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Adaptive Extended Kalman Filtering for Online Monitoring of Concrete Structures Subject to Impacts †

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Abstract

Structures are susceptible to external impacts under long-term service, resulting in various types of damage. Online accurate assessment of the severity of damage is the basis for formulating subsequent maintenance and reinforcement plans. In this work, an online damage identification method based on the Adaptive Extended Kalman Filter (AEKF) is proposed. Initially, the vibration signals of a concrete-filled steel tubular (CFST) test structure subject to multiple lateral impacts are processed, and signals before and after damage inception are spliced to track damage evolution. Subsequently, the natural frequencies extracted from the signals before and after damage inception, and the amplitude of the damage itself are integrated into the state vector, to build a nonlinear state transfer and observation model and allow estimation of the dynamic flexural stiffness of the structure. To further improve the problem solution in the presence of signal losses caused by detachment or breakage of the sensors when damage occurs, the reconstruction of missing signals is accomplished by way of the weighted Matrix Pencil (MP), which ensures the continuity and stability of the AEKF filtering process. By comparing the results with the real damage state, the proposed method is shown to effectively track the gradual reduction of the flexural stiffness, and verifies the feasibility of the proposed method to provide a reliable support for online monitoring and damage assessment.

Keywords: concrete-filled Steel Tube (CFST); impact-induced damage; structural health monitoring (SHM); Adaptive Extended Kalman Filter (AEKF); Matrix Pencil (MP); dynamic flexural stiffness

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1. Introduction

Concrete-filled steel tubes (CFSTs) combine the properties of steel tubes and concrete, to achieve high strength and durability. This type of composite structures is characterized by an extremely high load-bearing capacity, and has widely used as major structural components in large-scale structures and infrastructures [1].

During their service life, CFST structures are susceptible to different external factors including loads (sometimes exceeding the design levels), and temperature variations, which can lead to a series of issues gradually worsening over time. Moreover, extreme conditions due to impacts or earthquakes [2,3], they can experience a damage of varying severity, threatening the safety of the entire structure. In practice, damaged CFST structures are not able to attain anymore their ultimate failure level, though they still have a residual bearing capacity. There are several types of damage that may occur in the core concrete part, such as core debonding, crushing, or cracking, all of which leading to a significant reduction of the relevant flexural stiffness to affect the safe service. Therefore, an accurate assessment of the damage state is a prerequisite for determining appropriate repair measures, if needed.

The Extended Kalman Filter (EKF) [4,5] has been extensively used for nonlinear state estimation in structural dynamics. It works by linearizing the system equations around the current estimate, and it recursively updates the state vector. However, its accuracy strongly depends on the correct specification of the process and measurement noise covariances, which are often unknown or time-varying in practice. To overcome this limitation, the Adaptive Extended Kalman Filter (AEKF) [6,7] has been developed to adaptively updated the noise covariances during the filtering process, thus improving robustness to modeling uncertainties and measurement losses.

In this study, impact tests were conducted on CFST beam–column specimens. Vibration signals were acquired before and after the impacts, and the signals were spliced artificially to simulate real service states. An AEKF, see e.g., [8] was then adopted for the real-time damage identification. Finally, to also address potential data loss induced by structural damage, a weighted Matrix Pencil (MP) method was proposed for reconstructing possibly missing data. Through the comparison with the real damage states observed in the laboratory experiments, the effectiveness of the proposed approach is here validated.

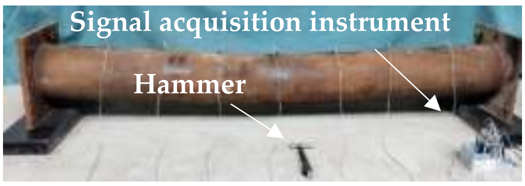
2. Experimental Modal Testing on CFST Specimens

To understand the influence of damage on the vibration properties of CFST structures, a series of CFST specimens were fabricated and a modal testing program was conducted before and after lateral impact tests [9].

2.1. Modal Testing Setup and Data Splicing

Before and after the lateral impact tests, hammer-induced vibration tests were performed on the instrumented CFST specimens. Each specimen was placed on rubber supports [10], and vibration responses in both the pristine and damaged states were recorded for subsequent analysis. Impacts were sequentially applied with a force hammer at fifteen predefined points marked on the top surface of the steel tube. In modal tests, 7 sensors were installed evenly spaced along the top surface of the specimen, and force and acceleration signals were sampled at 25 kHz. To minimize noise effects, each CFST specimen underwent five full repetitions of the entire impact sequence. The experimental setup is illustrated in Figure 1.

