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An emergence of formal logic induced by an internal agent

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Abstract: *In this paper, we mainly address three issues: externality of an agent, purpose of an agent, and a kind of "softness" of components in a system. Agents are independent of a system in an ordinary multi-agent model, hence the behavior of a system is not autonomous but influenced by the agents. If a multi-agent model is considered as a completely autonomous one, agents in the model are inevitably deprived of their externality and independence from the model. In order to treat of the completely autonomous transition of a system, we introduce an agent which is a part of a system, and has a purpose which is independent from a system. The interaction between a system and an agent transforms a random graph corresponding to the system into the graph which represents formal logic adequately. In the emergent graph, there are many complete subgraphs, which can be regarded as conceptualized things. We modify the definition of a conceptualized thing into a subgraph which is a cycle of arrows, and regard the density of arrows of each conceptualized thing as validness. We define this object with the density as a soft object. A complete graph has maximum number of arrows, hence is the most reliable soft object. In a similar way, we call an arrow with the validness a soft arrow, and treat of the relation between soft objects and soft arrows. The argument of this paper is relevant to dynamical formal logic, and at the same time, is intended to serve as a basis for an agent model.*

Keywords: Formal logic, Directed graph, Multi-agent model, Internal measurement

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1. Introduction

We propose a novel model of dynamical formal logic, especially, elaborate the emergence of formal logic. Dynamical transition of formal logic was dealt by Gunji et al. (2004) in the context of Informorphism by Barwise and Seligman (1997). Gunji et al. (2006) also proposed another model based on lattice theory (Davey & Priestley, 2002). We (2007, 2008) also already presented it in the form of a multi-agent model (Wooldridge, 2009). However in this paper, we raise a problem with a multi-agent model as below.

A multi-agent model premises at least one agent by definition. What is an agent? As an answer to this question, firstly we assume that an agent is what is simply transformed in a system. If agents of a system are completely independent of, and external to the system, the behavior of the system can be attributed to the behaviors of agents. Thus, we must check up the property of agents in order to argue about the property of a multi-agent model. This may lead to infinite regress. Responding to this situation, instead of external agents, we introduce an agent which exists inside a system, in other words, is a part of a system. The model which we propose is an internal measurement model of formal logic, where internal measurement proposed by Matsuno (1989). We call an agent which is inside a system completely an internal agent, and also call the model Internal Agent Model.

Another major characteristic of an agent is its autonomy. We define a guiding principle which is inherent in each agent and leads to the autonomous ability, and call it “purpose”. Thus an agent in Internal Agent Model has two main characteristics: the internal and the autonomous.

Classical propositional logic can be composed only of negation and implication. Though here we treat only implication represented by a directed graph (Harary, 1969), it is sufficient because one directed edge between two nodes represents an implicational relation between them, and the absence of a directed edge represents the negation of the implicational relation. However, not every directed graph represents adequately formal logic. We observe the emergence of a directed graph which represents formal logic by the action of an internal agent. Gunji and Higashi (2001) also argued exactly about the relation between directed graphs and category theory (Mac Lane, 1998).

We here make the purpose of an internal agent as the origination of the transitive law of implication. Ordinarily the fundamental property of a logical system is given in the form of an axiom thetically, and the same applies to the transitive law of implication. Instead of this situation, we introduce the transitive law into the formal system as the purpose of an internal agent. This kind of introduction means differentiation or localization of the axiom. A formal system in which a law stands in the whole of the system simultaneously is the system without time for the law. In addition, this introduction enables the system to transform itself continuously, in contrast with the ordinary axiomatic systems which vary discontinuously according to which axioms are adopted.

In addition, we also argue about logical objects in the process of observing the transition of a directed graph. The object in formal logic is obvious, for example, has the property of the reflexive law: X is X. In contrast with the obviousness, there is a critical problem such as Russell’s paradox (Whitehead & Russell, 1925). We present an attempt to solve this problem by introducing the notion “softness” into logical objects.

While we regard the system as a mere graph out of context, the internal agent is nothing more than a subgraph. That is to say, the interaction between the system and the internal agent which we propose in the paper is the interaction between a graph and its subgraph. Moreover, from the definition of the purpose of the internal agent, we can regard the model as the independent applications of the transitive law to either the whole or the part of a system. In a similar way, the notion of softness of an object leads to the uncertainty of the reflexive law (the obviousness of the object). In short, we aim to observe the dynamical feature of formal logic in which the fundamental laws are either deprived or partially adopted.

