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Determination of Conditions of Divergence for Antenna Array Measurements due to Changes in Satellite Attitude

Marcello Asciolla[1,a], Angela Cratere[1,b], Francesco Dell'Olio[1,c]

[1] Micro Nano Sensor Group, Politecnico di Bari – Bari, Italy

[a] m.asciolla@phd.poliba.it, [b] a.cratere@phd.poliba.it, [c] francesco.dellolio@poliba.it

Introduction and Aim of the Study

The study focuses on the determination of the conditions of the divergence of the variance of the measurement for antenna array able to perform measurements of direction of electromagnetic waves. The payload of the study is a cross array of antennas, able to perform measurements of direction through array beamforming and Angle of Arrival (AOA) technology. In particular, starting from the modeling of satellite kinematics in terms of position and attitude combined with its relative position with respect to an emitter of electromagnetic waves located on the surface of the Earth, the study gives the mathematical fundamentals to determine potential cases that lead to the divergence in the variance of the estimation of the position of the emitter of signal. The numerical predictions, conducted through the evaluation of the metrics of Cramér-Rao Lower Bound (CRLB), are on the angles of Azimuth, Elevation, and broadside through the generation of errors in the attitude with Monte Carlo simulations. Recent advancements on the miniaturization of the electronics make these studies of particular interest for a new set of technological demonstrators equipped with payloads composed of antenna array. Applications of interest are on Earth-scanning missions with exemplary cases of search and rescue or spectrum monitoring of jamming in E1/L1 band for GNSS.

Mathematical Modelling

It is supposed that an emitter of signal E is on the surface of the Earth, and a satellite platform S is in Low Earth Orbit (LEO)

$$\mathbf{x}_E\big|_{LLH} = \begin{pmatrix}\Lambda\\ \lambda\\ h\end{pmatrix} \qquad \mathbf{x}_S\big|_{PQW} = r\begin{pmatrix}\cos(\nu)\\ \sin(\nu)\\ 0\end{pmatrix}$$

$$\mathbf{x}_E\big|_{ECEF} = \begin{pmatrix}(r_\oplus + h)\cos(\Lambda)\cos(\lambda)\\ (r_\oplus + h)\cos(\Lambda)\sin(\lambda)\\ (r_\oplus + h)\sin(\Lambda)\end{pmatrix}$$

$$\mathbf{x}_E\big|_{ECI} = \mathbf{R}_{(\alpha_{GR}(t),3)}\,\mathbf{x}_E\big|_{ECI}$$

$$\mathbf{x}_S\big|_{ECI} = \mathbf{R}_{(\Omega,3)}\mathbf{R}_{(i,1)}\mathbf{R}_{(\omega,3)}\,\mathbf{x}_S\big|_{PQW}$$

It is supposed that the payload is composed of antenna array able to perform measurement of direction, denoted as Angle of Bearing ζ. Given a cross antenna array, it is possible to have two measurements ζ_x and ζ_y.

$$\begin{cases}\cos(\zeta_x) = \mathbf{u}_{LOB}\cdot\mathbf{u}_{Ar,x} = \dfrac{\mathbf{x}_E - \mathbf{x}_S}{\|\mathbf{x}_E - \mathbf{x}_S\|}\cdot\mathbf{u}_{Ar,x}\\[2ex] \cos(\zeta_y) = \mathbf{u}_{LOB}\cdot\mathbf{u}_{Ar,y} = \dfrac{\mathbf{x}_E - \mathbf{x}_S}{\|\mathbf{x}_E - \mathbf{x}_S\|}\cdot\mathbf{u}_{Ar,y}\end{cases}$$

Given the measurements ζ_x, ζ_y, and ζ_z, it is possible to retrieve the locals' angles of Azimuth and Elevation α_{Az} and α_{El}.

$$\cos(\zeta_x) = \mathbf{u}_{Ar,x}\cdot\mathbf{u}_{LOB} = \begin{pmatrix}1\\0\\0\end{pmatrix}\cdot\begin{pmatrix}\cos(\alpha_{El})\cos(\alpha_{Az})\\ \cos(\alpha_{El})\sin(\alpha_{Az})\\ \sin(\alpha_{El})\end{pmatrix} = \cos(\alpha_{El})\cos(\alpha_{Az})$$

$$\cos(\zeta_y) = \mathbf{u}_{Ar,y}\cdot\mathbf{u}_{AOA} = \begin{pmatrix}0\\1\\0\end{pmatrix}\cdot\begin{pmatrix}\cos(\alpha_{El})\cos(\alpha_{Az})\\ \cos(\alpha_{El})\sin(\alpha_{Az})\\ \sin(\alpha_{El})\end{pmatrix} = \cos(\alpha_{El})\sin(\alpha_{Az})$$

Their estimations are:

$$\begin{cases}\hat{\alpha}_{Az}(\hat{\zeta}_x,\hat{\zeta}_y) = \arctan\left(\dfrac{\cos(\hat{\zeta}_y)}{\cos(\hat{\zeta}_x)}\right)\\[2ex] \hat{\alpha}_{El}(\hat{\zeta}_x,\hat{\zeta}_y) = \sqrt{\cos^2(\hat{\zeta}_x) + \cos^2(\hat{\zeta}_y)}\end{cases}$$

The direct influence of the Attitude Determination and Control System (ADCS) can be observed in the matrices of roll θ_1, pitch θ_2, and yaw θ_3 in the conversion from body to orbital coordinate system.

