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Entropy and Copula Theory in Quantum Mechanics

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Abstract: In classical mechanics, we have individual particle and invariant density in the phase space. In quantum mechanics, any particle is sensitive in a different way from all other particles, for its position and also to the measure process. Thus, we substitute the classical probability in the phase space with the conditional probability in the network of communicating particles. Any probability and entropy are functions of the phase position conditioned by the position of the other particles. Therefore, for different measures we have different conditional entropies. The space of the entropies is a curved and possible torque multidimensional space where the derivative is the covariant derivative on a manifold of the entropic space. At the zero quantum field, the covariant derivative commutes and Fisher matrix is part of the kinetic terms in the Lagrangian where the derivative is the covariant derivative. With Lagrange minimum condition and the entropic space it is possible to show a connection between entropy space and Bohm potential in quantum mechanics. Entropy multidimensional space includes dependence and entanglement as geometric structure of the entropy. Now we can create a non-zero quantum field approach when the covariant derivative does not commute so we have curvature and torsion. The non-zero quantum field can be the Casimir field of forces. Therefore, Casimir force as gravity in the space-time is modelled by curvature and torsion of the entropic space. Useful connection between dependence and covariant derivatives are obtained by copula (dependence measure) and quantum mechanics.

Keywords: Fisher Information; Bohm Potential; Quantum Entropy; Copula Theory; Entropic Geometry; Quantum Fluctuations.

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1. Introduction

In classical mechanics, we have individual particle and invariant density in the phase space. In quantum mechanics, any particle is sensitive in different way from all other particles, for its position and to the measure process. In quantum mechanics, we substitute the classical probability in the phase space with the conditional probability in the network of communicating particles. Therefore, we have the set of conditional density functions or likelihood functions one for each incompatible variable such as momentum and position

$$\rho_j(x_1, x_2, .. x_m \mid \theta_1, \theta_2, .., \theta_n) \qquad (1)$$

That is the probabilities of the observed outcomes given certain parameter values. For example, the parameters are the ordinary standard deviations of position and momentum [1]. In many cases, it is useful the log-likelihood that for the classical thermodynamic equilibrium is connected with the Boltzmann entropy. So, we have

$$S = k \log \rho \qquad (2)$$

Another example is the communication network with electrons that connects atoms in chemical system (valence bond). It is determined by the conditional probabilities of the output events measured on the set of electrons in the i atoms given the input events preparation on the set of electrons in one j atom [2]. The conditional density matrix is

$$\rho_{i,j} = \rho_{i,j}(x_i \mid \theta_j) \qquad (3)$$

Any probability and entropy are functions of the phase position conditioned by the position of the other particles. Therefore, for different measures we have different conditional entropies. The space of the entropies by Boltzmann entropy is given by the multidimensional entropic form

$$S_i = k \log \rho_i(x_i \mid \theta_j) \qquad (4)$$

So for different measures we have the system of the entropies

$$\begin{cases} S_1 = k \log \rho_1 = \xi_1(x_1, .., x_n \mid \theta_1, \theta_2, ..., \theta_m) \\ S_2 = k \log \rho_2 = \xi_2(x_1, .., x_n \mid \theta_1, \theta_2, ..., \theta_m) \\ \quad \ldots \\ S_N = k \log \rho_N = \xi_N(x_1, .., x_n \mid \theta_1, \theta_2, ..., \theta_m) \end{cases} \qquad (5)$$

The (5) is a change of references from entropic reference as Cartesian reference to parametric reference as non- Euclidean reference. Because the metric (square of the entropic distance) for the entropic reference is

$$ds^2 = d\xi_1^2 + d\xi_2^2 + ... + d\xi_N^2 = dS_1^2 + dS_2^2 + ... + dS_N^2 \quad (6)$$

For the parameters reference we have

$$d\xi_j = \frac{\partial \xi_j}{\partial \theta_1} d\theta_1 + \frac{\partial \xi_j}{\partial \theta_2} d\theta_2 + ... + \frac{\partial \xi_j}{\partial \theta_m} d\theta_m \quad (7)$$

So

$$ds^2 = d\xi_1^2 + d\xi_2^2 + ... + d\xi_N^2 =$$
$$\left(\frac{\partial \xi_1}{\partial \theta_1} d\theta_1 + \frac{\partial \xi_1}{\partial \theta_2} d\theta_2 + ... + \frac{\partial \xi_1}{\partial \theta_m} d\theta_m\right)^2 + \left(\frac{\partial \xi_N}{\partial \theta_1} d\theta_1 + \frac{\partial \xi_N}{\partial \theta_2} d\theta_2 + ... + \frac{\partial \xi_N}{\partial \theta_m} d\theta_m\right)^2 \quad (8)$$

That can be written in this way

$$ds^2 = \begin{bmatrix} d\theta_1 \\ d\theta_2 \\ .... \\ d\theta_m \end{bmatrix}^T \begin{bmatrix} \sum_j \frac{\partial \xi_j}{\partial \theta_1} \frac{\partial \xi_j}{\partial \theta_1} & \sum_j \frac{\partial \xi_j}{\partial \theta_1} \frac{\partial \xi_j}{\partial \theta_2} & ... & \sum_j \frac{\partial \xi_j}{\partial \theta_1} \frac{\partial \xi_j}{\partial \theta_m} \\ \sum_j \frac{\partial \xi_j}{\partial \theta_2} \frac{\partial \xi_j}{\partial \theta_1} & \sum_j \frac{\partial \xi_j}{\partial \theta_2} \frac{\partial \xi_j}{\partial \theta_2} & ... & \sum_j \frac{\partial \xi_j}{\partial \theta_2} \frac{\partial \xi_j}{\partial \theta_m} \\ ... & ... & ... & ... \\ \sum_j \frac{\partial \xi_j}{\partial \theta_m} \frac{\partial \xi_j}{\partial \theta_1} & \sum_j \frac{\partial \xi_j}{\partial \theta_m} \frac{\partial \xi_j}{\partial \theta_2} & ... & \sum_j \frac{\partial \xi_j}{\partial \theta_m} \frac{\partial \xi_j}{\partial \theta_m} \end{bmatrix} \begin{bmatrix} d\theta_1 \\ d\theta_2 \\ .... \\ d\theta_m \end{bmatrix} \quad (9)$$

Or

$$ds^2 = \sum_{i,k} G_{i,k} d\theta^i d\theta^k \qquad (10)$$

where

$$G_{i,k}(x_1,...x_n) = \sum_j \frac{\partial \xi_j}{\partial \theta_i} \frac{\partial \xi_j}{\partial \theta_k} \qquad (11)$$

is the metric tensor for the parameter space. The entropic space is a curved space where the derivative is the covariant derivative on a manifold of the entropic space. At the zero quantum field, the covariant derivative commutes and the Fisher matrix is part of the kinetic terms in the Lagrangian where the derivative is the covariant derivative. With Lagrange minimum condition and the entropic space it is possible to show a connection between entropy space and Bohm potential in quantum mechanics. Entropy multidimensional space includes dependence and entanglement as a geometric structure of the entropy. Now we can create non-zero quantum field approach when the covariant derivative does not commute so we have curvature and torsion. The non-zero quantum field can be the Casimir field of forces. So Casimir force as gravity [3] in the space time is modelled by curvature and torsion of the entropic space. Useful connection between peculiar quantum dependence and covariant derivatives can be obtained by copula theory (dependence measure by multivariate probability distribution) within the traditional structure of quantum mechanics