The paper is organized as follows: firstly we define an internal agent inside a system. An internal agent differs from a part of the system only in that it has a purpose, that is, an internal agent is nothing more than a mere part of the system which has a purpose. Next, we schematize the purpose of an internal agent, and define the interaction between a system and an internal agent. In Section 3, we observe the emergence of a directed graph which represents formal logic adequately out of the interaction, and look into the results under some various conditions. We also observe some distinctive features of the emergent graph. In order to elaborate these features, we define the notion of softness of both an object and an arrow in Section 4. And then we check up some results from particular cases in order to discuss the softness of both an object and an arrow, especially the influence of soft arrows on soft objects. At the last we sum up the difference of tendency among the values of some parameters, however in any case, all the emergent graphs can be regarded as formal logic.

We here present only an emergence of formal logic premised on the purpose of an agent, though we have an interest of an opposite direction also, that is, a representation of the purpose of an agent by means of manipulation of formal logic.

2. Internal Agent Model

2.1. System and internal agent

Hereafter, we treat only the implicational fragment of propositional logic as mentioned in the preceding section. We concern only a directed graph (Harary, 1969), which can represent naturally a set of implication. We represent a system composed of objects and arrows between objects by a directed graph. Naturally, an object and an arrow correspond to a node and a directed edge in a directed graph, respectively. There is no arrow from an object to itself, and there is at most only one arrow between an ordered pair of objects. These settings are for convenience of explanation.

In an ordinary multi-agent model, an agent exists independently outside of the world which is represented by the whole of the system. That is, the agent is an observer and the world is the observable. There is a rigid distinction between them. However, we consider that the externality of an agent is a mere postulate. The agent obviously requires things of the world, which it thinks about or treats. The knowledge which the agent has consists of the components of the world, hence we can regard an aggregate of the components as an agent itself. Thus we set out an agent inside the world. For instance, when the world is represented by a directed graph, we regard a particular subgraph as an agent. Fig. 1 shows an example. Due to this setting, we can treat an agent and objects which are observable things of the world on the same level. We are in state of denial of discrimination of an agent from a system in order to describe completely independent transitions of a system. In addition, an agent becomes nothing more than an object which can observe from the standpoint of internal measurement (Matsuno, 1989). An agent as an object can be naturally influenced by a system. Therefore, there may be interaction between an agent and a system. Now we call such a part of a system an internal agent. We sometimes abbreviate internal agent to agent hereafter.

Another main characteristic of an agent is its autonomy. In general, agents are treated as if agents consider autonomously in a system. The autonomous behavior of an agent requires a guiding principle which is inherent in the agent and can vary according to circumstances, though it may not be seen. We call the guiding principle a purpose. The system which is the outside of the agent cannot concern the purpose of the agent by definition. Indeed we give a purpose to a part of a system and regard the part as an internal agent, and the purpose is independent of the system.

Based on the above understanding, in this paper we define an internal agent as an object of world which has purpose.

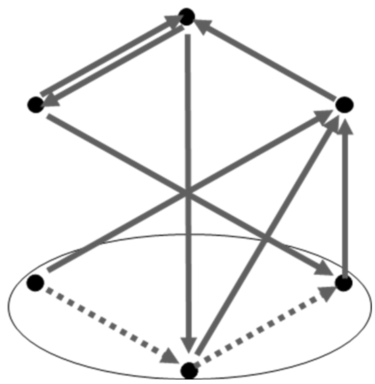


Figure 1: An example of a system and an internal agent. While the whole directed graph represents a system, an internal agent is the part of the graph represented by dashed arrows.

2.2. Purpose of agent

An arbitrary directed graph does not necessarily hold all the properties of formal logic. Hence not every directed graph represents adequately formal logic. We pay notice to the transitive law in this paper as well as our previous papers (Sawa & Gunji, 2007, 2008). We define an index to show the emergence of the transitive law, as follows.

Definition 1 (*Transitivity rate*). Given a directed graph G, TR is defined as

$$\mathrm{TR} := |G| \,/\, |G'|,$$

where $|G|$ is the number of directed edges in G, and G' is the graph transformed from G, in which the transitive law holds completely by adding minimum number of requisite directed edges.

Fig. 2 shows an example. Transitivity rate (TR) is one of measures of reliability of a directed graph as formal logic. Here, the increase of TR is defined as the purpose of an agent.

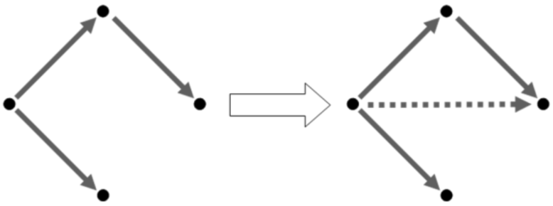


Figure 2: An example of calculating TR. For a given directed graph composed of 3 arrows (left), one dashed arrow must be added in order to hold the transitive law completely (right). Hence the value of TR of a given directed graph is 3/4 = 0.75.

