric distance is the classed collection cognition and the topological distance is the individual cognition, we can interpret the different aspects of its role in scale-free correlation in addition to sharing information for the large domain. Based on the results shown in Fig.5, the scale-free correlation has some connection with the movement of the flock as a whole because the correlation domain extends or compresses its shape corresponding to the flock size. To adjust the size of the correlation domain, it is natural to consider that each agent has information of the whole flock, whose size is over the agent's interaction range. The SPPs model would partly answer this question. The density dependence phase transition is widely seen for the SPPs model. The agent in the SPPs model rapidly aligns in a uniform direction when the density of the agents exceeds a certain value. Once the phase transition occurs, all agents are uniformly aligned in the same direction because many fixed neighborhoods overlap each other. This aspect of SPPs models indicates that each agent knows some information about the flock as a whole, despite the fact that each agent interacts only with agents in its neighborhood. Indeed, the SPPs model is often discussed as an example of self-organization. However, the phase transition of SPPs is difficult to recognize due to fact that it uses the range of the flock for its periodic boundary condition. Cavagna et al. suggested that the SPPs model is not in conflict with the scale-free correlation based on the work of Czirok et al. that showed that the correlation function shows a power law similar
to the scale-free correlation (Czirok and Vicsek, 2006). However, they also implied that the validity of the power law of the SPPs model was dubious because the correlation function was given using unbounded flocks (Cavagna et al., 2009). The SPPs model, therefore, works to partly explain the scale-free correlation but lacks the concept of the flock as a whole in principle. On the other hand, if we interpret the scale-free correlation in the context of the metric-topological interaction model, the classed collection and individual cognitions correspond to the roles in the scale-free correlation. From a review of the SPPs model, the classed collection cognition contributes to the correlated or anti-correlated domain because the classed collection cognition corresponds to the metric distance (that is, the neighborhood in the SPPs model) and each neighborhood of the classed collection cognition overlaps with each other such as in the SPPs model. The correlated domain, thus, is what corresponds to the individual cognition. It is possible that the agents that use the individual cognition correspond to the un-correlated domain. Recall that the individual cognition emerges from the breaking of the neighborhood; there are many agents that have a different direction from each other in the un-correlated domain. Therefore, it can sort two cognitions respective to the scale-free domains into correlated domains or un-correlated domains, and it can suggest that the flock is maintained by adjusting the neighborhood or neighbors for each agent. However, this interpretation raises the following question: If a flock always has two regions that are correlated domains, why would the flock not collapse? This question is relevant because if the flock has an anti-correlated domain that means that at least two sub-flocks have different directions in one flock. Our answer to this question is that the agents that use the individual cognition play a role of the cohesion of these anti-correlated sub-flocks. Indeed, we previously showed that the flocks where the agents stopped, the individual cognitions collapse much easier than the normal metric-topological interaction model (Niizato and Gunji, 2010a).

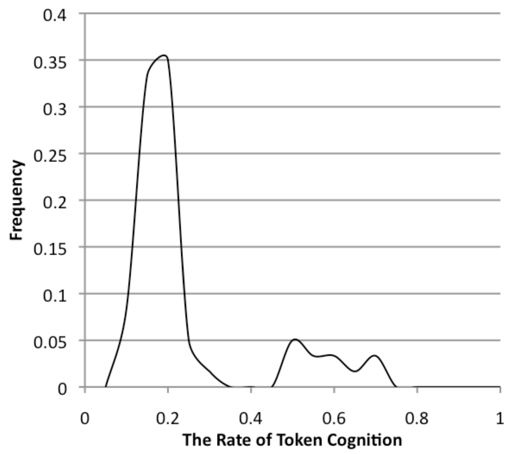

Figure 6: The graph of the relationship between the rate of the individual cognition and its frequency. The horizontal axis corresponds to the rate of the individual cognition and the vertical axis corresponds to its frequency. The number of agents is 60.