2. Zero field of quantum forces for Boltzmann entropic vector

In this chapter, we will show that classical quantum mechanics can be obtained by the substitution of the classical derivative with the covariant derivative in the Boltzmann entropic space. When we have zero field of quantum mechanics with the change of the derivative operator we can show that Boltzmann entropic geometric interpretation of the mutual non local influence of the particle gives us the quantum potential and, consequently, the Schrödinger equation. We use the Maxwell scheme to clarify the connection between Boltzmann entropic geometry and quantum mechanics [see 4-6]
Given a generic vector which components in the Cartesian space ξ_k are V_j we build its covariant components

$$V = v_k = \sum_j V_j \frac{\partial \xi^j}{\partial \theta^k} = V^j e_j \qquad (12)$$

e_j *is a covariant set of basis vectors*

The covariant set of basis vectors can be represented in this matrix form

$$e_j = e_{j,i}(x) = \begin{bmatrix} e_{1,1} & e_{12} & \cdots & e_{1N} \\ e_{21} & e_{22} & \cdots & e_{2N} \\ \cdots & \cdots & \cdots & \cdots \\ e_{N1} & e_{N2} & \cdots & e_{NN} \end{bmatrix} \quad (13)$$

The controvariant components are

$$v^i = \sum_j V_j \frac{\partial \theta^i}{\partial \xi^j} = V_j e^j \quad (14)$$

e^j is a controvariant set of basis vectors

Where

$$e^j e_k = \delta^j_k \quad (15)$$

Now we define the covariant derivative in this way

$$D_h V = D_h v_k = \frac{\partial v_k}{\partial \theta^h} = \frac{\partial V^j}{\partial \theta^h} \frac{\partial \xi_j}{\partial \theta^k} + V^j \frac{\partial^2 \xi_j}{\partial \theta^h \partial \theta^k} = \frac{\partial V^j}{\partial \theta^h} e_j + V^j \frac{\partial e_j}{\partial \theta^h} \quad (16)$$

In the traditional tensor calculus we have

$$\frac{\partial^2 \xi_j}{\partial \theta^h \partial \theta^k} = \frac{\partial e_j}{\partial \theta^h} = \Gamma^j_{kh} e_j \quad (17)$$

So

$$D_h V = D_h v_k = \frac{\partial V^j}{\partial \theta^h} e_j + V^j \frac{\partial e_j}{\partial \theta^h} = \frac{\partial V^j}{\partial \theta^h} e_j + V^j \Gamma^j_{kh} e_j = (\frac{\partial V^j}{\partial \theta^h} + V^j \Gamma^j_{kh}) e_j \quad (18)$$

The components of the covariant derivative are

$$(\frac{\partial V^j}{\partial \theta^h} + V^j \Gamma^j_{kh}) \quad (19)$$

2.1 Quantum form of the covariant derivative

for

$$e^j = \frac{\partial \theta^i}{\partial \xi_j} \tag{20}$$

$$D_k = \left(\frac{\partial}{\partial \theta^k} + \frac{\partial^2 \xi_j}{\partial \theta^k \partial \theta^h} e^j\right)$$

In fact

$$\begin{aligned} D_k(V^j e_j) &= D_k(e_j V^j) = \left(\frac{\partial}{\partial \theta^k} + \frac{\partial^2 \xi_j}{\partial \theta^k \partial \theta^h} e^j\right) e_j V^j \\ &= \left(\frac{\partial}{\partial \theta^k} e_j + \frac{\partial^2 \xi_j}{\partial \theta^k \partial \theta^h}\right) V^j = \frac{\partial V^j}{\partial \theta^k} e_j + \frac{\partial^2 \xi_j}{\partial \theta^k \partial \theta^h} V^j \end{aligned} \tag{21}$$

Properties of the covariant derivatives. Given

$$D_k = \frac{\partial}{\partial \theta^k} + \frac{\partial^2 \xi_j}{\partial \theta^k \partial \theta^p} \frac{\partial \theta^i}{\partial \xi_j} = \frac{\partial}{\partial \theta^k} + \frac{\frac{\partial^2 \xi_j}{\partial \theta^k \partial \theta^p}}{\frac{\partial \xi_j}{\partial \theta^i}} = \frac{\partial}{\partial x^k} + \frac{\frac{\partial^2 \xi_j}{\partial \theta^k \partial \theta^p}}{\frac{\partial \xi_j}{\partial \theta^i} \frac{\partial \xi_j}{\partial \theta^p}} \frac{\partial \xi_j}{\partial \theta^p} \tag{22}$$

The second derivatives of the entropy ξ can be written in this way

$$\begin{aligned} &\frac{\partial^2 \xi_j}{\partial \theta^k \partial \theta^p} = \frac{\partial^2 \log \rho_j}{\partial \theta^k \partial \theta^p} = \frac{\partial}{\partial \theta^k}\left(\frac{\partial \log \rho_j}{\partial \theta^p}\right) = \\ &\frac{\partial}{\partial \theta^k}\left(\frac{1}{\rho_j} \frac{\partial \rho_j}{\partial \theta^p}\right) = \frac{-1}{(\rho_j)^2} \frac{\partial \rho_j}{\partial \theta^k} \frac{\partial \rho_j}{\partial \theta^p} + \frac{1}{\rho_j} \frac{\partial^2 \rho_j}{\partial \theta^k \partial \theta^p} \\ &= -\frac{\partial \log \rho_j}{\partial \theta^k} \frac{\partial \log \rho_j}{\partial \theta^p} + \frac{1}{\rho_j} \frac{\partial^2 \rho_j}{\partial \theta^k \partial \theta^p} = -\frac{\partial \xi_j}{\partial \theta^k} \frac{\partial \xi_j}{\partial \theta^p} + \frac{1}{\rho_j} \frac{\partial^2 \rho_j}{\partial \theta^k \partial \theta^p} \\ &= -\frac{1}{\rho_j^2} \frac{\partial \rho_j}{\partial \theta^k} \frac{\partial \rho_j}{\partial \theta^p} + \frac{1}{\rho_j} \frac{\partial^2 \rho_j}{\partial \theta^k \partial \theta^p} \end{aligned} \tag{23}$$

2.2 The space of the parameters flat, no curvature when the parameters are independent one from the others

When the parameters in the density of probability are independent, we have

$$\begin{aligned}&\rho_j(x_1,x_2,..,x_N \mid \theta_1,\theta_2,....,\theta_N) = \rho(X \mid \theta_1,\theta_2,....,\theta_N)\\&= \rho_j(X \mid \theta_1)\rho_j(X \mid \theta_2).....\rho_j(X \mid \theta_N)\end{aligned} \tag{24}$$

