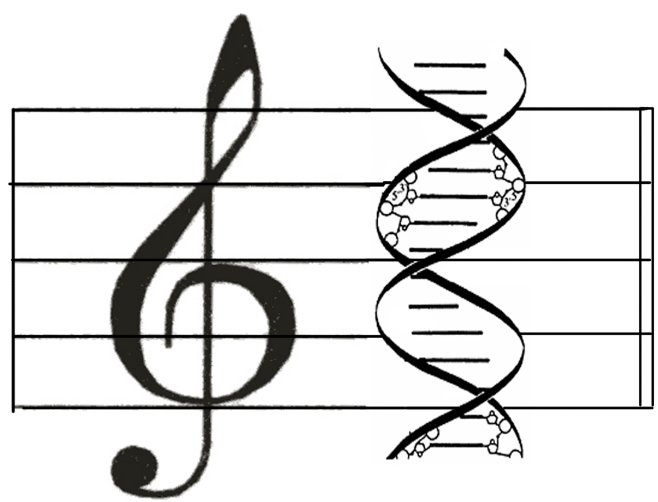
The Genetic Code, the Golden Section and Genetic Music

A.Koblyakov, S.Petoukhov, I.Stepanian

The Moscow P. I. Tchaikovsky Conservatory



The human brain does not possess a special center of music. The feeling of love to music seems to be dispersed in the whole organism. It is known that different emotions belong to inherited biological phenomena. It seems that many aspects of musical harmony also belong to inborn feelings and are connected with genetic phenomena.

Charles Darwin:

"... all the chief expressions exhibited by man are the same throughout the world. ... we may infer with much probability, that such expressions are **innate** or instinctive."

(http://www.bbc.co.uk/news/magazine-15600203)

From ancient times, understanding the phenomenon of music and musical structures were associated with mathematics. G.Leibniz declared:

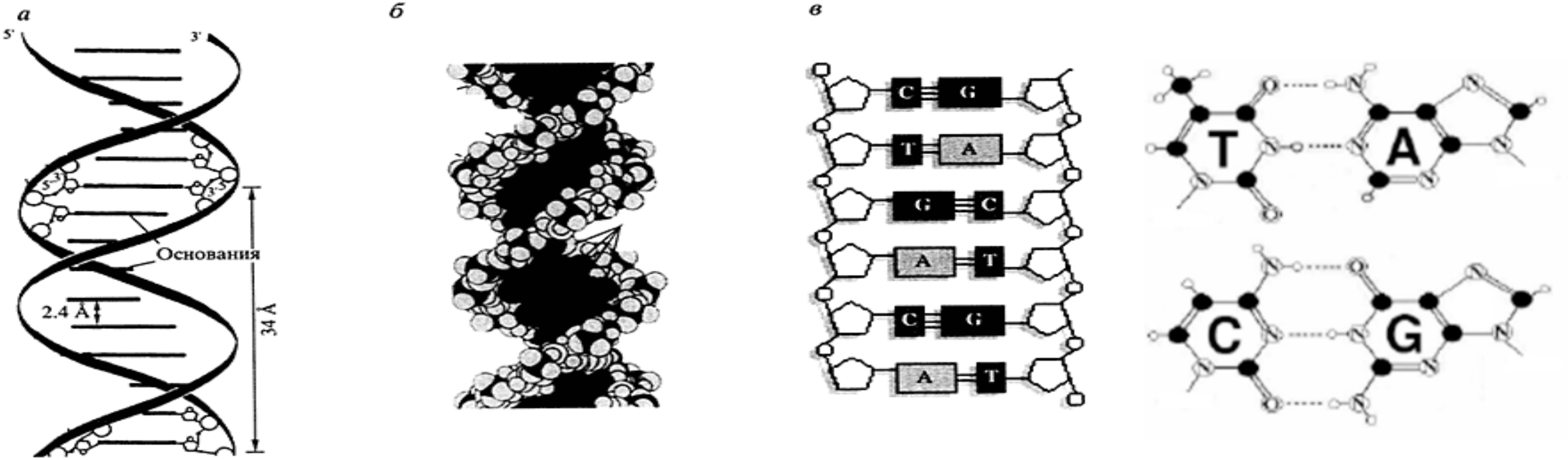
"***Music is a secret arithmetical exercise and the person who indulges in it does not realize that he is manipulating numbers***" and "***music is the pleasure the human mind experiences from counting without being aware that it is counting***".

