

Modeling coastal morphology with sediment transport and Kelvin–Voigt seabed behavior

Maria Antonietta Scarcella¹, Manuela Carini¹

¹Department of Environmental Engineering (DIAM), University of Calabria
ma.scarcella@unical.it; manuela.carini@unical.it

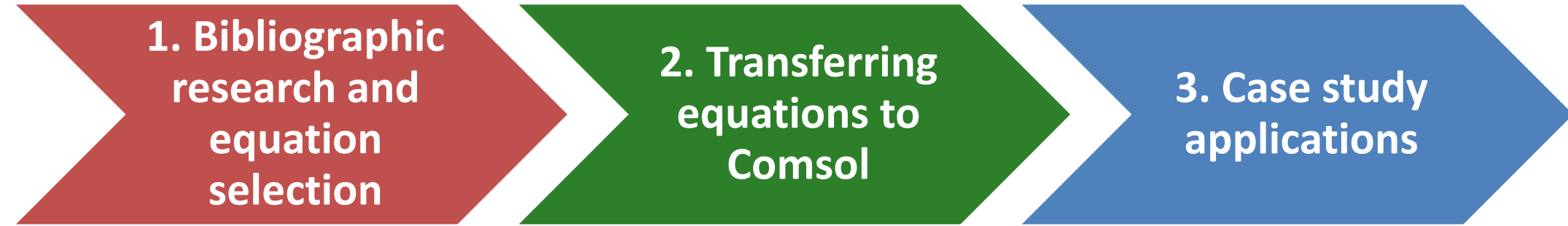
INTRODUCTION & AIM

PROBLEM. Coastal erosion is increasingly influenced by anthropogenic alterations to the sediment cycle and morphological transformations. Traditional shallow water models often neglect the mechanical behavior of the seabed and its rheological response to hydrodynamic forcing, limiting their accuracy in forecasting erosion patterns.

GOALS. To address these limitations this study extends the classical one-dimensional Saint-Venant (shallow water) model by incorporating effects of viscosity, frictional effects, sediment transport (Exner) and viscoelasticity (Kelvin–Voigt). Six idealized test cases (bed step/depression), solved in COMSOL Multiphysics, are used to examine how substrate elasticity and damping influence morphodynamic transients. Our aim is to improve erosion forecasts by embedding seabed mechanics into shallow-water morphodynamics and to identify regimes where viscoelastic effects are most relevant for coastal management.

RESULTS. Results show that viscoelasticity stabilizes bed evolution by attenuating oscillations and smoothing abrupt bathymetric variations.

METHOD



1. Bibliographic research and equation selection

Saint Venant/Exner/Kelvin–Voigt equations–1D

Continuity equation

$$\frac{\partial z}{\partial t} + \frac{\partial(zv)}{\partial x} = 0$$

Momentum equation

$$\frac{\partial(zv)}{\partial t} + \frac{\partial(zv^2)}{\partial x} + g \cdot z \cdot \left(\frac{\partial z}{\partial x} + \frac{dz_f}{\partial x} + S_f \right) - (\nu_c + E) \frac{\partial^2(zv)}{\partial x^2} = 0$$

Conservation of the mass of sediments equation

$$(1 - \rho) \frac{\partial(z_f)}{\partial t} + \frac{\partial(q_s)}{\partial x} = -k_b(z_f - z_{f0}) - \eta_b^{(v)} \frac{\partial z_f}{\partial t}$$

- ✓ $z = z(x, t)$ is the depth of the water column;
- ✓ $v = v(x, t)$ is the fluid velocity;
- ✓ g is the gravitational constant;
- ✓ $S_f = S_f(x, t)$ is the friction term;
- ✓ ν_c is the kinematic viscosity;
- ✓ E is the dispersion coefficient;
- ✓ z_f describes the topography of the seabed;
- ✓ ρ is the porosity of sediment;
- ✓ q_s is the volumetric bedload sediment transport rate;
- ✓ $k_b = E_b/H_b$ the elastic stiffness;
- ✓ E_b is the elastic modulus of the seabed;
- ✓ H_b is the effective deformable thickness of the active seabed layer participating in reversible deformation;
- ✓ $\eta_b^{(v)} = \eta_b/H_b$ is the viscous damping of the seabed;
- ✓ η_b is rate-dependent damping of the seabed.