In this case, we have

$$\begin{aligned}&-\frac{1}{\rho_j^2}\frac{\partial\rho_j}{\partial\theta^k}\frac{\partial\rho_j}{\partial\theta^p}+\frac{1}{\rho_j}\frac{\partial^2\rho_j}{\partial\theta^k\partial\theta^p}=0\\&\textit{and}\\&\frac{\partial^2\rho_j}{\partial\theta^k\partial\theta^p}=\frac{1}{\rho_j}\frac{\partial\rho_j}{\partial\theta^k}\frac{\partial\rho_j}{\partial\theta^p}\end{aligned} \tag{25}$$

And for the (24) :

$$\frac{\dfrac{\partial^2\xi_j}{\partial\theta^k\partial\theta^p}}{\dfrac{\partial\xi_j}{\partial\theta^i}\dfrac{\partial\xi_j}{\partial\theta^p}}\frac{\partial\xi_j}{\partial\theta^p}=\frac{-\dfrac{\partial\xi_j}{\partial\theta^k}\dfrac{\partial\xi_j}{\partial\theta^p}+\dfrac{1}{\rho_j}\dfrac{\partial^2\rho_j}{\partial\theta^k\partial\theta^p}}{\dfrac{\partial\xi_j}{\partial\theta^i}\dfrac{\partial\xi_j}{\partial\theta^p}}\frac{\partial\xi_j}{\partial\theta^p} \tag{26}$$

$$\begin{aligned}&=\frac{\dfrac{1}{\rho_j}\dfrac{\partial^2\rho_j}{\partial\theta^k\partial\theta^p}}{\dfrac{\partial\xi_j}{\partial\theta^i}\dfrac{\partial\xi_j}{\partial\theta^p}}\frac{\partial\xi_j}{\partial\theta^p}-\frac{\partial\xi_j}{\partial\theta^p}=\frac{\dfrac{1}{\rho_j}\dfrac{\partial^2\rho_j}{\partial\theta^k\partial\theta^p}}{\dfrac{1}{\rho_j^2}\dfrac{\partial\rho_j}{\partial\theta^i}\dfrac{\partial\rho_j}{\partial\theta^p}}\frac{\partial\log\rho_j}{\partial\theta^p}-\frac{\partial\log\rho_j}{\partial\theta^p}=\\&\frac{1}{\dfrac{1}{\rho_j}\dfrac{\partial\rho_j}{\partial\theta^i}\dfrac{\partial\rho_j}{\partial\theta^p}}\left(\frac{\partial^2\rho_j}{\partial\theta^k\partial\theta^p}-\frac{1}{\rho_j}\frac{\partial\rho_j}{\partial\theta^k}\frac{\partial\rho_j}{\partial\theta^p}\right)\frac{\partial\log\rho_j}{\partial\theta^p}\end{aligned} \tag{27}$$

For the independent parameters we have

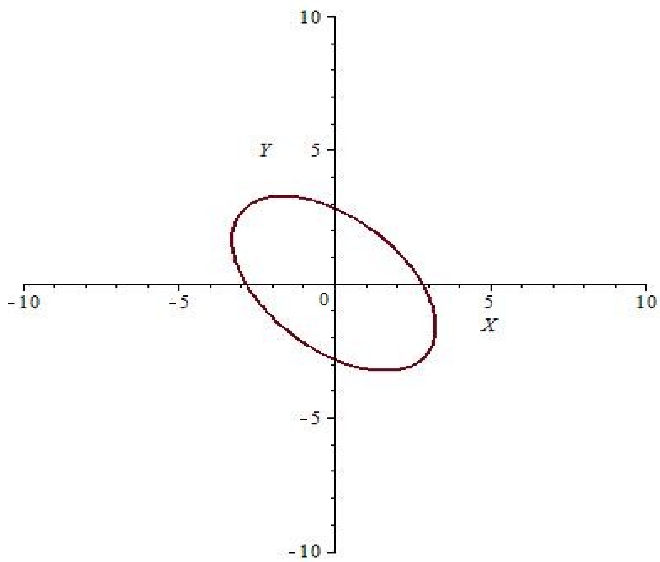
$$D_k=\frac{\partial}{\partial\theta^k}+\frac{\partial^2\xi_j}{\partial\theta^k\partial\theta^p}\frac{\partial\theta^i}{\partial\xi_j}=\frac{\partial}{\partial\theta^k} \tag{28}$$

Given the distribution of probability

$$\rho(x_1, x_2 \mid \theta_1, \theta_2) = k \exp(-[\frac{(x_1-\theta_1)^2 + (x_2-\theta_2)^2 + (x_1-\theta_1)(x_2-\theta_2)}{\sigma^2}] \quad (29)$$

Where the quadratic form is given by the correlation ellipse (fig.1) between parameters

Figure 1. Correlation ellipse



For the previous density of probability, we have

$$D_k = \frac{\partial}{\partial\theta^k} + \frac{\partial^2\xi_j}{\partial\theta^k\partial\theta^p}\frac{\partial\theta^i}{\partial\xi_j} = \frac{\partial}{\partial\theta^k} + \rho\frac{\partial\log\rho}{\partial\theta^k} \quad (30)$$

3. Copula definition

Given the joint probability

$$p(x_1, x_2, ..., x_n) = c(F_1, F_2, ..., F_n) p_1(x_1) p_2(x_2) .... p_n(x_n) \quad (31)$$

Now we want to compute a special transformation for which we can eliminate the marginal probabilities $p(x_j)$ from the previous joint probability. In fact the (1) can be written in this way

$$p_j(x_j)=\frac{dF_j(x_j)}{dx_j}$$

and (32)

$$p(x_1,x_2,...,x_n)=c(F_1,F_2,...,F_n)\frac{dF_1(x_1)}{dx_1}\frac{dF_2(x_2)}{dx_2}.....\frac{dF_n(x_n)}{dx_n}$$

Now we make the integral at the left and at the right of the previous equation so we have

$$\int_0^{u_1}....\int_0^{u_n} p(x_1,x_2,...,x_n)dx_1....dx_n=\int_0^{u_1}....\int_0^{u_n} c(F_1,F_2,...,F_n)\frac{dF_1(x_1)}{dx_1}dx_1....\frac{dF_n(x_n)}{dx_n}dx_n$$
$$=F(u_1,u_2,...,u_n)=\int_0^{u_1}....\int_0^{u_n} c(F_1,F_2,...,F_n)dF_1(x_1)....dF_n(x_n) \tag{33}$$

Now we have

$$u_j=F^{-1}(x_j)$$
$$=F(F^{-1}(x_1),F^{-1}(x_2),...,F^{-1}(x_n))$$
$$=\int_0^{u_1}....\int_0^{u_n} c(F_1,F_2,...,F_n)dF_1....dF_n=C(u_1,u_2,....,u_n) \tag{34}$$

For two variables we have the composition rule

$$C_{1,2}=C(u_1,u_2) \tag{35}$$

We remark that in the function C we lose the product of the marginal functions $p_j(x_j)$ and we have only the function $C(F_1,...,F_n)$ that we denote copula.