The range of human sound perception contains an infinite set of sound frequencies. Pythagoras has discovered that certain mathematical rules allow separating - from this infinite set of frequencies - a discrete set of frequencies, which determine the harmonious sound set.

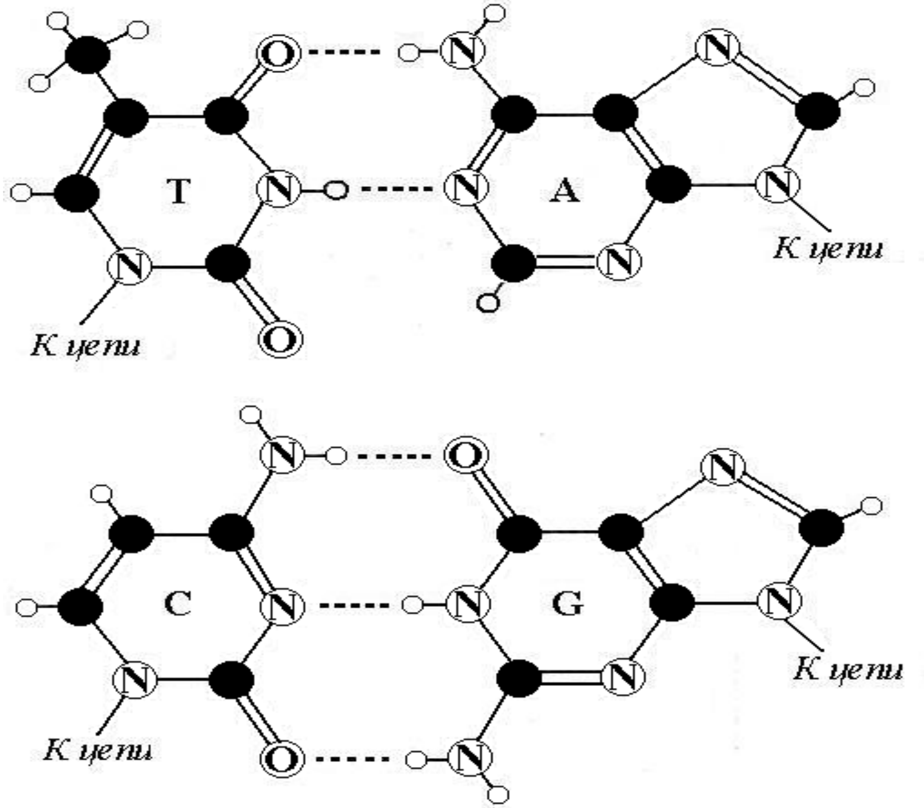
But **Pythagor said nothing about the fact that other discrete sets of sound frequencies may exist, which will also form harmonious sets of sounds**.

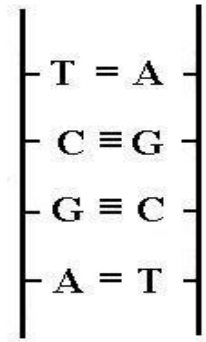
Many scientists of different centuries (including Kepler, Descartes, Leibniz, Euler) have tried to find new musical scales but they didn’t know about the genetic code.

This presentation is devoted to “genetic musical scales”, which are based on symmetric features of molecular ensembles of genetic systems. We present our study of these scales very briefly here.



It is known that DNA-molecules of heredity contain a sequence of **4 letters** (adenine A, cytosine C, guanine G, thymine T) along their filaments. **64 triplets = 4^3** (or combination of three genetic letters: CGA, TAG, etc.) encode 20 amino acids (and punctuation signs), a sequence of which defines a primary structure of proteins.





Letters A-T and C-G form complementary pairs with **2 and 3** hydrogen bonds correspondingly.

<u>Matrix presentation of genetic alphabets</u>.

In computers, information is stored in a form of matrices and is processed by means of tensor (or Kronecker) multiplications of matrices. One can present a set of 4 genetic letters in a form of a square matrix [C T; A G]. Then each complete set of **<u>4^n</u>** polyplets with a length "n" can be represented algorithmically by a matrix $[C\ T;\ A\ G]^{(n)}$, where (n) is a tensor (Kronecker) power. For example, a (8x8)-matrix $[C\ T;\ A\ G]^{(3)}$ contains all 64 triplets in a strong order.