(**a**) Before impact

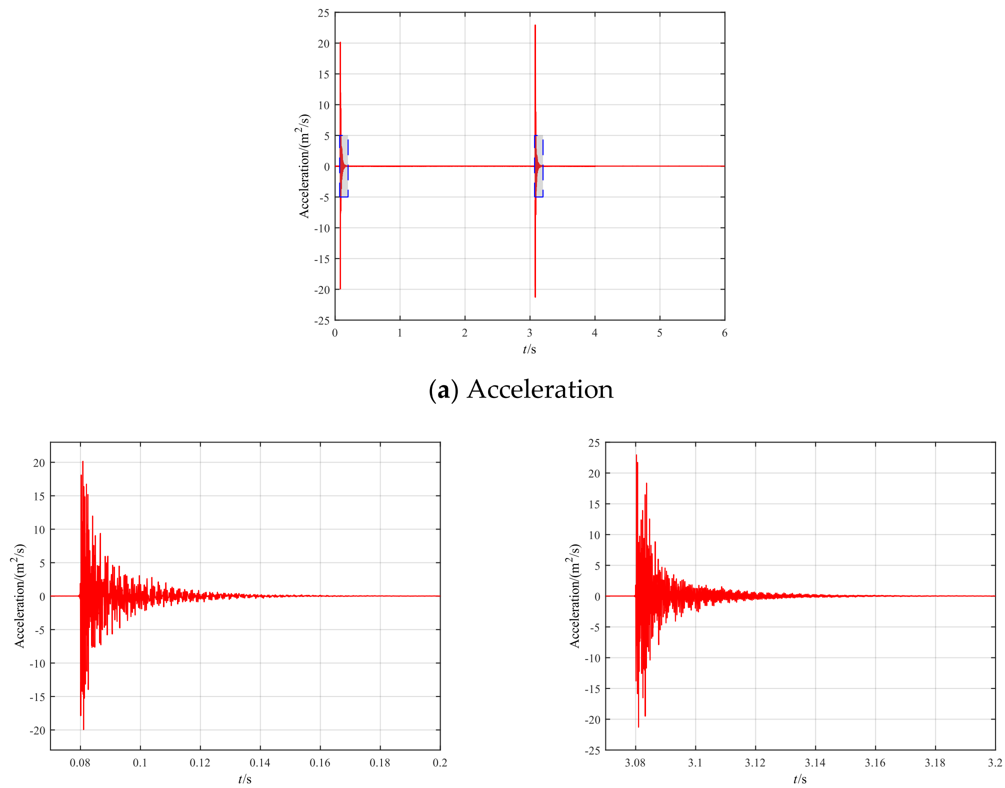


(**b**) After impact

Figure 1. Modal testing setup.

Subsequently, vibrations in 1 s, as obtained before and after the impact, were artificially spliced. At the interval between the intact and damaged signals, the frequency spectra of the final 1000 data points of the baseline signals and the initial 1000 data points of the damaged signals were merged to form a 2 s noise segment. Furthermore, by replicating the spectra of the final 1000 data points of the damaged signals, an additional 2 s noise segment was finally added at the end of the dataset. Therefore, the signal used for online update lasts 6 s. The artificially spliced vibration time history is depicted in Figure 2.

It should be noted that, when damage occurs, the installed sensors may detach from the outer surface of the specimen or even break, leading to a loss of data. In these conditions, reconstructing the missing data is essential to ensure the continuous operation of the online damage identification process. In this paper, an MP method was employed, see [11–13], and a standard Vandermonde matrix [14–17] was constructed to apply weighting to data regions which need a specific focus, that are the tail part of the time domain data and the mode emphasized in the frequency domain.



(**a**) Acceleration

(**b**) Acceleration before damage (**c**) Acceleration after damage

Figure 2. Time history of a vibration signal.

In a discrete time series, data x_k can be represented as a linear superposition of r exponential modes, according to:

$$x_k = \sum_{i=1}^{r} \alpha_i \lambda_i^k \quad (1)$$

where λ_i is i-th mode, α_i is the relevant amplitude, and r is the total number of considered modes.