The Copula theory deals with the connection between different random distributions or marginal distributions [for a general recent scenario see: 7]

In figure 2 we show the probability distribution and its cumulative function F which values u have homogeneous distribution.

Figure 2. Dependence between two distribution

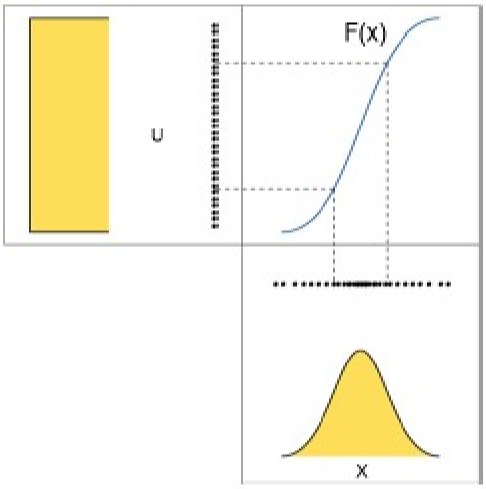


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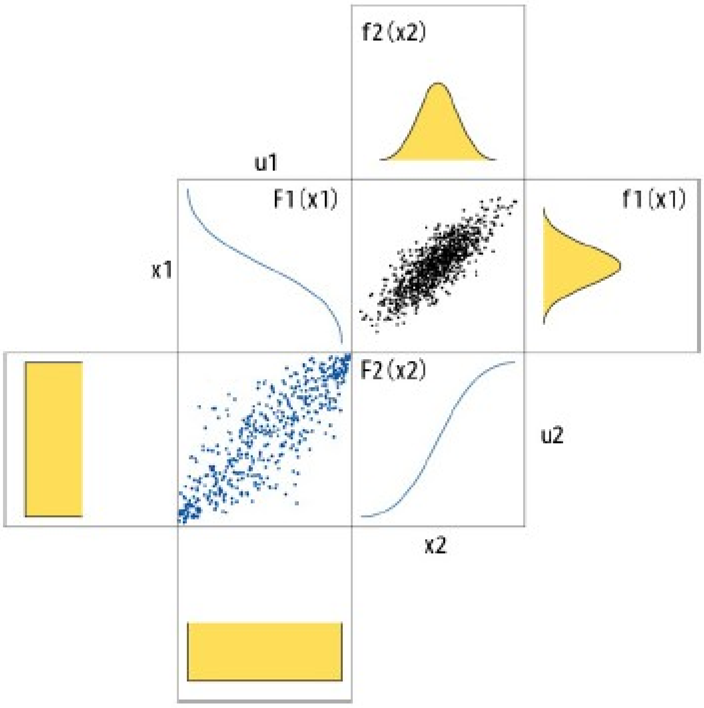
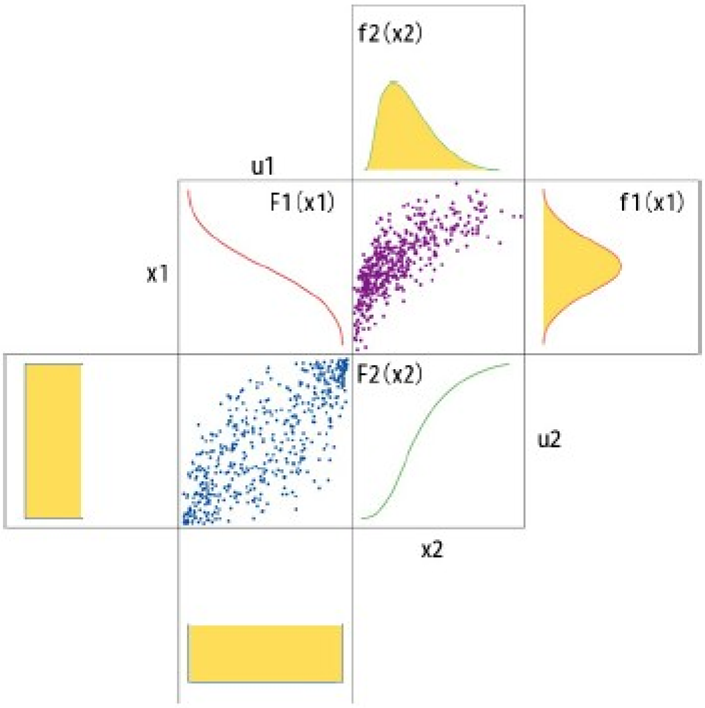
Figure 3. Property of the copula as dependence



In this figure 3, we show one of the most important property of copula. When we have individual different distributions, we know that dependence between elements can be the same. Now, because Copula does not deal with the individual distribution, but only the inter-media dependence the copula does not change its form but only its intensity as we can see in fig.4:

Figure 4. Other example of dependence



We remark that

$$c(u_1,....,u_n)=\frac{\partial^n C(u_1,....,u_n)}{\partial u_1 \partial u_2 ... \partial u_n} \qquad (36)$$

and

$$p(x_1,x_2,....,x_n)=c(u_1,u_2,....,u_n)p_1(x_1)p_2(x_2).....p_n(x_n) \qquad (37)$$

3.1 Copula for two photons entangled in opposite direction

By quantum mechanics, we know that the probability for two photons is given by the expression

$$p(\alpha_1,\alpha_2)=k\sin^2(\alpha_1-\alpha_2) \qquad (38)$$

The cumulative joint function is

$$F(\alpha_1,\alpha_2)=k\int\int\sin^2(\alpha_1-\alpha_2)d\alpha_1 d\alpha_2 = k\sin^2[\frac{1}{2}(\alpha_1^2\alpha_2-\alpha_2^2\alpha_1)] \qquad (39)$$

For the marginal cumulative function we have

$$F_1(\alpha_1) = F(\alpha_1, \frac{\pi}{2}) = k\sin^2[\frac{1}{2}(\alpha_1^2 \frac{\pi}{2} - (\frac{\pi}{2})^2 \alpha_1)]$$
$$F_2(\alpha_2) = F(\frac{\pi}{2}, \alpha_2) = k\sin^2[\frac{1}{2}((\frac{\pi}{2})^2 \alpha_2 - \alpha_2^2 \frac{\pi}{2})] \qquad (40)$$