[C T; A G] =

C	T
A	G

; **$[C\ T;\ A\ G]^{(2)}$ =**

CC	CT	TC	TT
CA	CG	TA	TG
AC	AT	GC	GT
AA	AG	GA	GG

$[C\ T;\ A\ G]^{(3)}$ =

CCC	CCT	CTC	CTT	TCC	TCT	TTC	TTT
CCA	CCG	CTA	CTG	TCA	TCG	TTA	TTG
CAC	CAT	CGC	CGT	TAC	TAT	TGC	TGT
CAA	CAG	CGA	CGG	TAA	TAG	TGA	TGG
ACC	ACT	ATC	ATT	GCC	GCT	GTC	GTT
ACA	ACG	ATA	ATG	GCA	GCG	GTA	GTG
AAC	AAT	AGC	AGT	GAC	GAT	GGC	GGT
AAA	AAG	AGA	AGG	GAA	GAG	GGA	GGG

Quantities of hydrogen bonds (2 and 3) of complementary DNA-bases A-T, C-G have an important meaning in the genetic scheme. Let us replace each n-plet in $[C\ T;\ A\ G]^{(n)}$ by the product of numbers of hydrogen bonds: C=G=3, A=T=2. For instance, due to such operation, the triplet CGA is replaced by 3x3x2=18. As a result, $[C\ T;\ A\ G]^{(n)} \rightarrow$ $\mathbf{[3,\ 2;\ 2,\ 3]^{(n)}}$. As an example, Figure demonstrates the numeric matrix $\mathbf{[3,\ 2;\ 2,\ 3]^{(3)}}$:

27	18	18	12	18	12	12	8
18	27	12	18	12	18	8	12
18	12	27	18	12	8	18	12
12	18	18	27	8	12	12	18
18	12	12	8	27	18	18	12
12	18	8	12	18	27	12	18
12	8	18	12	18	12	27	18
8	12	12	18	12	18	18	27

$$\begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix}^{(1)}; \qquad \begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix}^{(2)} = \begin{vmatrix} 9 & 6 & 6 & 4 \\ 6 & 9 & 4 & 6 \\ 6 & 4 & 9 & 6 \\ 4 & 6 & 6 & 9 \end{vmatrix};$$

$$\begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix}^{(3)} = \begin{vmatrix} 27 & 18 & 18 & 12 & 18 & 12 & 12 & 8 \\ 18 & 27 & 12 & 18 & 12 & 18 & 8 & 12 \\ 18 & 12 & 27 & 18 & 12 & 8 & 18 & 12 \\ 12 & 18 & 18 & 27 & 8 & 12 & 12 & 18 \\ 18 & 12 & 12 & 8 & 27 & 18 & 18 & 12 \\ 12 & 18 & 8 & 12 & 18 & 27 & 12 & 18 \\ 12 & 8 & 18 & 12 & 18 & 12 & 27 & 18 \\ 8 & 12 & 12 & 18 & 12 & 18 & 18 & 27 \end{vmatrix}$$

These family of numerical genomatrices **$[3, 2; 2, 3]^{(n)}$** have interesting mathematical properties.

<u>A connection between genetic matrices and the golden section</u>. The genomatrices **[3, 2; 2, 3]$^{(n)}$** have a hidden relation with the famous golden section $\varphi = (1+5^{0.5})/2 = 1{,}618\ldots$ If we take the square root from any genomatrix **[3, 2; 2, 3]$^{(n)}$**, the result is a new matrix **$([3, 2; 2, 3]^{(n)})^{1/2}$** = **$[\varphi, \varphi^{-1}; \varphi^{-1}, \varphi]^{(n)}$**, all elements of which are equal to the golden section φ in different powers. In this way a new tensor family of matrices **$[\varphi, \varphi^{-1}; \varphi^{-1}, \varphi]^{(n)}$** arises:

$$\begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix}^{0.5} = \begin{vmatrix} \varphi & \varphi^{-1} \\ \varphi^{-1} & \varphi \end{vmatrix}; \qquad \left(\begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix}^{(2)}\right)^{0.5} = \begin{vmatrix} \varphi^{2} & \varphi^{0} & \varphi^{0} & \varphi^{-2} \\ \varphi^{0} & \varphi^{2} & \varphi^{-2} & \varphi^{0} \\ \varphi^{0} & \varphi^{-2} & \varphi^{2} & \varphi^{0} \\ \varphi^{-2} & \varphi^{0} & \varphi^{0} & \varphi^{2} \end{vmatrix}$$