2.3. Interaction between system and internal agent

The internal agent influences the system through pursuit of its purpose. To be more precise, the arrow satisfying the conditions as below can be added to the system. Note that the arrow is added not to the internal agent but to the system. The conditions for adding a new arrow at certain time are set up as follows:

- The arrow can increase TR of the agent if it exists;
- It does not exist in the system at the time (by definition, it inevitably does not exist in the agent);
- It shares at least one node with arrows of the agent.

On the other hand, the agent is a mere object in the system, hence there may also be the influence of a system on an agent as well as ordinary objects. We also set up the influence of the system on the agent. The detail conditions are similar to the influence in the opposite direction:

- The arrow can increase TR of the system if it exists;
- It does not exist in the agent at the time;
- It shares at least one node with arrows of the agent.

In this way, we introduce interaction between the system and agent in the model. We call this interaction S-IA interaction. The system and agent influence each other alternately, and a couple of influences in both directions conduct at each time instant. Fig. 3 shows an example of the transitions by S-IA interaction. The added arrow is randomly chosen in each case. If the finite number of searches of an arrow satisfying the conditions is conducted though the arrow is not found, the influence in that direction is skipped so that no arrows are added. Thus the number of arrows in the system increases monotonically as time proceeds, and the same applies to the agent. The maximum number of arrows is obviously n (n-1) in the directed graph which consists of n nodes, actually the transitions is halted at lower number of arrows in almost every case as we present in the next section. We call this model Internal Agent Model.

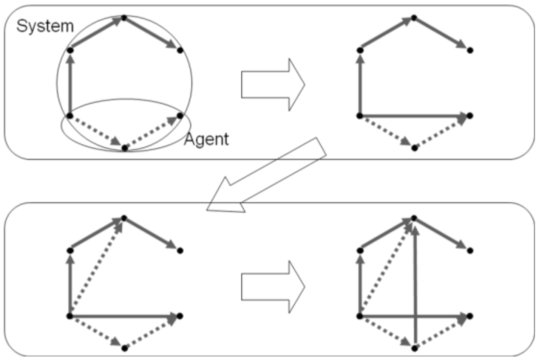
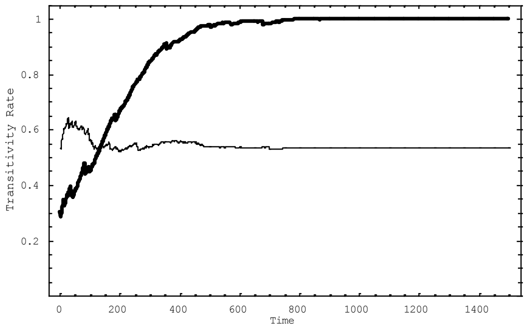


Figure 3: An example of time transitions by S-IA interaction. Dashed arrows represent the agent and all arrows (solid and dashed arrows) represent the system.

3. Results

The initial graph of the system is given at random, and the arrows of the graph of the agent are picked up from the graph of the system at random likewise. Note that the graph of the agent is a subgraph of the system. Fig. 4 shows the results under the conditions that are as follows: 50 nodes, the number of iterations of interactions 1500, the rate of the number of arrows of the initial system to the number of all possible arrows 0.02, and the rate of the numbers of arrows of the initial agent to the number of arrows of the initial system 0.5.

(A)



(B)

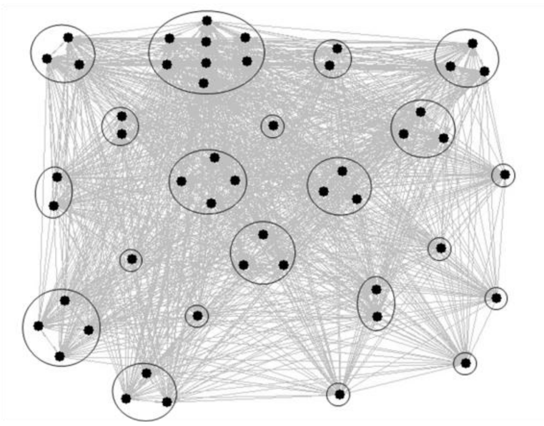


Figure 4: (A) Time transitions of TRs of a system (bold line) and an agent (solid line). Only TR of the system converges at 1. (B) Final state of the graph of a system. Complete subgraphs (including ones composed only of one node) are framed by ovals. There also are “complete” arrows among the complete subgraphs. These complete arrows have the property of the transitive law.