Now we change - within the cumulative joint probability - the variables (α_1, α_2) into the variables $(F(\alpha_1), F(\alpha_2))$ so we must solve the equation

$$k\sin^2[\frac{1}{2}(\alpha_1^2 \frac{\pi}{2} - (\frac{\pi}{2})^2 \alpha_1)] = F_1(\alpha_1) \; or \sin[\frac{1}{2}(\alpha_1^2 \frac{\pi}{2} - (\frac{\pi}{2})^2 \alpha_1)] = \frac{1}{k}\sqrt{F_1(\alpha_1)}$$
$$and \;\; \alpha_1^2 \frac{\pi}{2} - (\frac{\pi}{2})^2 \alpha_1) = 2\arcsin(\frac{1}{k}\sqrt{F_1(\alpha_1)}) \; and \;\; \alpha_1^2 - \frac{\pi}{2}\alpha_1 - \frac{4}{\pi}\arcsin(\frac{1}{k}\sqrt{F_1(\alpha_1)}) = 0$$
$$and \qquad (41)$$
$$\alpha_1 = \frac{\frac{\pi}{2} \pm \sqrt{(\frac{\pi}{2})^2 + \frac{16}{\pi}\arcsin(\frac{1}{k}\sqrt{F_1(\alpha_1)})}}{2}$$

And

$$k\sin^2[\frac{1}{2}((\frac{\pi}{2})^2 \alpha_2 - \alpha_2^2 \frac{\pi}{2})] = F_2(\alpha_2) \; or \sin[\frac{1}{2}((\frac{\pi}{2})^2 \alpha_2 - \alpha_2^2 \frac{\pi}{2})] = \frac{1}{k}\sqrt{F_2(\alpha_2)}$$
$$and \;\; ((\frac{\pi}{2})^2 \alpha_2 - \alpha_2^2 \frac{\pi}{2}) = 2\arcsin(\frac{1}{k}\sqrt{F_2(\alpha_2)}) \; and \; -\alpha_2^2 + \frac{\pi}{2}\alpha_2 - \frac{4}{\pi}\arcsin(\frac{1}{k}\sqrt{F(\alpha_2)}) = 0$$
$$and$$
$$\alpha_2 = \frac{-\frac{\pi}{2} \pm \sqrt{(\frac{\pi}{2})^2 + \frac{16}{\pi}\arcsin(\frac{1}{k}\sqrt{F_2(\alpha_2)})}}{-2} \qquad (42)$$

So we have the copula

$$C(F(\alpha_1), F(\alpha_2)) = k\sin^2[\frac{1}{2}((\frac{\frac{\pi}{2} \pm \sqrt{(\frac{\pi}{2})^2 + \frac{16}{\pi}\arcsin(\frac{1}{k}\sqrt{F_1(\alpha_1)})}}{2})^2 \frac{\frac{\pi}{2} \mp \sqrt{(\frac{\pi}{2})^2 + \frac{16}{\pi}\arcsin(\frac{1}{k}\sqrt{F_2(\alpha_2)})}}{2} +$$
$$-(\frac{\frac{\pi}{2} \mp \sqrt{(\frac{\pi}{2})^2 + \frac{16}{\pi}\arcsin(\frac{1}{k}\sqrt{F_2(\alpha_2)})}}{2})^2 \frac{\frac{\pi}{2} \pm \sqrt{(\frac{\pi}{2})^2 + \frac{16}{\pi}\arcsin(\frac{1}{k}\sqrt{F_1(\alpha_1)})}}{2})] \qquad (43)$$

The copula for discrete values between 0 and 1 for the cumulative marginal functions is given by the table

$$C = \begin{pmatrix} 0 & 0.125 & 0.25 & 0.375 & 0.5 & 0.625 & 0.75 & 0.875 & 1 \\ 0.125 & 0 & 0.027 & 0.091 & 0.18 & 0.292 & 0.428 & 0.599 & 0.912 \\ 0.25 & 0.027 & 0 & 0.02 & 0.073 & 0.157 & 0.275 & 0.44 & 0.808 \\ 0.375 & 0.091 & 0.02 & 0 & 0.018 & 0.07 & 0.161 & 0.308 & 0.693 \\ 0.5 & 0.18 & 0.073 & 0.018 & 0 & 0.018 & 0.077 & 0.195 & 0.57 \\ 0.625 & 0.292 & 0.157 & 0.07 & 0.018 & 0 & 0.021 & 0.101 & 0.439 \\ 0.75 & 0.428 & 0.275 & 0.161 & 0.077 & 0.021 & 0 & 0.031 & 0.3 \\ 0.875 & 0.599 & 0.44 & 0.308 & 0.195 & 0.101 & 0.031 & 0 & 0.154 \\ 1 & 0.912 & 0.808 & 0.693 & 0.57 & 0.439 & 0.3 & 0.154 & 0 \end{pmatrix}$$

The Copula is symmetric and the extreme value is connected with the classical logic expression $y = x_1 \neq x_2$ which table is

$$y = x_1 \neq x_2 = \begin{bmatrix} & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \qquad (44)$$

So y is logic expression for the “exclusive or”; y is true or one or total dependence when the polarization is opposite and is false (no dependence) when polarization is equal for the two photons. Between 0 and 1 we have other logic values and degrees of dependence. The logic table

$$MM = \begin{pmatrix} 0 & 0.125 & 0.25 & 0.375 & 0.5 & 0.625 & 0.75 & 0.875 & 1 \\ 0.125 & 0 & 0.125 & 0.25 & 0.375 & 0.5 & 0.625 & 0.75 & 0.875 \\ 0.25 & 0.125 & 0 & 0.125 & 0.25 & 0.375 & 0.5 & 0.625 & 0.75 \\ 0.375 & 0.25 & 0.125 & 0 & 0.125 & 0.25 & 0.375 & 0.5 & 0.625 \\ 0.5 & 0.375 & 0.25 & 0.125 & 0 & 0.125 & 0.25 & 0.375 & 0.5 \\ 0.625 & 0.5 & 0.375 & 0.25 & 0.125 & 0 & 0.125 & 0.25 & 0.375 \\ 0.75 & 0.625 & 0.5 & 0.375 & 0.25 & 0.125 & 0 & 0.125 & 0.25 \\ 0.875 & 0.75 & 0.625 & 0.5 & 0.375 & 0.25 & 0.125 & 0 & 0.125 \\ 1 & 0.875 & 0.75 & 0.625 & 0.5 & 0.375 & 0.25 & 0.125 & 0 \end{pmatrix}$$

is topological isomorphic to the copula. The copula C and MM has the same behavior also if the values are different one from the others. So, for the dependence structure we can take the logic expression MM and with local deformation we can came back to the original copula.