$$\left(\begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix}^{(3)}\right)^{0.5} = \begin{vmatrix} \varphi^{3} & \varphi^{1} & \varphi^{1} & \varphi^{-1} & \varphi^{1} & \varphi^{-1} & \varphi^{-1} & \varphi^{-3} \\ \varphi^{1} & \varphi^{3} & \varphi^{-1} & \varphi^{1} & \varphi^{-1} & \varphi^{1} & \varphi^{-3} & \varphi^{-1} \\ \varphi^{1} & \varphi^{-1} & \varphi^{3} & \varphi^{1} & \varphi^{-1} & \varphi^{-3} & \varphi^{1} & \varphi^{-1} \\ \varphi^{-1} & \varphi^{1} & \varphi^{1} & \varphi^{3} & \varphi^{-3} & \varphi^{-1} & \varphi^{-1} & \varphi^{1} \\ \varphi^{1} & \varphi^{-1} & \varphi^{-1} & \varphi^{-3} & \varphi^{3} & \varphi^{1} & \varphi^{1} & \varphi^{-1} \\ \varphi^{-1} & \varphi^{1} & \varphi^{-3} & \varphi^{-1} & \varphi^{1} & \varphi^{3} & \varphi^{-1} & \varphi^{1} \\ \varphi^{-1} & \varphi^{-3} & \varphi^{1} & \varphi^{-1} & \varphi^{1} & \varphi^{-1} & \varphi^{3} & \varphi^{1} \end{vmatrix}$$

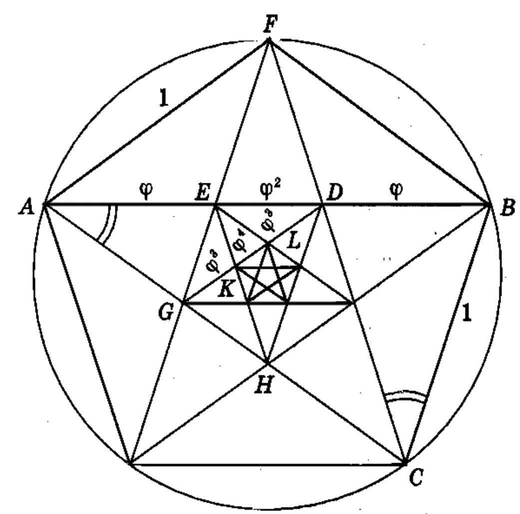
$$\begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix}^{\mathbf{0.5}} = \begin{vmatrix} \varphi & \varphi^{-1} \\ \varphi^{-1} & \varphi \end{vmatrix}; \qquad \left(\begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix}^{\mathbf{(2)}}\right)^{\mathbf{0.5}} = \begin{vmatrix} \varphi^{2} & \varphi^{0} & \varphi^{0} & \varphi^{-2} \\ \varphi^{0} & \varphi^{2} & \varphi^{-2} & \varphi^{0} \\ \varphi^{0} & \varphi^{-2} & \varphi^{2} & \varphi^{0} \\ \varphi^{-2} & \varphi^{0} & \varphi^{0} & \varphi^{2} \end{vmatrix}$$

$$\left(\begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix}^{\mathbf{(3)}}\right)^{\mathbf{0.5}} = \begin{vmatrix} \varphi^{3} & \varphi^{1} & \varphi^{1} & \varphi^{-1} & \varphi^{1} & \varphi^{-1} & \varphi^{-1} & \varphi^{-3} \\ \varphi^{1} & \varphi^{3} & \varphi^{-1} & \varphi^{1} & \varphi^{-1} & \varphi^{1} & \varphi^{-3} & \varphi^{-1} \\ \varphi^{1} & \varphi^{-1} & \varphi^{3} & \varphi^{1} & \varphi^{-1} & \varphi^{-3} & \varphi^{1} & \varphi^{-1} \\ \varphi^{-1} & \varphi^{1} & \varphi^{1} & \varphi^{3} & \varphi^{-3} & \varphi^{-1} & \varphi^{-1} & \varphi^{1} \\ \varphi^{1} & \varphi^{-1} & \varphi^{-1} & \varphi^{-3} & \varphi^{3} & \varphi^{1} & \varphi^{1} & \varphi^{-1} \\ \varphi^{-1} & \varphi^{1} & \varphi^{-3} & \varphi^{-1} & \varphi^{1} & \varphi^{3} & \varphi^{-1} & \varphi^{1} \\ \varphi^{-1} & \varphi^{-3} & \varphi^{1} & \varphi^{-1} & \varphi^{1} & \varphi^{-1} & \varphi^{3} & \varphi^{1} \\ \varphi^{-3} & \varphi^{-1} & \varphi^{-1} & \varphi^{1} & \varphi^{-1} & \varphi^{1} & \varphi^{1} & \varphi^{3} \end{vmatrix}$$