Subsequently, an $L \times M$ Hankel matrix H is constructed [18–20], with H_1 and H_2 being the submatrices obtained by deleting the last and first column of H, respectively:

$$H = \begin{bmatrix} x_0 & x_1 & \cdots & x_{M-1} \\ x_1 & x_2 & \cdots & x_M \\ \vdots & \vdots & \ddots & \vdots \\ x_{L-1} & x_L & \cdots & x_{L+M-2} \end{bmatrix}, H_1 = \begin{bmatrix} x_0 & x_1 & \cdots & x_{M-2} \\ x_1 & x_2 & \cdots & x_{M-1} \\ \vdots & \vdots & \ddots & \vdots \\ x_{L-1} & x_L & \cdots & x_{L+M-2} \end{bmatrix}, H_2 = \begin{bmatrix} x_1 & x_2 & \cdots & x_{M-1} \\ x_2 & x_3 & \cdots & x_M \\ \vdots & \vdots & \ddots & \vdots \\ x_L & x_{L+1} & \cdots & x_{L+M-2} \end{bmatrix} \quad (2)$$

At this stage, the MP is defined as:

$$\mathrm{P}(z) = H_2 - zH_1 \tag{3}$$

where z is a generalized eigenvalue of the matrix pencil H_1 and H_2. If there exists a non-zero vector v that satisfies the following eigenproblem:

$$(H_2 - zH_1)v = 0 \tag{4}$$

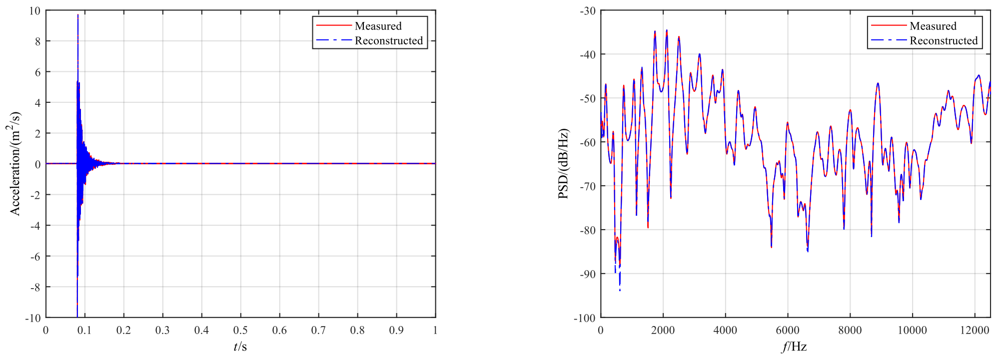
z turns out to be the mode of the signal.

Once the modal parameter identification is completed, the following standard Vandermonde matrix is constructed:

$$V = \begin{bmatrix} \lambda_1^0 & \lambda_2^0 & \cdots & \lambda_r^0 \\ \lambda_1^1 & \lambda_2^1 & \cdots & \lambda_r^1 \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_1^{N-1} & \lambda_2^{N-1} & \cdots & \lambda_r^{N-1} \end{bmatrix} \tag{5}$$

In this matrix, each column corresponds to a modal parameter λ_i and each row corresponds to a discrete k-th time point. The obtained Vandermonde matrix can be then employed to apply weighting to both the time domain and the frequency domain modal components.

A comparison between the measured and reconstructed data is shown in Figure 3. From the graphs, it can be clearly observed that the reconstructed vibration signal is very similar to the measured data, both in the time and frequency domains, to verify the effectiveness of the proposed weighed MP method.



(**a**) Time domain (**b**) Frequency domain

Figure 3. Comparison between the ground-truth and reconstructed signals.

2.2. Adaptive Extended Kalman Filter

Extended Kalman filtering (EKF) [21–23] has been extensively applied in real-time system state estimation, demonstrating excellent performance. In what follows, the discrete EKF and AEKF for structural state estimation are briefly recalled.

Several studies have been devoted to enhancing EKF techniques by adaptively estimating unknown or time-varying system parameters. Among these approaches, maximum likelihood (ML)-based methods iteratively update the measurement noise covariance using optimization techniques, such as sequential quadratic programming [24]. Random-weighting based strategies, including adaptive random-weighted H-infinity filtering and random weighting Kalman filters, have been proposed to handle process and measurement uncertainties under limited prior knowledge [25,26]. Windowing strategies, such as moving-window adaptive fitting H-infinity filters and moving horizon estimation-based adaptive unscented Kalman filters, have been applied to capture temporal variations of noise statistics in nonlinear systems [27,28]. In addition, advanced nonlinear Gaussian filters, including the adaptive unscented Kalman filter (UKF) [29] and adaptive cubature Kalman filter (CKF) [30], have been widely investigated in navigation, sensor fusion, and target tracking applications. These filters integrate online noise covariance estimation with nonlinear transformations of the state distribution, yielding improved estimation accuracy in complex systems.