3.2 Two slits copula and logic equivalence in many valued logic

For the two slits experiment we have the probability

$$p(\alpha_1, \alpha_2) = k\cos^2(\alpha_1 - \alpha_2) \qquad (45)$$

With the same method of two photons entangled in opposite direction, we have the copula

$$C(F(\alpha_1),F(\alpha_2))=k\sin^2[\frac{1}{2}(\frac{\frac{\pi}{2}\pm\sqrt{(\frac{\pi}{2})^2+\frac{16}{\pi}\arccos(\frac{1}{k}\sqrt{F_1(\alpha_1)})}}{2})^2\frac{\frac{\pi}{2}\mp\sqrt{(\frac{\pi}{2})^2+\frac{16}{\pi}\arccos(\frac{1}{k}\sqrt{F_2(\alpha_2)})}}{2}+$$

$$-(\frac{\frac{\pi}{2}\mp\sqrt{(\frac{\pi}{2})^2+\frac{16}{\pi}\arccos(\frac{1}{k}\sqrt{F_2(\alpha_2)})}}{2})^2\frac{\frac{\pi}{2}\pm\sqrt{(\frac{\pi}{2})^2+\frac{16}{\pi}\arccos(\frac{1}{k}\sqrt{F_1(\alpha_1)})}}{2})] \qquad (46)$$

Which has the table form

$$M1=\begin{pmatrix} 1 & 0.846 & 0.7 & 0.561 & 0.43 & 0.307 & 0.192 & 0.088 & 0 \\ 0.846 & 1 & 0.969 & 0.899 & 0.805 & 0.692 & 0.56 & 0.401 & 0.125 \\ 0.7 & 0.969 & 1 & 0.979 & 0.923 & 0.839 & 0.725 & 0.572 & 0.25 \\ 0.561 & 0.899 & 0.979 & 1 & 0.982 & 0.93 & 0.843 & 0.708 & 0.375 \\ 0.43 & 0.805 & 0.923 & 0.982 & 1 & 0.982 & 0.927 & 0.82 & 0.5 \\ 0.307 & 0.692 & 0.839 & 0.93 & 0.982 & 1 & 0.98 & 0.909 & 0.625 \\ 0.192 & 0.56 & 0.725 & 0.843 & 0.927 & 0.98 & 1 & 0.973 & 0.75 \\ 0.088 & 0.401 & 0.572 & 0.708 & 0.82 & 0.909 & 0.973 & 1 & 0.875 \\ 0 & 0.125 & 0.25 & 0.375 & 0.5 & 0.625 & 0.75 & 0.875 & 1 \end{pmatrix}$$

Which have the same form of the equivalent classical logic $y=x_1\neq x_2$ which table is

$$y=x_1\equiv x_2=\begin{bmatrix} & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (47)$$

We can compare the copula of the two photons in opposite direction with the copula for two photons in the same directions (two slits experiment).

4. Copula and dependence to define covariant derivative in quantum mechanics

For

$$-\frac{1}{\rho_j^2}\frac{\partial\rho_j}{\partial\theta^k}\frac{\partial\rho_j}{\partial\theta^p}+\frac{1}{\rho_j}\frac{\partial^2\rho_j}{\partial\theta^k\partial\theta^p}=\frac{1}{\rho_j}(\frac{\partial^2\rho_j}{\partial\theta^k\partial\theta^p}-\frac{1}{\rho_j}\frac{\partial\rho_j}{\partial\theta^k}\frac{\partial\rho_j}{\partial\theta^p})$$

and (48)

$$\frac{\partial^2\rho_j}{\partial\theta^k\partial\theta^p}=\frac{\partial}{\partial\theta^k}(\frac{\partial\rho_j}{\partial\theta^p})=\frac{\partial}{\partial\theta^k}(\frac{\partial c_j}{\partial\theta^p}\rho(\theta_1)...\rho(\theta_N)+c\rho(\theta_1)...\frac{\partial\rho(\theta_p)}{\partial\theta_p}....\rho(\theta_N))$$

and

$$= \frac{\partial^2 c_j}{\partial\theta^k \partial\theta^p}\rho(\theta_1)...\rho(\theta_N) + \frac{\partial c_j}{\partial\theta^p}\rho(\theta_1)...\frac{\partial\rho(\theta_k)}{\partial\theta_k}...\rho(\theta_N) + \frac{\partial c_j}{\partial\theta^k}\rho(\theta_1)...\frac{\partial\rho(\theta_p)}{\partial\theta_p}....\rho(\theta_N) +$$

$$c\rho(\theta_1)...\frac{\partial\rho(\theta_p)}{\partial\theta_p}...\frac{\partial\rho(\theta_k)}{\partial\theta_k}....\rho(\theta_N))$$

$$\frac{1}{\rho_j}\frac{\partial\rho_j}{\partial\theta^k}\frac{\partial\rho_j}{\partial\theta^p} =$$

$$\frac{1}{c\rho(\theta_1)...\rho(\theta_N)}(\frac{\partial c_j}{\partial\theta^k}\rho(\theta_1)...\rho(\theta_N) + c\rho(\theta_1)...\frac{\partial\rho(\theta_k)}{\partial\theta_k}....\rho(\theta_N))(\frac{\partial c_j}{\partial\theta^p}\rho(\theta_1)...\rho(\theta_N) + c\rho(\theta_1)...\frac{\partial\rho(\theta_p)}{\partial\theta_p}....\rho(\theta_N))$$

we have

$$\frac{1}{\rho_j}(\frac{\partial^2\rho_j}{\partial\theta^k\partial\theta^p} - \frac{1}{\rho_j}\frac{\partial\rho_j}{\partial\theta^k}\frac{\partial\rho_j}{\partial\theta^p}) = \frac{1}{\rho_j}((\frac{\partial^2 c_j}{\partial\theta^k\partial\theta^p} - \frac{1}{\rho_j}\frac{\partial c_j}{\partial\theta^k}\frac{\partial c_j}{\partial\theta^p})\rho_j +$$
$$\frac{\partial c_j}{\partial\theta^p}\rho(\theta_1)...\frac{\partial\rho(\theta_k)}{\partial\theta_k}...\rho(\theta_N) + \frac{\partial c_j}{\partial\theta^k}\rho(\theta_1)...\frac{\partial\rho(\theta_p)}{\partial\theta_p}....\rho(\theta_N) +$$
$$c\rho(\theta_1)...\frac{\partial\rho(\theta_p)}{\partial\theta_p}...\frac{\partial\rho(\theta_k)}{\partial\theta_k}....\rho(\theta_N)) - \frac{c}{\rho_j}(\frac{\partial c_j}{\partial\theta^k}\rho(\theta_1)...\frac{\partial\rho(\theta_p)}{\partial\theta_p}....\rho(\theta_N)\rho_j + \tag{49}$$
$$\frac{\partial c_j}{\partial\theta^p}\rho(\theta_1)...\frac{\partial\rho(\theta_k)}{\partial\theta_k}...\rho(\theta_N)\rho_j + c\rho(\theta_1)...\frac{\partial\rho(\theta_p)}{\partial\theta_p}...\frac{\partial\rho(\theta_k)}{\partial\theta_k}....\rho(\theta_N)))$$

5. Zero quantum field of forces and quantum potential

For the Fisher information we have

$$\frac{E[\frac{\partial^2 \log\rho_j}{\partial\theta^i\partial\theta^j}]}{E[\frac{\partial\log\rho_j}{\partial\theta^i}\frac{\partial\log\rho_j}{\partial\theta^i}]} = 1 \tag{50}$$