It is clear from Fig. 4 (A) that TR of the system converges at 1. This means an emergence of reliable formal logic in which the transitive law holds completely, however, it may be trivial by definition of S-IA interaction.

There are 1150 arrows in the final state of the graph of the system as shown in Fig. 4 (B). More importantly, 21 complete subgraphs are generated. These complete subgraphs do not overlap, and include 8 complete subgraphs composed only of one node. We call such a complete subgraph composed of one node a singleton. At the same time, the arrows between two arbitrary complete subgraphs are also “complete” ones. That is, there is maximum number of arrows in the same direction between two distinct complete subgraphs. For instance, given two complete subgraphs composed of m and n nodes respectively, there are mn arrows between the two complete subraphs. The transitive law holds among all complete arrows as evidenced by the result that the value of TR is 1.

Thus the distribution of the arrows becomes non-uniform and characteristic one in the final state. In short, the graph is divided into two parts: in which the arrows go in cycles (complete subgraph); in which the arrows flow uniformly (complete arrow). In our opinion, these complete subgraphs and arrows can be regarded as the “hardest” ones as ordinary logical components. A detailed account of this reason will be given in a later section, and here we turn our attention to the transformed graph in which the complete subgraph and arrow are compressed into one node and one arrow respectively.

Fig. 5 (A) shows the graph which is transformed from the graph shown in Fig. 4. Each complete graph including singletons is represented by one node. The arrows are distributed mostly in a line, so that the meaning as formal logic is not rich. That is, the structure of the directed graph is hardly a conjunctive and disjunctive structure, and there are hardly complements of each object. However, the other trial from different conditions can generate a richer structure. See Fig. 5 (B).

The interactional relation between the system and agent is fundamentally linked to the results. In fact, the control experiments show the following results. If the agent is influenced by the agent itself instead of by the system, TR of the system converges at 0.41 and only two cycles of arrows including big one composed of 35 nodes emerge. If the system is influenced by the system itself, TR of the system converges at 1, however, only 3 small complete subgraphs emerge. While the agent is fixed on the initial state and only the system is influenced by the agent, TR of the system converges at 0.29 and only 5 cycles of arrows including a big cycle emerge. In any case, we cannot observe the emergence of a directed graph, which is appropriate to be called formal logic.

In summary, we conclude that (1) S-IA interaction: the interaction between the system and agent inside the system itself yields formal logic; (2) the complete subgraph and arrow, which can be regarded as logical components, are inevitably induced by the emergence of formal logic.

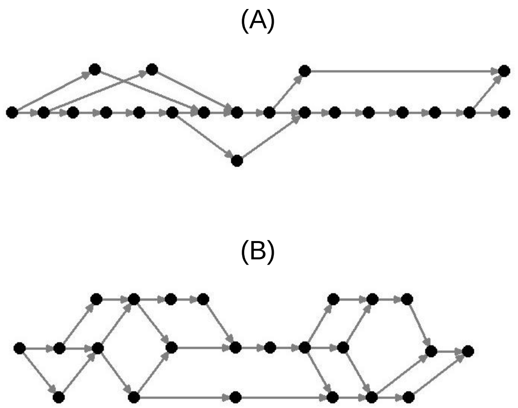


Figure 5: The transformed graphs in which complete subgraphs are compressed into one node. The compression of complete subgraphs naturally involves the compression of complete arrows between complete subgraphs. The transitive law holds in both graphs, therefore only minimum number of arrows is depicted in order to facilitate visualization. (A) The graph which is transformed from the graph of Fig. 4 by compressing of 21 complete subgraphs and corresponding complete arrows. (B) The graph from the other conditions: the rate of the numbers of arrows of the initial agent to the number of arrows of the initial system 0.75, and the others are same as the first trial. It exhibits a more complex structure than the graph of (A).

4. Softness of logical component

4.1. Why softness is needed?

In connection with the completeness of logical components above mentioned, we develop an argument about the inside of logical components. The complete component can be construed as the "hardest" one, while the inside of components is considered and "softness" is introduced into components as described later.

In an ordinary formal logic, the validness of an object is forced to be alternative of 1 or 0, that is, an object either exists or never exists. The intermediate state is unconsidered and not represented. The same applies to an arrow. For example in predicate calculus LK (Troelstra & Schwichtenberg, 2000), there are inevitably sequents such as $X \vdash X$ at the tops of the derivation, in which X denotes an atomic formula. This means that only atomic formulas and formulas composed of atomic formulas are valid, and the other cannot exist in LK. There is a clear distinction between the existence and the nonexistence, and no intermediate states. An atomic formula is a minimum unit in LK. In spite of the arbitrariness of an atomic formula, there is no doubt about the obviousness of an atomic formula. However in our opinion, an atomic formula must be a temporal minimum one for a superior argument. It is realized through the inspection of the inside of an object. For instance, "a dog" is absolutely "a dog", however, the thing "a dog" splits into "a brown dog", "a big dog" and so on, when we show our preference.