Moving to the problem of interest here in relation to CFST structures, the flexural stiffness EI_0 of each section can be determined as:

$$(EI)_0 = E_\mathrm{s} I_\mathrm{s} + E_\mathrm{c} I_\mathrm{c} \tag{6}$$

where subscripts s and c respectively refer to steel and concrete; E and I represent the Young's modulus and the moment of inertia of each material. After lateral impacts, a simplified model is adopted to allow for the general influence of concrete damage and steel-concrete debonding on the same flexural stiffness of the CFST specimens. It is assumed that the elastic modulus E_c of concrete is affected by a uniform reduction, so that the stiffness of the beam-columns can be determined as:

$$(EI)_\mathrm{d} = E_\mathrm{s} I_\mathrm{s} + \alpha E_\mathrm{c} I_\mathrm{c} \tag{7}$$

where α is a factor ruling the aforementioned reduction of E_c due to damage.

The equation of motion for a multi degrees of freedom (DOFs) CFST structure can be written as:

$$Mu(t) + Cv(t) + Ka(t) = F(t) \tag{8}$$

where M, C, and K are the mass, damping, and stiffness matrices, respectively; u, v and a are the displacement, velocity and acceleration vectors; F is the external force vector. In this study, the DOF of CFST specimens was set 63.

In the time-discrete version of the EKF, the nonlinear state transition equation and the observation equation are:

$$x_k = \mathbb{f}(x_{k-1}) + w_k \tag{9}$$

$$\mathbb{z}_k = \mathbb{H} x_k + v_k \tag{10}$$

where x_k is the state vector at time instant t_k, which gathers displacement, velocity, acceleration. In this study, it also includes the damage indicator α and the three lowest natural frequencies f_i of the specimen:

$$x_k = \begin{bmatrix} u \\ v \\ a \\ \alpha \\ f_i \end{bmatrix} \tag{11}$$

$\mathbb{z}_k$ is the observation vector at time t_k, which lists the acceleration and the lowest three natural frequencies f_i obtained from the online updating procedure. In Equation (11), $\mathbb{f}(\cdot)$ is the state transition function, which is derived by discretizing the multi DOF equation of motion using the Newmark–β method. $\mathbb{H}$ denotes instead the observation matrix, which provides a map between the measurement and the state variables. In this study, $\mathbb{H}$ is a Boolean matrix used to select the measured accelerations in 7 channels, and the natural frequencies. Finally, w_k and v_k are zero-mean Gaussian processes, with covariance matrices Q_k and R_k, respectively.

In the initial stage of the EKF, the state vector is initialized as follows:

$$\hat{x}_{0|0} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \tag{12}$$

In the prediction stage, the estimate of the predicted state is obtained as:

$$\hat{x}_{k|k-1} = \mathbb{f}\left(\hat{x}_{k-1|k-1}\right) \tag{13}$$

At the same time, the Jacobian matrix is computed as follows:

$$\mathbb{F}_{k-1} = \frac{\partial \mathbb{f}}{\partial x}|_{\hat{x}_{k-1|k-1}} \tag{14}$$

Therefore, the predicted error covariance is given by:

$$P_{k|k-1} = \mathbb{F}_{k-1} P_{k-1|k-1} \mathbb{F}_{k-1}^{T} + Q_{k-1} \tag{15}$$

The innovation, defined as the difference between the measurement and the current prediction, is expressed in the following form:

$$y_k = \mathbb{z}_k - \mathbb{H}\hat{x}_{k|k-1} \tag{16}$$

The innovation covariance and the Kalman gain are defined as:

$$S_k = \mathbb{H} P_{k|k-1} \mathbb{H}^T + R_k \tag{17}$$

$$K_k = P_{k|k-1} \mathbb{H}^T S_k^{-1} \tag{18}$$

and the updated state estimate and corresponding error covariance are given by the following expressions:

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k y_k \tag{19}$$

$$P_{k|k} = (1 - K_k \mathbb{H}) P_{k|k-1} \tag{20}$$

Practical applications of the EKF still have some limitations. While the measurement noise covariance R can typically be determined through repeated experimentation, an incorrectly chosen process noise covariance Q can introduce substantial errors into the structural state estimation. Hence, an accurate selection of Q is critical to ensure a reliable online evaluation of structural state via the EKF. An adaptive, innovation-based method [31,32] for the online estimation of Q is here adopted, according to:

$$Q_k = \left(1 - \beta_{q,k-1}\right) Q_{k-1} + \beta_{q,k-1} K_k y_k y_k^T K_k^T \tag{21}$$

with the initial value of β_q set to 0.25.