For the average theorem, or theorem of the mass probability we have

$$\frac{E[\frac{\partial^2 \log\rho_j}{\partial\theta^i\partial\theta^j}]}{E[\frac{\partial\log\rho_j}{\partial\theta^i}\frac{\partial\log\rho_j}{\partial\theta^i}]} = \frac{\frac{\partial^2 \log\rho_j(c\,|\,\theta)}{\partial\theta^i\partial\theta^j}}{\frac{\partial\log\rho_j(c\,|\,\theta)}{\partial\theta^i}\frac{\partial\log\rho_j(c\,|\,\theta)}{\partial\theta^i}} = 1 \tag{51}$$

Where c is the center of mass or average value. In this situation we have

$$D_k = \frac{\partial}{\partial x^k} + \frac{\partial \xi_j}{\partial \theta^k} = \frac{\partial}{\partial x^k} + \frac{\partial \log \rho_j}{\partial \theta^k} \qquad (52)$$

When the fluctuation near to the average are little we have the gauge relation

$$D_k = \frac{\partial}{\partial \theta^k} + \frac{\partial^2 \xi_j}{\partial \theta^k \partial \theta^p}\frac{\partial \theta^i}{\partial \xi_j} = \frac{\partial}{\partial \theta^k} - \frac{\partial \log \rho_j}{\partial \theta^h} \qquad (53)$$

In this case the field

$$[D_k, D_h] = F_{k,h} = 0 \qquad (54)$$

in fact

$$[D_k, D_h]V_i =$$

$$[(\frac{\partial}{\partial \theta^k} - \frac{\partial \log \rho_j}{\partial \theta^k})(\frac{\partial}{\partial \theta^h} - \frac{\partial \log \rho_j}{\partial \theta^h}) - (\frac{\partial}{\partial \theta^h} - \frac{\partial \log \rho_j}{\partial \theta^h})(\frac{\partial}{\partial \theta^k} - \frac{\partial \log \rho_j}{\partial \theta^k})]V_i = 0 \qquad (55)$$

Because we have

$$\begin{aligned}
&\frac{\partial^2 V_i}{\partial \theta^k \partial \theta^h} - \frac{\partial^2 V_i}{\partial \theta^h \partial \theta^k} = 0 \\
&(\frac{\partial}{\partial \theta^h}(\frac{\partial \log \rho_j}{\partial \theta^k} V_i) + \frac{\partial \log \rho_j}{\partial \theta^h}\frac{\partial V_i}{\partial \theta^k}) - (\frac{\partial}{\partial \theta^k}(\frac{\partial \log \rho_j}{\partial \theta^h} V_i) + \frac{\partial \log \rho_j}{\partial \theta^k}\frac{\partial V_i}{\partial \theta^h}) = \\
&= \frac{\partial^2 \log \rho_j}{\partial \theta^h \partial \theta^k} V_i + \frac{\partial \log \rho_j}{\partial \theta^k}\frac{\partial V_i}{\partial \theta^h} + \\
&+ \frac{\partial \log \rho_j}{\partial \theta^h}\frac{\partial V_i}{\partial \theta^k} - \frac{\partial^2 \log \rho_j}{\partial \theta^k \partial \theta^h} V_i - \frac{\partial \log \rho_j}{\partial \theta^h}\frac{\partial V_i}{\partial \theta^k} - \frac{\partial \log \rho_j}{\partial \theta^k}\frac{\partial V_i}{\partial \theta^h} = 0 \\
&\frac{\partial \log \rho_j}{\partial \theta^h}\frac{\partial \log \rho_j}{\partial \theta^k} - \frac{\partial \log \rho_j}{\partial \theta^k}\frac{\partial \log \rho_j}{\partial \theta^h} = 0
\end{aligned} \qquad (56)$$

We can prove the (29) so the field of quantum forces is equal to zero. In the next chapter we show the connection between zero point quantum force and quantum mechanics. *The previous covariant* *derivative has a very important relation with quantum potential and quantum mechanics.* In fact we have the deformation of the derivative for the non- Euclidean geometry by the expression

$$\frac{\partial}{\partial x^k}+\frac{\partial^2 \xi_j}{\partial x^k \partial x^p}\frac{\partial x^i}{\partial \xi_j}=\frac{\partial}{\partial x^k}-\frac{\partial \log \rho_j}{\partial x_h}=\frac{\partial}{\partial x^k}+A_h^j \qquad (57)$$

Where A_h is like Weyl gauge potential [8, 9]. Now in the classical mechanics the equation of the motion can be written by the definition of the action

$$S=\int \rho[\frac{\partial S}{\partial t}+\frac{1}{2m}\frac{\partial S}{\partial x^i}\frac{\partial S}{\partial x^j}+V]dtd^n x \qquad (58)$$

For quantum mechanics, we have a deformation of the momenta for the change of the geometry. We change the kinetic part of Lagrangian in analogy to the Lagrangian for the electromagnetic vector potential. So we have

$$S=\int \rho[\frac{\partial S}{\partial t}+\frac{1}{2m}(p_i p_j+A_i A_j)+V]dtd^n x=$$

$$S=\int \rho[\frac{\partial S}{\partial t}+\frac{1}{2m}(p_i p_j+\frac{\partial \log \rho_k}{\partial x_i}\frac{\partial \log \rho_k}{\partial x_j})+V]dtd^n x=$$

$$\int \rho[\frac{\partial S}{\partial t}+\frac{1}{2m}p_i p_j+V)dtd^n x+\frac{1}{2m}\int \rho\frac{\partial \log \rho_k}{\partial x_i}\frac{\partial \log \rho_k}{\partial x_j})]dtd^n x \qquad (59)$$

The movement of particles in first approximation is given by the classical mechanics where we assume that the dependence of the particles is so little that we can eliminate it. With a higher approximation we can introduce a new term in the kinetic energy that is proportional to the quantum effect of particle non-isolation measured by Fisher information [10,11; see also: 12] With the Euler Lagrange minimum condition we have that the Fisher information or quantum action assumes the minimum value when

$$\delta S=0$$

For

$$\delta\int \rho[\frac{\partial S}{\partial t}+\frac{1}{2m}p_i p_j+V)dtd^n x=0 \qquad (62)$$

so

$$\frac{\partial S}{\partial t}+\frac{1}{2m}p_i p_j+V+\frac{1}{2m}(\frac{1}{\rho^2}\frac{\partial \rho}{\partial x_i}\frac{\partial \rho}{\partial x_j}-\frac{2}{\rho}\frac{\partial^2 \rho}{\partial x_i \partial x_j})=\frac{\partial S}{\partial t}+\frac{1}{2m}p_i p_j+V+Q \qquad (60)$$

Where Q is the Bohm quantum potential that is a consequence for the extreme condition of Fisher information (minimum or maximum condition for the Fisher information). We know that the quantum potential as real part and the continuous equation as the imaginary part can generate the Schrödinger equation starting from the Boltzmann entropic geometry. We can also use the Schrödinger equation and came back to the Fisher information and to the pure conditional probability interpretation of the quantum mechanics.