2. Transferring equations to Comsol

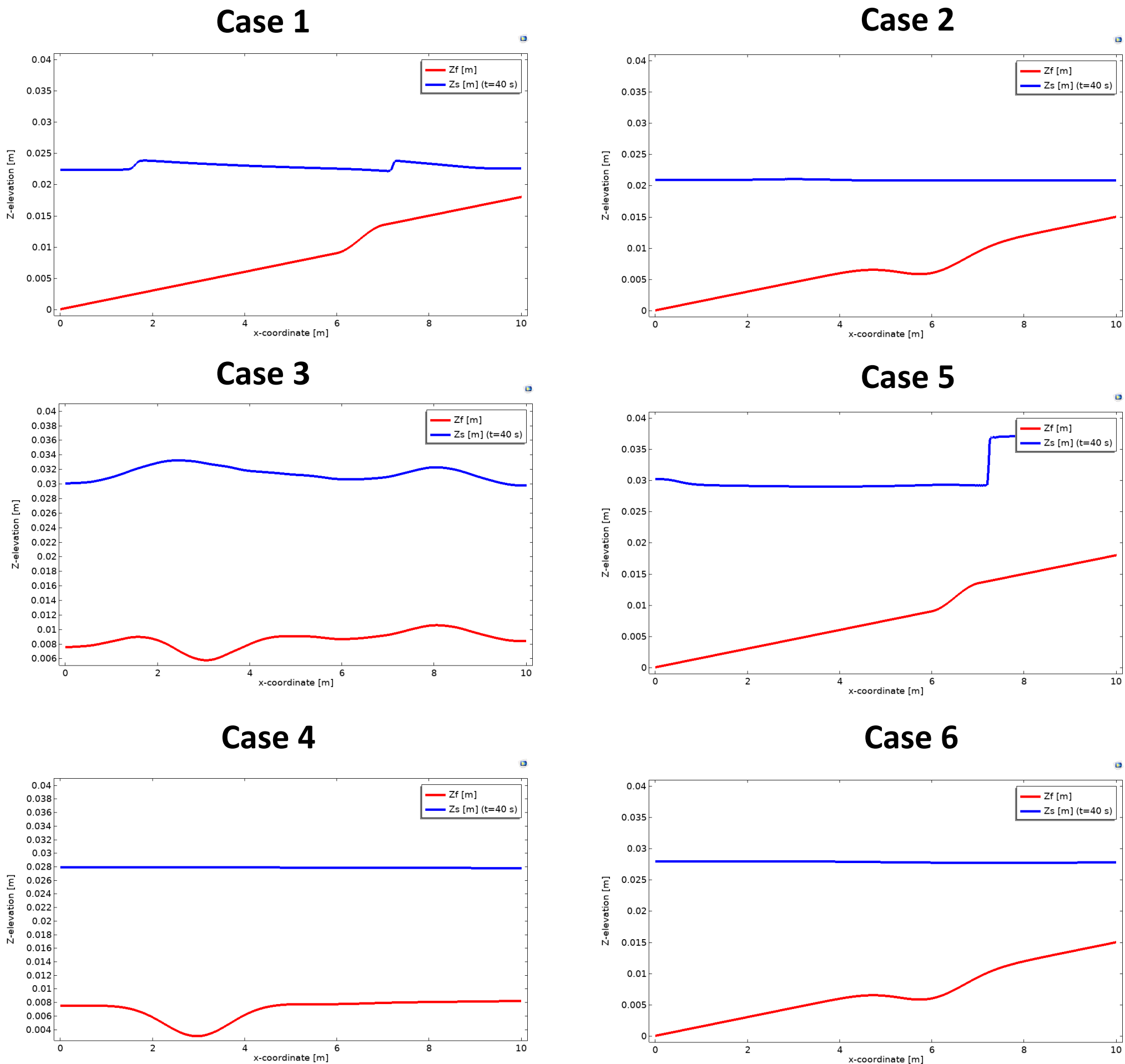
ea $\frac{\partial^2 u}{\partial t^2} + \text{da} \frac{\partial u}{\partial t} + \nabla \cdot \Gamma = F$

- ✓ ea is a zero matrix of order 3;
- ✓ da is an identity matrix of order 3;
- ✓ u is a column vector whose components are z, zv, zf;
- ✓ Γ is the conservative flux;
- ✓ F is the force due to external factors.

3. Case study applications

Case	Topography	Sediment Transport	Viscoelastic bottom	Friction	Dispersion
1	Step	No	No	No	No
2	Hollow	No	No	Yes	Yes
3	Step	Yes	No	No	No
4	Hollow	Yes	No	Yes	Yes
5	Step	Yes	Yes	No	No
6	Hollow	Yes	Yes	Yes	Yes

RESULTS & DISCUSSION



Cases 1 vs 2. Adding friction and dispersion smooths the free surface and damps small instabilities induced by topography.

Cases 3 vs 5. Kelvin–Voigt seabed stabilizes the system vs rigid bed—transients damped, bed-change amplitudes reduced.

Cases 4 vs 6. With viscoelasticity the response is slower and more regular, with attenuated extrema of both free surface and bed.

CONCLUSION

This study has presented an extended formulation of the classical one-dimensional Saint-Venant shallow water model, incorporating key physical processes such as sediment transport, bottom friction, wave dispersion, and the viscoelastic response of the seabed. The numerical simulations demonstrated that the inclusion of viscoelasticity provides a stabilizing influence on the seabed.

FUTURE WORK / REFERENCES

A future development of the present work could involve the inclusion of vegetation effects within the shallow water framework. Vegetation introduces additional drag forces that modify the momentum balance.

$$F_{veg} = \frac{1}{2} \rho C_D h_V b_V N_V v |v| \quad \text{Morrison equation}$$

where ρ is the water density, C_D is the drag coefficient, h_V is the vegetation stem height, b_V is the stem width, N_V is the number of stems per unit horizontal area, v is the local flow velocity.

This research was funded by the Next Generation EU - Italian NRRP, Mission 4, Component 2, Investment 1.5, call for the creation and strengthening of 'Innovation Ecosystems', building 'Territorial R & D Leaders' (Directorial Decree n. 2021/3277) - project Tech4You - Technologies for climate change adaptation and quality of life improvement, n. ECS0000009. This work reflects only the authors' views and opinions, neither the Ministry for University and Research nor the European Commission can be considered responsible for them.