A preliminary trial-and-error process allowed to observe that the selection of β_q influences the online updating performance. Therefore, a real-time updating method for β_q was also defined as follows:

$$\beta_{q,k} = \beta_{q,k-1} \times \left(1 + \tanh\left(\frac{y_k^T y_k}{\mathrm{trace}(S_k)} - 1\right)\right) \tag{22}$$

to provide $\beta_{q,k}$ through multiplication by a scaling factor. This latter factor is computed as the ratio of the squared norm of the innovation vector and the trace of the innovation covariance, and then mapped to (−1, 1) through the hyperbolic tangent function. If this ratio is larger than 1, the scaling factor becomes greater than 1, indicating that the system uncertainty is underestimated; conversely, if the ratio is less than 1, the estimated uncertainty is too high.

3. Results

The accuracy of the proposed AEKF method for estimating the reduction of the elastic modulus of the core concrete in CFST beam–column specimens subjected to impacts is now investigated. Simultaneously, data loss scenarios, caused by sensor detachment or breakage when damage occurs, were also simulated. The proposed weighted MP method was then applied for the online reconstruction of the missing data, and these reconstructed data were subsequently used to continue the AEKF update process.

3.1. AEKF Update Results Without Data Loss

In this test case, the initial values of state vector, error covariance matrix, process noise covariance matrix Q, and observation noise covariance matrix R are defined as follows:

$$\hat{x}_{0|0} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}_{193\times 1} \tag{22}$$

$$P_{0|0} = \mathrm{diag}(1\times 10^{-12}, 1\times 10^{-12}, 1\times 10^{-12}, 1\times 10^{-12}, 1\times 10^{-12})_{193\times 193} \tag{23}$$

$$Q_0 = \mathrm{diag}(1\times 10^{-1}, 1\times 10^{-1}, 1\times 10^{-1}, 1\times 10^{-1}, 1\times 10^{-1})_{193\times 193} \tag{24}$$

$$R_0 = \mathrm{diag}(1\times 10^{-1}, 1\times 10^{-1})_{10\times 10} \tag{25}$$

The real-time results of the AEKF are presented in Figure 4 in terms of the time evolution of the damage factor α. From the plot it clearly emerges that the proposed AEKF method promptly detects the damage, and finally converges to the value of 0.637 in no more than 2 s. The said damage value is close to the target one of 0.634, with an error of only 0.47% to ascertain the effectiveness of the proposed approach.

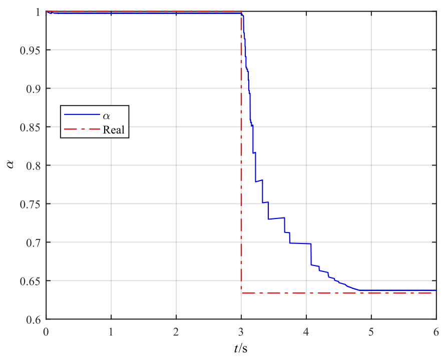
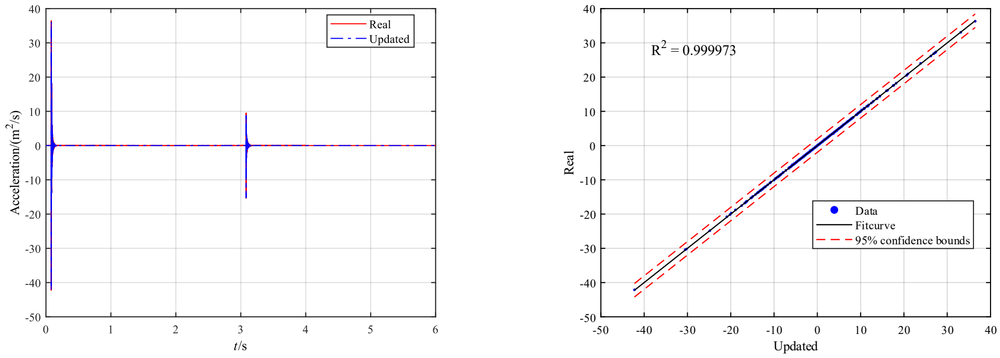


Figure 4. AEKF results: time evolution of the damage factor α with no data loss.