$$\begin{pmatrix}x_{Orb}\\ y_{Orb}\\ z_{Orb}\end{pmatrix} = \mathbf{R}_{(\theta_3,3)}\mathbf{R}_{(\theta_2,2)}\mathbf{R}_{(\theta_1,1)}\begin{pmatrix}x_B\\ y_B\\ z_B\end{pmatrix}$$

The indirect influence of the Attitude Determination and Control System (ADCS) can be observed in the expression of the standard deviation of the single measurement. The best prediction of the variance is in the form of CRLB:

$$\sigma^2_{\zeta_x} = \frac{12}{(2\pi)^2 SNR\left(\dfrac{M_x+1}{M_x-1}\right)M_x\left(\dfrac{L_x}{\lambda}\right)^2 \sin^2\left(\zeta_x(\theta_1,\theta_2,\theta_3)\right)}$$

$$\sigma^2_{\zeta_y} = \frac{12}{(2\pi)^2 SNR\left(\dfrac{M_y+1}{M_y-1}\right)M_y\left(\dfrac{L_y}{\lambda}\right)^2 \sin^2\left(\zeta_y(\theta_1,\theta_2,\theta_3)\right)}$$

Where M is number of antenna elements, L is the length of the array, λ is the wavelength, and SNR is the Signal-to-Noise ratio.

The divergence of the variance of the measurement can be formulated as

$$\sigma^2_\zeta \to \infty \quad\Leftrightarrow\quad \frac{1}{\sin^2(\zeta(\theta_1,\theta_2,\theta_3))} \to \infty \quad\Leftrightarrow\quad \zeta \to 0 + k\pi \qquad k \in \mathbb{Z}$$

This means that the divergence happens when

$$\cos(\zeta(\theta_1,\theta_2,\theta_3)) \to \pm 1$$

Conditions of divergence for bearing angle ζ_x

$$\cos(\alpha_{El})\cos(\alpha_{Az}) \to \pm 1$$
$$\alpha_{El} \to k_1\pi \cup \alpha_{Az} \to k_2\pi \qquad k_1,k_2 \in \mathbb{Z}$$

Conditions of divergence for bearing angle ζ_y

$$\cos(\alpha_{El})\sin(\alpha_{Az}) \to \pm 1$$
$$\alpha_{El} \to k_1\pi \cup \alpha_{Az} \to \frac{\pi}{2} + k_2\pi \qquad k_1,k_2 \in \mathbb{Z}$$

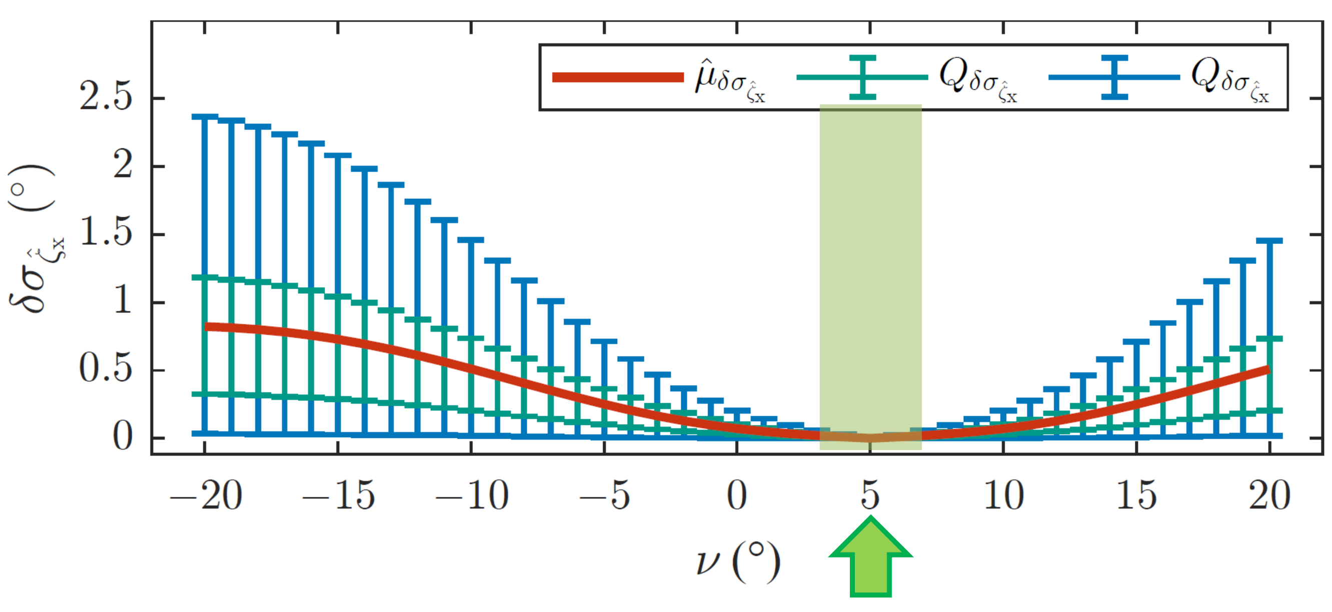
Example of Monte Carlo Simulation

Emitter of signal:
$\Lambda = 5°$, $\lambda = 5°$, $h_E = 0$ m.

Signal and payload:
f = 1575.42 MHz, SNR = 15dB
M = 7, L = 0.31 m

Satellite:
$h_S = 450$ km, e = 0
i = 90°, $\Omega = 0°$
Nadiral pointing, $\sigma_\theta = 10^{-3}$ rad

Measurement OK



Divergence in the measurement