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Mathematical Modeling, Derivation, and Analysis of Polymer Melts Transportation in Extrusion Molding

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INTRODUCTION & AIM

Extrusion molding is one of the most widely used processes for shaping polymeric materials such as plastics and rubbers. Unstable flow phenomena such as sharkskin defects, melt fracture, and turbulent transitions often arise, leading to reduced product quality and inconsistent processing performance. Previous studies have primarily relied on empirical or semi-empirical observations, providing valuable rheological data but limited theoretical insight.

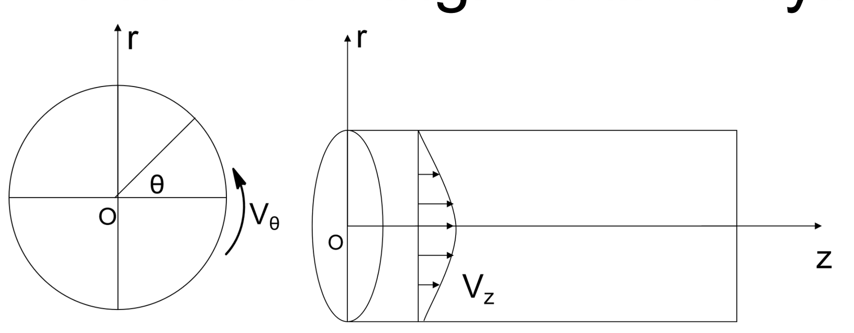
This study aims to: 1. Develop mathematical models that describe polymer transport during extrusion molding using both single- and double-concentric cylinder representations of the screw mechanism; 2. Derive and analyze governing equations for the transport of polymer melts (melting and metering zones); 3. Evaluate boundary conditions—including velocity, flow rate, shear rate, and shear stress—to better understand the mechanisms controlling polymer motion and potential instability development.

METHOD

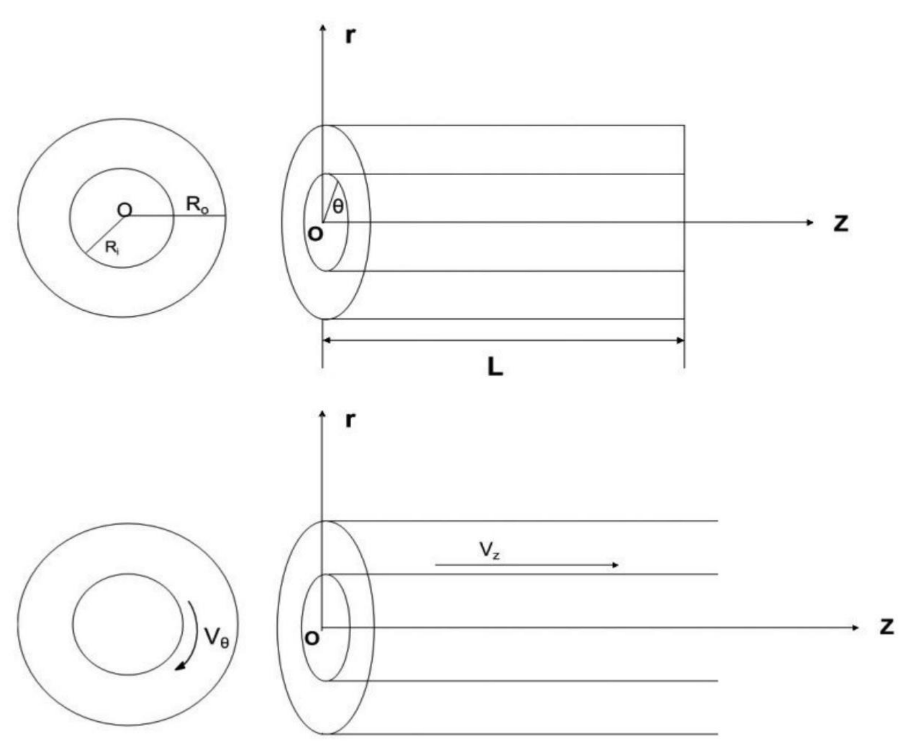
Sensible assumptions are made as following:

1. The fluid is fully developed and in a steady state;
2. The pressure gradient is a constant;
3. Other external forces (like gravity and inertia force) are ignored;
4. The fluid is Newtonian fluid and incompressible;
5. The leak of fluids is ignored.