Furthermore, this setting of LK realizes that an infinite decomposition of a formula is not permitted, while the infinite composition is permitted. That is to say, a one-way infinity is permitted and there is asymmetry of the decomposition and composition of a formula. In addition, a set of objects is sharply distinguished from objects themselves in terms of logical status (Whitehead & Russell, 1925). In our opinion, such a property of objects also is a mere postulate.

Instead of minimum objects and their hierarchical structure, we here present the argument about the inside of each component in the form of the introduction of softness as hereafter defined. Due to this introduction, the validness of each component is permitted to become the intermediate value between 1 and 0. The existing component of an ordinary formal logic corresponds to the complete one in the preceding section, and we can consider an intermediate state. It follows that the complete component becomes the “hardest” one.

Moreover, we can address the disconnection of a transition of formal logic. It is sometimes difficult that we represent the emergence of a component consecutively. The reason of the emergence never exists inside the system itself. We cannot help regarding the system as what is either dependent on some other thing, or random. In this way, the system implicitly comes to require its outside. The preceding state of the system is not intrinsically related to the following state when we regard the system as an independent one. We consider that this disconnection between the preceding and the following is caused by composing of components whose interior is ignored. While we consider the inside and the softness of components, the transition of the system becomes a kind of continuous one and the outside of the system may not be required.

4.2. Soft object and soft arrow

In predicate logic, each formula has the property of the reflexive law: $X \to X$, where “ $\to$ ” denotes implication. If a cycle of implication (e.g. $X \to Y$, $Y \to Z$, and $Z \to X$) exists, there are implicational relations between two arbitrary objects in the cycle under the transitive law. It follows that every object in the cycle can be assigned to both sides of logical connective “$\to$”, for example, $X \to Z$, $Y \to Y$, and so on. This assignability enables the cycle itself to be regarded as one extended object. Indeed, there is a bundle of arrows in the same direction from the new extended object to itself, and this situation is similar to the reflexive law. Fig. 6 clearly illustrates the similarity by a diagram. Thus we define the set of objects regarded as one unit, as a soft object.

Definition 2 (*Soft object*). In a given directed graph, we call a set of nodes which has the following property a soft object: the set consists of at least 2 nodes, and there is at least one sequence of directed edges in the same direction between every ordered pair of two nodes of the set. Moreover, a node which is not the component of any soft objects composed of multiple nodes is also called a soft object.

The latter type of soft object is a singleton. Soft objects differ in fragility according to the number of arrows in each soft object. A soft object composed of many arrows is more difficult to break than one composed of fewer arrows. See Fig. 7. By definition a complete graph is a soft object, moreover, is actually the “hardest” one.

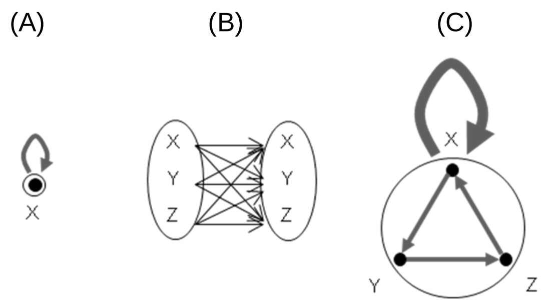


Figure 6: (A) Diagram of the reflexive law: $X \to X$. Cyclic implications $X \to Y$, $Y \to Z$, and $Z \to X$ lead to the situation shown by the diagram of (B) under the transitive law. This can be depicted in the diagram of (C) while the bundle of arrows from the set {X, Y, Z} to itself is represented by one arrow. The left and right diagrams are similar hence we regard the set composed of three nodes as one unit, and call it a soft object.

The introduction of softness enables objects to be divided or united in a nonhierarchical way, and also resolves the asymmetry of the decomposition and composition of a formula in theory. Each soft object has its size, which can increase or decrease.

In a similar way, we define a bundle of arrows in the same direction as a soft arrow.

Definition 3 (*Soft arrow*). In a given directed graph, and given two non-empty sets of nodes, we call a bundle of arrows in the same direction from the set of nodes to another one a soft arrow.

Note that a soft arrow is not between two soft objects but between two sets of nodes. We can also consider the softness of a soft arrow in the same manner of a soft object. The number of arrows in a soft arrow represents the softness: a soft arrow becomes harder in proportion to the increase of arrows in the soft arrow. The maximum number of arrows in a soft arrow is *mn*, if the numbers of nodes of two sets are *m* and *n* respectively.