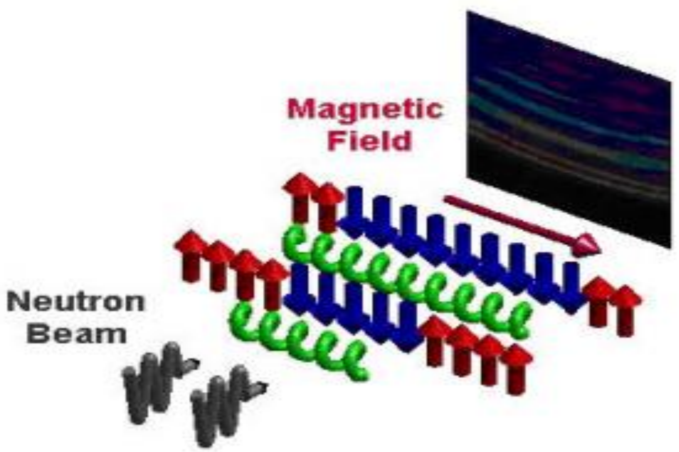
$$([3, 2; 2, 3]^{(3)})^{1/2} = \begin{vmatrix} \varphi^{3} & \varphi^{1} & \varphi^{1} & \varphi^{-1} & \varphi^{1} & \varphi^{-1} & \varphi^{-1} & \varphi^{-3} \\ \varphi^{1} & \varphi^{3} & \varphi^{-1} & \varphi^{1} & \varphi^{-1} & \varphi^{1} & \varphi^{-3} & \varphi^{-1} \\ \varphi^{1} & \varphi^{-1} & \varphi^{3} & \varphi^{1} & \varphi^{-1} & \varphi^{-3} & \varphi^{1} & \varphi^{-1} \\ \varphi^{-1} & \varphi^{1} & \varphi^{1} & \varphi^{3} & \varphi^{-3} & \varphi^{-1} & \varphi^{-1} & \varphi^{1} \\ \varphi^{1} & \varphi^{-1} & \varphi^{-1} & \varphi^{-3} & \varphi^{3} & \varphi^{1} & \varphi^{1} & \varphi^{-1} \\ \varphi^{-1} & \varphi^{1} & \varphi^{-3} & \varphi^{-1} & \varphi^{1} & \varphi^{3} & \varphi^{-1} & \varphi^{1} \\ \varphi^{-1} & \varphi^{-3} & \varphi^{1} & \varphi^{-1} & \varphi^{1} & \varphi^{-1} & \varphi^{3} & \varphi^{1} \\ \varphi^{-3} & \varphi^{-1} & \varphi^{-1} & \varphi^{1} & \varphi^{-1} & \varphi^{1} & \varphi^{1} & \varphi^{3} \end{vmatrix}$$

For instance, the matrix $([3, 2; 2, 3]^{(3)})^{1/2} = [\varphi, \varphi^{-1} ; \varphi^{-1}, \varphi]^{(3)}$ has only two pairs of inverse numbers: φ^{1} and φ^{-1}, φ^{3} and φ^{-3}. The golden section is a mathematical symbol of a self-reproduction for many centuries (Leonardo da Vinci, J.Kepler, etc). It is well known that the golden section is shown by many authors in genetically inherited physiological systems: cardio-vascular system, respiratory system, electric activities of brain, *etc*.

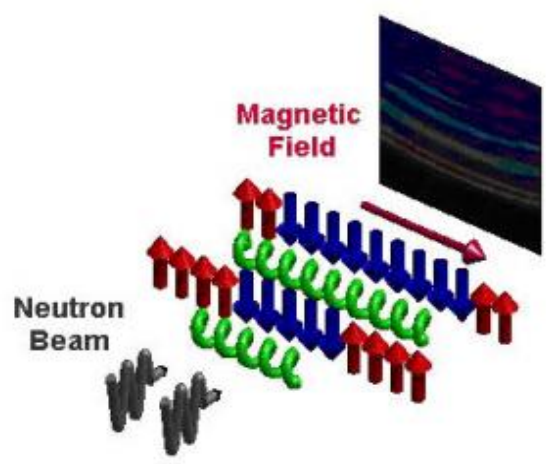
The golden section exists in 5-symmetrical figures, which are presented widely in living nature. Many objects of generalized crystallography have the golden section: quasi-crystalls by Nobel Prize winner D.Shechtman, R.Penrose’s mosaics, fullerenes, dodecahedrons of ensembles of water molecules, biological phyllotaxis laws, etc.