#1 Model: Single hallow cylinder model:



#2 Model: Double-concentric cylinder model:



RESULTS & DISCUSSION

#1 Model: Single hallow cylinder model:

$$V_\theta = N * r \qquad V_z = \frac{(P_L - P_0)}{4\mu L'} * r^2 - \frac{(P_L - P_0)}{4\mu L'} * R^2$$

$$P = \frac{P_L - P_0}{L'} * z + P_0$$

$$Q_m = \int_0^R V_z * 2\pi r * dr$$

$$= 2\pi \int_0^R \left[\frac{(P_L - P_0)}{4\mu L'} r^3 - \frac{(P_L - P_0)}{4\mu L'} R^2 * r\right] dr = \frac{\pi (P_0 - P_L)}{8\mu L'} R^4$$

$$\gamma_{r\theta} = \frac{dV_\theta}{dr} = N \qquad \gamma_{rz} = \frac{dV_z}{dr} = \frac{(P_L - P_0)}{2\mu L'} r$$

$$\tau_{r\theta} = -\mu \gamma_{r\theta} = -\mu * N \qquad \tau_{rz} = -\mu \gamma_{rz} = -\frac{(P_L - P_0)}{2L'} r$$

#2 Model: Double-concentric cylinder model:

$$V\theta = \frac{Ro^2 \times Ri^2 \times N}{Ri^2 - Ro^2} \times r - \frac{Ro^4 \times Ri^2 \times N}{Ri^2 - Ro^2} \times \frac{1}{r}$$

$$Vz = 2\pi \times D \times \cos\theta \times N \times \left(\frac{\ln\frac{r}{Ro}}{\ln\frac{Ri}{Ro}}\right) + \frac{Ro^2}{4\mu}\left(\frac{P0 - PL}{L}\right)\left[1 - \left(\frac{r}{Ro}\right)^2 - (1 - \alpha^2)\frac{\ln\frac{r}{Ro}}{\ln\frac{Ri}{Ro}}\right]$$

$$\Pr = \frac{1}{2} \times \left(\frac{Ro^2 Ri^2 N}{Ri^2 - Ro^2}\right)^2 \times r^2 - \left(\frac{Ro^4 Ri^2 N}{Ri^2 - Ro^2}\right)^2 \times \frac{1}{r} - \frac{2Ro^6 Ri^4 N^2}{(Ri^2 - Ro^2)^2} \times \ln r + C$$

$$Qm = \int_{Ri}^{Ro} Vz \times 2\pi r \times \mathrm{d}r$$

$$= \int_{Ri}^{Ro} \left\{ 2\pi \times D \times \cos\theta \times N \times \left(\frac{\ln\frac{r}{Ro}}{\ln\frac{Ri}{Ro}}\right) + \frac{Ro^2}{4\mu}\left(\frac{P0 - PL}{L}\right)\left[1 - \left(\frac{r}{Ro}\right)^2 - (1 - \alpha^2)\frac{\ln\frac{r}{Ro}}{\ln\frac{Ri}{Ro}}\right] \right\} \times 2\pi r \times \mathrm{d}r$$

$$= \frac{\pi Ro^4 (P0 - PL)\left[1 - \left(\frac{Ri}{Ro}\right)^2\right]}{8\mu L}\left[1 + \left(\frac{Ri}{Ro}\right)^2 + \frac{1 - \left(\frac{Ri}{Ro}\right)^2}{\ln\frac{Ri}{Ro}}\right] - 2\pi^2 Ro^2 Ri \cos\theta \times N\left[\left(\frac{Ri}{Ro}\right)^2 + \frac{1 - \left(\frac{Ri}{Ro}\right)^2}{2\ln\frac{Ri}{Ro}}\right]$$

$$\begin{cases} \gamma r\theta = \dfrac{\mathrm{d}V\theta}{\mathrm{d}r} = \dfrac{Ro^2 Ri^2 \times N}{Ri^2 - Ro^2} + \dfrac{Ro^4 Ri^2 \times N}{Ri^2 - Ro^2} \times \dfrac{1}{r^2} \\ \tau r\theta = -\mu \times \gamma r\theta = -\mu \times \left(\dfrac{Ro^2 Ri^2 \times N}{Ri^2 - Ro^2} + \dfrac{Ro^4 Ri^2 \times N}{Ri^2 - Ro^2} \times \dfrac{1}{r^2}\right) \end{cases};$$

$$\begin{cases} \gamma rz = \dfrac{\mathrm{d}Vz}{\mathrm{d}r} = \dfrac{2\pi \times Ri \times \cos\theta \times N}{\ln\frac{Ri}{Ro}} \times \dfrac{1}{r} - \dfrac{(P0 - PL)}{4\mu L}\left[2r - \dfrac{\left(1 - \left(\frac{Ri}{Ro}\right)^2\right)}{\ln\frac{Ri}{Ro}} \times \dfrac{R0^2}{r}\right] \\ \tau rz = -\mu \times \gamma rz = -\mu \left\{ \dfrac{2\pi \times Ri \times \cos\theta \times N}{\ln\frac{Ri}{Ro}} \times \dfrac{1}{r} - \dfrac{(P0 - PL)}{4\mu L}\left[2r - \dfrac{\left(1 - \left(\frac{Ri}{Ro}\right)^2\right)}{\ln\frac{Ri}{Ro}} \times \dfrac{R0^2}{r}\right] \right\} \end{cases}$$

CONCLUSION & FUTURE WORK

This study establishes a unified theoretical framework that elucidates the transport behavior of polymers during extrusion molding, providing a foundation for predicting and mitigating flow instabilities. Future work will extend the model to include thermal coupling and non-Newtonian rheology for broader applicability across polymer systems.