The time history of the acceleration is next shown in Figure 5. The AEKF method shows outstanding performance in such an online acceleration prediction and further prove the effectiveness of the method.



(**a**) Time domain (**b**) Scatter comparison

Figure 5. AEKF results: time evolution of the acceleration signal with no data loss.

3.2. AEKF Update Results with Data Loss

During the service life of the structure, when damage occurs, sensors may detach or break, leading to data loss. Under such circumstances, it is necessary to reconstruct the missing data online and in real-time. In this section, missing damage data are artificially induced and reconstructed using the proposed weighted MP method; after that stage, the AEKF is employed to estimate the damage state. As illustrated in Figure 6, for data reconstructed by the weighted MP method, the proposed AEKF approach still promptly captures the damage features and get online estimation of the elastic modulus reduction with high accuracy.

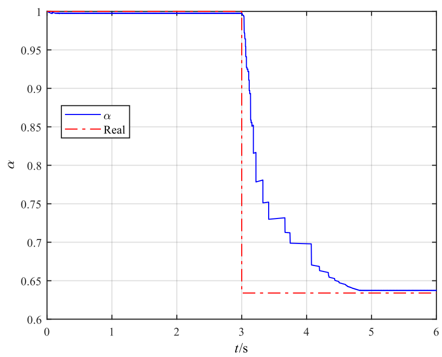
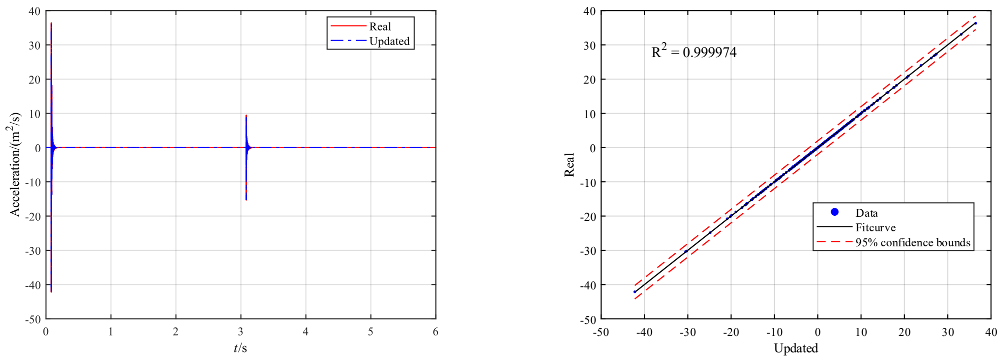


Figure 6. AEKF results: time evolution of the damage factor α in the presence of data.

The online acceleration estimation of the reconstructed data is illustrated in Figure 7, which shows that the AEKF method can estimate the acceleration still with high accuracy. This testifies that the structural state can be captured in real-time by updating the structural parameters.



(**a**) Time domain (**b**) Scatter comparison

Figure 7. AEKF results: time evolution of the acceleration signal in the presence of data loss.

4. Conclusions

In this study, to obtain an online, real-time estimation of structural damage states, an online damage identification method based on the AEKF has been proposed. Simultaneously, to solve the problem of data loss due to damage, a weighted MP method has been proposed to reconstruct the missing data. Based on the obtained results, conclusions can be drawn as follows.

1. By adaptively updating Q and the scale factor β_q, the proposed method avoids the estimation fluctuation problem caused by time invariant Q like in the traditional EKF, so that the filter process remains stable and accurate at varying damage state.
2. The study incorporated the lowest 3 natural frequencies and the damage parameter α into the state vector, and updated them in the nonlinear state transfer and observation model. It can therefore automatically track a gradual degradation process of the structural flexural stiffness, if any.
3. When acceleration data are missing, the weighted MP method can be used and weights were applied to the time domain and the target mode to complete data reconstruction. The reconstructed data were shown to be highly consistent with the real measurements in both time and frequency domains, and the subsequent AEKF process can output the effective stiffness reduction in close agreement with the true value. The proposed methodology has been shown to lead to high accuracy and prompt response to the changing state, as well as tolerance to sensor failures in actual operational conditions.

In future research, the focus will be on extending the proposed AEKF-based monitoring strategy to other structural systems and loading conditions. Furthermore, applications to large-scale structures with multi-sensor fusion and real operational uncertainty will be also explored. Finally, the combination of adaptive filtering with machine learning-based uncertainty evaluation methods will be pursued to enhance structural health monitoring applications.

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