From the standpoint of soft object and arrow, the results in the preceding section showed the emergence of the soft objects and soft arrows among soft objects, which are in the hardest state. These results reinforce the validness of formal logic induced by S-IA interaction.

(A) (B)

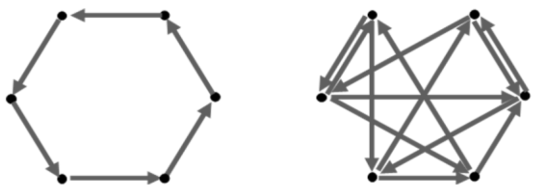


Figure 7: Examples of soft object. (A) If one arbitrary arrow is removed, the soft object breaks into 6 soft objects (singletons), and 5 arrows among them. (B) Removing an arbitrary arrow cannot break up the soft object into smaller ones. Soft object of (A) is “softer” than one of (B).

4.3. Transition of formal logic induced by soft arrows

We conduct a following experiment in order to observe the relation between the softness of objects and arrows. As an initial graph, we give a directed graph composed of 25 nodes. The nodes are divided into 5 sets of 5 nodes. The sets of nodes are linearly-arranged and adjacent two sets are linked by a soft arrow. We emphasize that there are no soft arrows between unadjacent sets, and also no arrows inside a set of nodes. Fig. 8 clearly illustrates an example of the directed graph and its adjacent matrix.

(A)

(B)

(C)

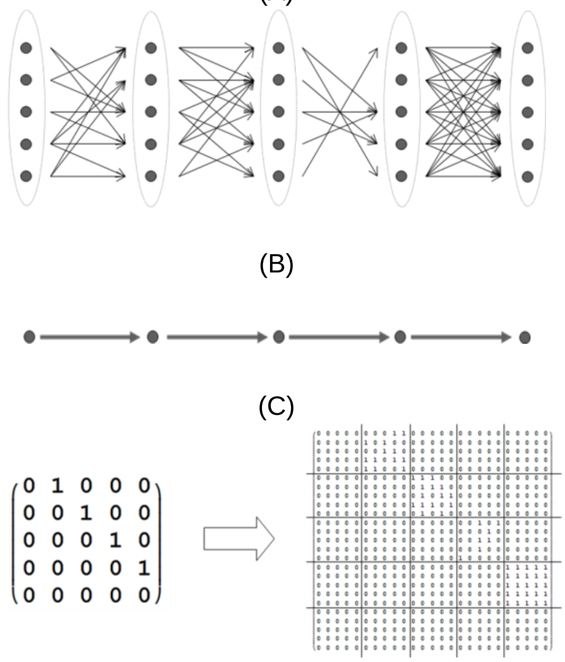


Figure 8: (A) An example of the initial directed graph. There are 5 sets of 5 nodes framed by ovals, and 4 soft arrows among the sets of nodes. This is a graph which represents the directed graph of (B). The softness of soft arrows are various, for example, the number of the rightmost soft arrows is 25 so that the hardest one. (C) The adjacent matrices of the directed graphs of (A) (right), and of (B) (left). This matrix is formed out of the left matrix, by replacements of 0 by a zero submatrix, 1 by a submatrix including one or more 1. These replacements imply the inspection of the inside of each object.

The initial directed graph is formed considering the inside of each object and arrow of the graph composed of 5 objects and 4 arrows. Each set of nodes represents a “latent” object. We mean by “latent” that each set of nodes is not a soft object, however can be regarded as one unit only by force of soft arrows. Soft arrows obviously represent arrows of the former graph. There are soft arrows only between adjacent sets (latent objects) hence the soft arrows do not hold the transitive law.

The graph is consecutively transformed by S-IA interaction, where an initial system is composed of all given arrows and an initial agent is a part of them. The arrows of the initial system are randomly chosen from the graph composed of 4 hardest soft arrows. Note that the “hardest” means that there are 25 arrows between 2 latent objects composed of 5 nodes. The arrows of the initial agent are also randomly chosen from the arrows of the initial system. The rate of the number of arrows of the initial system to the graph composed of 4 hardest soft arrows (*p*), and the rate of the initial agent to the initial system (*q*) are given as initial conditions. After a sufficient number of transitions (at most about 200 transitions), the graph converges in the form of reliable formal logic. That is to say, soft objects and soft arrows emerge as the hardest ones; the transitive law is satisfied among soft arrows; and TR of system is naturally 1. However, the emergence of soft objects from the latent objects involves some errors: some latent objects are divided into smaller soft objects; some soft arrows are also divided into smaller soft arrows and new soft arrows emerge inside of a latent object; new soft objects which are composed of nodes of different latent objects emerge. It is clear from the difference between two results in Fig. 9 that the frequency of the error increases in proportion to the softness of soft arrows of the initial graph of the system (*p*), and also of the agent (*q*). However in either case, the emergent graph represents formal logic adequately, and the logical structure expected from the former graph (the size of each object, and the direction of each arrow and so on) is roughly retained.