We know that the term field means a physical system with an infinite number of degrees of freedom. The generalized coordinates q_i for mechanical system with a finite number of degrees of freedom are replaced by field functions $\rho_j(x_k)$ where the variables x_k are continuous variables. In the entropic approach to quantum mechanics the density of probability assumes a new meaning. So, the density of probability is a field that generalizes the mechanical general coordinates q_i of the different parts of a mechanical system. In the quantum mechanics the finite degree of freedom is substitute with the infinite degree of freedom of the density of probability as a field that has new properties respect to the traditional fields as gravity, electricity and so on. The field of probability is a field of information [13]

6. Non zero field Force (fluctuations) for Boltzmann entropic vector and Casimir force

For the entropic approach, we can assume that the quantum fluctuations are not near to zero [14], so in this case we have

$$D_k = \frac{\partial}{\partial\theta^k} + \frac{1}{\frac{1}{\rho_j}\frac{\partial\rho_j}{\partial\theta^i}\frac{\partial\rho_j}{\partial\theta^p}}\left(\frac{\partial^2\rho_j}{\partial\theta^k\partial\theta^p} - \frac{1}{\rho_j}\frac{\partial\rho_j}{\partial\theta^k}\frac{\partial\rho_j}{\partial\theta^p}\right)\frac{\partial\log\rho_j}{\partial\theta^p} = \frac{\partial}{\partial\theta^k} + \Gamma^i_{k,p} \tag{61}$$

$$= \frac{\partial}{\partial\theta^k} + K^i_{k,p}(\rho_j)\frac{\partial\log\rho_j}{\partial\theta^p}$$

Where $\Gamma^i_{k,p}$ are the Christoffel symbols in the curved parametric space $\theta = (\theta_1, \theta_2, \ldots\ldots, \theta_m)$.

For non-zero field quantum mechanics we have a deformation of the momenta for the change of the geometry so we have

$$S = \int\rho\left[\frac{\partial S}{\partial t} + \frac{1}{2m}(p_i + \Gamma^i_{k,p})(p_j + \Gamma^j_{k.p}) + V\right]dtd^n x =$$

$$S = \int\rho\left[\frac{\partial S}{\partial t} + \frac{1}{2m}(p_i p_j + p_i\Gamma^j_{k.p} + p_j\Gamma^i_{k,p} + \Gamma^i_{k,p}\Gamma^j_{k,p}) + V\right]dtd^n x =$$

$$S = \int\rho\left[\frac{\partial S}{\partial t} + \frac{1}{2m}(p_i p_j + K^i_{k,p}(p)\frac{\partial\log\rho}{\partial x_i}\frac{\partial\log\rho}{\partial x_j}) + V\right]dtd^n x =$$

$$\int\rho\left[\frac{\partial S}{\partial t} + \frac{1}{2m}p_i p_j + V)dtd^n x + \frac{1}{2m}\int\rho K^i_{k,p}(\rho)K^j_{k,p}(\rho)\frac{\partial\log\rho}{\partial x_i}\frac{\partial\log\rho}{\partial x_j})\right]dtd^n x \tag{62}$$

For the differential geometry we have the commutator forms

$$[\nabla_{\mu},\nabla_{\upsilon}]V_{\alpha} = -R^{\lambda}_{\alpha\mu\nu}V_{\lambda} \qquad (63)$$

Where the Riemann tensor is

$$R^{\lambda}_{\alpha\mu\nu} = \partial_{\mu}\ \Gamma^{\lambda}_{\alpha\nu}\ -\ \partial_{\nu}\ \Gamma^{\lambda}_{\alpha\mu} + \Gamma^{\lambda}_{\sigma\nu}\Gamma^{\sigma}_{\alpha\mu}\ -\ \Gamma^{\lambda}_{\sigma\mu}\Gamma^{\sigma}_{\alpha\nu} \qquad (64)$$

For the like-Maxwellian Compensative model [15,16, 17] we put the hypothesis that the commutator will be the Casimir force field. With the double commutator, we have the sources of the quantum dynamics.

$$[\nabla_{\mu},[\nabla_{\alpha},\nabla_{\beta}]]K_{\nu} = [\nabla_{\mu}[\nabla_{\alpha},\nabla_{\beta}]]K_{\nu} - [\nabla_{\alpha},\nabla_{\beta}](\nabla_{\mu}K_{\nu}) =$$
$$= R^{\lambda}_{\mu\alpha\beta}(\nabla_{\mu}K_{\nu}) - (\nabla_{\mu}R^{\lambda}_{\mu\alpha\beta})K_{\lambda} = \chi J_{\mu\alpha\beta}K_{\nu} \qquad (65)$$

Where R is the Riemann Tensor , ∇_k is the covariant derivative , K_{ν} is the vacuum field and $J_{\mu\alpha\beta}$ are the quantum currents or sources. For the conservation of the quantum current we have, after contractions, the equation

$$\nabla_{\mu}[R^{\mu\nu} + \chi(T^{\mu\nu} + \frac{1}{2}g^{\mu\nu}T]K_{\nu} + R^{\mu\nu}(\nabla_{\mu}K_{\nu}) = 0 \qquad (66)$$

When $\nabla_k K_{\nu} = 0$ we have the like Einstein quantum equations not in the space time but in the space of the parameters. Most applications of differential geometry assume that the connection is “torsion free” that is: vectors do not rotate during parallel transport. Because some extensions do include torsion, it is useful to see how torsion appears in standard geometrical definitions and formulas in modern language. The torsion corresponds intuitively to the condition that vectors not be rotated by parallel transport. Such a condition is natural to impose, and the theory of general relativity itself includes this assumption. However, differential geometry is equally well-defined with torsion as without it. We define the torsion tensor by the Christoffel symbols in this way

$$\Gamma^{c}_{ab} - \Gamma^{c}_{ba} = T^{c}_{ab} \qquad (67)$$

Where T^{c}_{ab} is the torsion tensor. In the Boltzmann entropic geometry for quantum mechanics , we are free to take Christoffel symbols with torsion.

7. Conclusion

The key element of the paper is to define affine transformations of the statistical parameters and entropies of the observers (Entropic Transformation) that perturb the physical system by measure. The affine transformations define a non- Euclidean geometry of the statistical parameters as deformation of the original Euclidean space of the entropies. Entropic transformations give the dynamical equation of the Christoffel symbols for the non-Euclidean geometry of the stochastic parameters. In the space of the statistical parameters, derivative is deformed by the particular geometry so we can show that the new derivative can commute or not. By means of the commuting derivative we can find the Bohm quantum potential and Schrodinger equation. With non-commuting derivative we can compute new wave equation for quantum mechanics without the wave function but only by Entropic principle or information for quantum phenomena In conclusion the global interaction of physical objects generates a stochastic form (diffusion) in structured space by geometrical constrains. The form of the entropy or information is generated by all mutual dependences of any object in the universe included the human measures. The Universe itself is not an external object with its law, but it is the result of all possible interactions that we represent by a geometric form.

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