Therefore, there the agent of the individual cognition always plays a role of the gluing of two anti-correlated domains in the classed collection cognition flock. The flock maintains its shape due to the spontaneous emergence of agents that use the individual cognition. To see this aspect, we examined the distribution of the number of the individual cognition agents in a set interval. Fig.6

shows the distribution rate of agents using individual cognition in 1,000 steps. There are roughly two peaks in this graph. The left peak means that the rate of agents using the individual cognition in certain steps is 20 percent; in other words, about 12 agents use the individual cognition 200 times in 1,000 steps. Therefore, the agents in this area use the classed collection cognition 80 percent of the time. On the other hand, the right peak of Fig.6 means that there are agents that use the individual cognition 700 times in 1,000 steps. These contrasts are quite often observed for other results as well. It could be considered therefore, that there are always some agents using the individual cognition in the flock. These agents glue correlated domains together and help sustain the flock. The agent of the metric-topological interaction model, thus, spontaneously shares a role of the classed collection (i.e., correlated domain) and the individual (i.e., un-correlated domain) cognition.

3. Discussion

The problem of modeling collective behavior was originally interesting because of its statistical mechanics. The density dependence or noise dependence phase transition seems to be highly relevant to the biological collective phenomenon. Indeed, the density dependence phase transition was largely confirmed for locusts (Buhl et al., 2006) and fishes (Parrich, 1999). In the context of biological phenomena, the noise of an agent was taken as the degree of the freedom from the movement of the whole or the determined social rule. However, the notion of topological distance should be accepted as the meaning of the “density” for an agent in its flock. To explain the topological distance, the agent must be aware of adjusting his neighborhood to keep a constant coherence with its neighbors. However, this dynamic aspect of the topological distance was neglected, and only the statistic aspect of the model was emphasized due to the fact that the agent interacts with only seven neighbors (Giardina, 2008). Unfortunately, there are several concerns with the idea of topological distance. For example, the limit of fish or birds ability to count is three to four (Agrillo et al., 2009; Hunt et al., 2008). The topological distance, therefore, was accepted as the limit of the animal cognition, and the concept of the collective behavior as the statistical mechanics was preserved.

This problem also arises in scale-free correlation. Cavagna et al. tried to explain the scale-free correlation in bird flocks using the context of statistical mechanics and finally concluded that the scale-free correlation may be interpreted as a noise critical phenomenon (Cavagna et al., 2009). In other words, they suggested that if the each agent’s noise was too low, the flock became more stable and it was harder to change the whole flock’s motion. If the noise of the agent was too high, the correlation of the birds would collapse from its noise. This caused them to consider the concept of a “soft degree of freedom”. The agent must not be either too constrained or unconstrained within its flock. They called this state of the agents the “noise critical phenomenon”. This interpretation does not accurately indicate the importance of the scale-free correlation. The scale-free correlation not only encompasses the problem of the adjusting freedom (noise) for the whole flock’s movement but also involves the wholeness that each agent dynamically makes. To understand the scale-free correlation, the agent needs information from a larger range that it can collect from its neighborhood (the metric distance) or neighbors (the topological distance) in the flock. The noise critical phenomenon, in the context of statistical mechanics, cannot explain how the agent shares information from such a large range or how it can dynamically change the range of shared information in proportion to its flock size. If one takes the stance of the noise critical phenomenon, we would miss these important elements of the scale-free correlation.

We proposed the metric-topological interaction model to explain these ideas of a neighborhood in the context of classed collection and individual cognitions. We first confirmed that the flock in the metric-topological interaction model rapidly changed directions without any external noise. Instead, the noise emerged from adjustments between the classed collection and individual cognitions of each agent. In this model, each agent used information from its neighborhood with the assumption that the neighborhood was correct. By using this approach, the agent’s degree of freedom was relevant to the flock as a whole because the agent of the metric-topological interaction model made an assumption about where its neighbors would go. The metric-topological interaction model clearly showed scale-free correlation, as seen in Fig.3. Moreover, it demonstrated that the correlation length was also proportional to the flock size when a flock was followed for one series, and it indicated that the scale-free correlation helped the flock change direction. In this paper, we demonstrated the flaws in using terms of statistical mechanics to understand a biological collective phenomenon. There are certainly some links between them, but one should not confuse them. We proposed the metric-topological interaction model as another possible interpretation of collective behavior and showed that it could demonstrate scale-free correlation without the notion of noise critical phenomena. We believe that the metric-topological interaction model will help us to understand the dynamic properties of collective behavior.

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