This is a time development derived by the softness of arrows, in other words, a transition of formal logic induced by the inspection of the inside of logical components. The candidate of an object (latent object) which is not defined clearly is transformed into a valid object by the relation between the candidates. We note for comparison that a transformation by S-IA interaction from the graph composed of 5 nodes cannot lead to the similar result. It leads to a simpler logic, for example, in which only two soft objects exist. Thus we consider that the inspection of the inside of logical components is fundamental to transition of formal logic.

While we consider that the result in Section 3 represents the first emergence of logic, the result in this section represents the second transformation of logic which has been already established. That is to say, if we remove all arrows of each soft object in the emergent graph, and conduct time transitions by S-IA interaction once again, the graph representing formal logic must be newly transformed. This following experiment in this section is a simple and partial example of this second transformation.

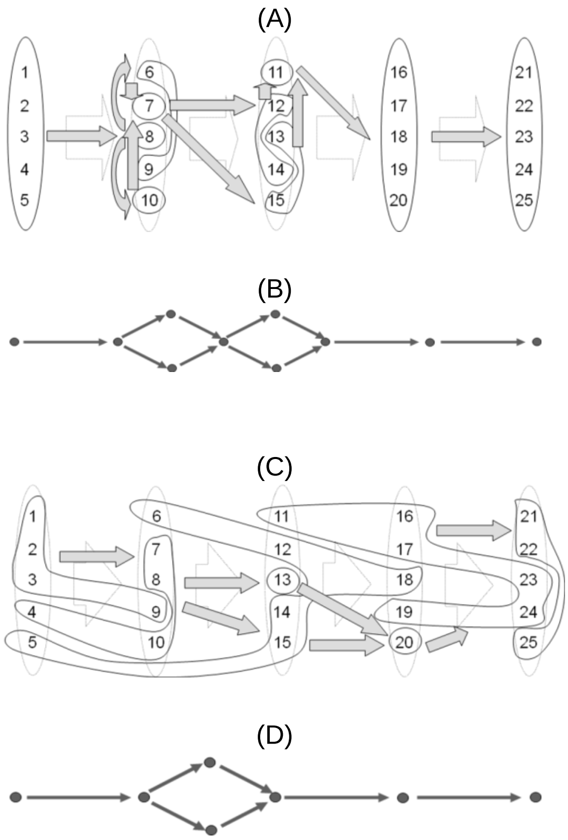


Figure 9: (A, C) Final states of graphs transformed from different initial graphs. Each number represents a node. Soft objects are framed by curved lines. The minimum number of arrows is depicted by heavy arrows. (A) The rates p (for the initial system) and q (for the initial agent) are 1.0 and 0.75 respectively. Though the second and third sets of nodes (latent objects) break into some soft objects, the other ones become soft objects expected from soft arrows. (C) The rates p and q are both 0.5. Soft objects emerge in the different forms than expected ones, however the whole graph represents formal logic. (B, D) The graphs in which soft objects and soft arrows of (A) or (C) are represented by nodes and arrows. As compared with the former graph (Fig. 8 (B)), arrows are split into some ones.

5. Discussions

In short, Internal Agent Model is driven by considering the reflexive and transitive laws, which are the fundamental properties of formal logic. S-IA interaction is indeed the succession of the applications of the transitive law to two parts: the system and agent. When we regard formal logic as what is already formed and rigid one, the transitive law is nothing more than a consequence. However in our opinion, it is also a cause of the transition of logic as well as being a consequence. If the transitive law has such a double meaning, it can impel formal logic to become a dynamical one which responds to diverse situations. And at the same time, ordinary formal logic becomes a snapshot of the dynamical formal logic.

On the other hand, the transition of logic from latent objects as observed in Section 4 is a transition due to an invalidation of the reflexive law. A soft object except the hardest one (a complete graph) is an object in which the reflexive law is partially invalidated, and a latent object is the most completely invalidated one. All nodes are directly connected to all nodes except themselves without the transitive law in the hardest soft object. Though the fragility of a soft object correlates with a lack of arrows, it can be obtained by the application of the transitive law. Thus the transitive law realizes the satisfaction of the reflexive law.

A fundamental concept of mathematics, equivalence law consists of reflexive law, transitive laws and symmetric law ($A \rightarrow B$ implies $B \rightarrow A$). Equivalence law is the condition that a set is treated as one unit. Shinohara et al. (2007) pay notice to symmetrical bias of human cognition (also see Takahashi et al., 2010). While we associate the symmetrical bias with symmetric law, Internal Agent Model is related to these studies with a central focus on equivalence law.

While an object in formal logic represents a concept in the world which formal logic represents, introducing the notion of soft object enables us to represent the conceptualization of objects. Some concepts are united into one new concept, which is treated in the same manner as the constitutive concepts.

Moreover, in formal logic, all formulas are homogeneous, especially from the standpoint of the relations among formulas. That is to say, there are no cases that one formula is associated to many other formulas, and the other is associated to the fewer. Formal logic originally deals with the relations of concepts hence it is natural that formulas are deprived of their individual characteristics. However, while we consider that formal logic is derived from our natural linguistic behavior, we may deal with the individual characteristics in the early stages of the emergence of formal logic. We obtain the transitive law as an axiom in the whole of formal logic and lose the individual characteristic of each formula in process of changing the view of the “natural” formal logic to mere ordinary formal logic.

Due to the simple definition of TR, a new arrow can appear not only at requisite places which are from a source to a target of a sequence of arrows in the same direction, but also at the other diverse places in a directed graph. This positional diversity leads to the emergence of soft objects. The purpose of both the agent and system, which is the guiding principle to transformation, was the increase of TR. We introduce another guiding principle to transformation as substitute for TR as a further experiment, however, the obtained result is not similar to the result by TR, that is, the soft object does not emerge and the distribution is not remarkable. The new guiding principle enables a directed graph to satisfy the transitive law microscopically. That is, as it were, the minimum agents which are unevenly distributed and have no memory. From this result and the results of the control experiments mentioned in Section 3, we conclude that it is necessary for the emergence of formal logic that the agent is sufficiently large in comparison with the size of the system, and can retain an appropriate memory.

We treat only the implicational fragment of propositional logic, however, it is sufficient to represent formal logic. As seen in Fig. 9 (B) and (D), arrows are split into some ones as compared with the former graph depicted in

Fig.8 (B). This split of an arrow in Internal Agent Model corresponds to the situation in formal logic as shown in Fig. 10. That is, an implicational relation $X \rightarrow Y$ is transformed into a set of some implicational relations by observations of $X = A \wedge B$ and $Y = A \vee B$. These observations can be represented by means of soft objects that enable us to inspect the inside of an object. In addition, universal quantifier also can be represented by the potentiality of contraction/expansion of soft objects (Sawa & Gunji, in press). The other topics, for example, complement, and more detailed analysis remain to be solved.

In Internal Agent Model, both a soft object and an agent are mere subgraphs of whole of a system. A soft object is an alternative to an ordinary object: a nonhierarchical, divisible, and incorporable object. Meanwhile, an agent as a mere subgraph at each time instant, however, has purpose when the progress of time is taken into consideration. The agent in the model purposes the adequacy of the system as formal logic. As shown in Section 3, we have the result that soft objects emerge assuming the purpose of an agent. Moreover, we consider that we can treat of the opposite direction: the emergence of the purpose of an agent assuming soft objects, by the argument of the positional relation or inclusive relation among soft objects. That is to say, the purpose of an agent is the temporal property of an object, and the soft object is the spatial property of an object. Although this relation between the purpose and the soft object is within the scope of this study, we leave it as a task for future studies.

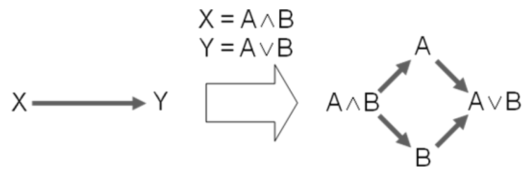


Figure 10: A transformation of an implicational relation due to observations of objects.

6. Conclusions

The local and stepwise applications of the transitive law to a system transform the system to one which represents formal logic. The objects of the system are divided into some sets of objects. There is an arrow between every arbitrary ordered pair of objects in each emergent set, so that each set can be regarded as a logical unit from the aspect of the reflexive law. These are the main features of the model from the standpoint of formal logic.

On the other hand, the model can be construed as a model of interaction between a system and an internal agent which is a part of the system. The system can become a consistent one by virtue of the internal agent. The system is influenced only by the internal agent, hence the transition of the system is completely autonomous one in contrast with a normal multi-agent model. A multi-agent model premises outside of a system, by the name of agent.

In addition, the interaction between the system and the internal agent can fluctuate the structure of formal logic, while we deprive the obviousness of objects. The rate of change from former logic to latter one is related to the softness of both the system and the internal agent. However in either case the transformed structure represents formal logic adequately.

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