Whether such vibrational systems exist in the Nature, whose resonant frequencies are associated with the golden section φ ? Yes, the article "**Golden ratio discovered in quantum world**: Hidden symmetry observed for the first time in solid state matter" has been published in «Science Daily» on 07.01.2010 (http://www.sciencedaily.com/releases/2010/01/100107143909.htm)



Researches of cobalt niobate, which has magnetic properties, have revealed that "*the chain of atoms acts like a nanoscale guitar string. ... The tension comes from the interaction between spins causing them to magnetically resonate. For these interactions we found a series of resonant notes: the first two notes show a perfect relationship with each other.* ***Their frequencies (pitch) are in the ratio of 1.618…, which is the golden ratio famous from art and architecture***".



A connection between genetic matrices and Pythagorean musical scale ("genetic music")

The genomatrices have a close relation with Pythagorean (or quint) musical scale based on the quint ratio 3:2 (the perfect fifth). Genomatrices **$[3, 2; 2, 3]^{(n)}$** demonstrate a quint principle of their structure because they have the quint ratio 3:2 at different levels: between numerical sums in top and bottom quadrants, sub-quadrants, sub-sub-quadrants, etc. including quint ratios between adjacent numbers in them.

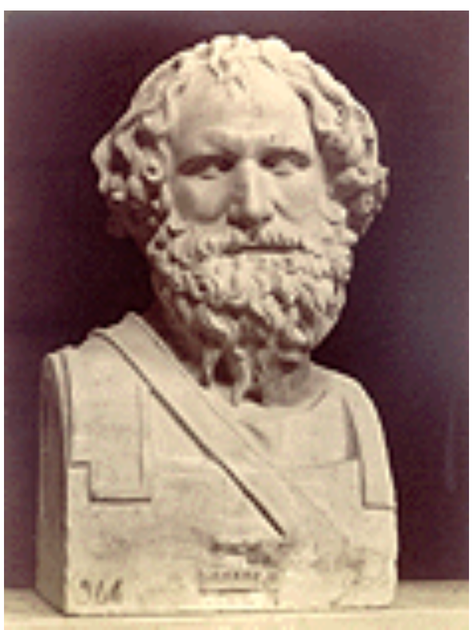
27	18	18	12	18	12	12	8
18	27	12	18	12	18	8	12
18	12	27	18	12	8	18	12
12	18	18	27	8	12	12	18
18	12	12	8	27	18	18	12
12	18	8	12	18	27	12	18
12	8	18	12	18	12	27	18
8	12	12	18	12	18	18	27

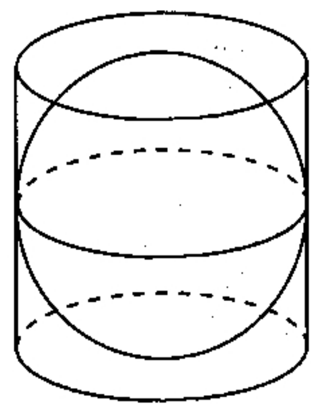
For example, $\mathbf{[3, 2; 2, 3]^{(3)}}$ contains only 4 numbers – 27, 18, 12, 8 - with the quint ratio between them: 27/18=18/12=12/8=**3/2**. Such genomatrices can be named “quint genomatrices”.

27	18	18	12	18	12	12	8
18	27	12	18	12	18	8	12
18	12	27	18	12	8	18	12
12	18	18	27	8	12	12	18
18	12	12	8	27	18	18	12
12	18	8	12	18	27	12	18
12	8	18	12	18	12	27	18
8	12	12	18	12	18	18	27

It is known that the ancient Greek Pythagorean scale was basically identical with the old Chinese music scale. Both of them were based on quint ratio 3/2. In Europe this music scale is known as Pythagorean scale. In Ancient China this music scale had a cosmic meaning connected with the book "I Ching": numbers 2 and 3 were named "numbers of Earth and Heaven" and they were the basis of Chinese arithmetic. After Ancient China, Pythagoreans considered numbers 2 and 3 as the female and male numbers, which can give birth to new musical tones in their interconnection.

Ancient Greeks attached an extraordinary significance to search of the quint 3:2 in natural systems because of their thoughts about musical harmony in the organization of the world. For example, Archimedes considered as the best result of his life a detection of the quint 3:2 between volumes and surfaces of a cylinder and a sphere entered in it. Just these geometrical figures with the quint ratio were pictured on his gravestone according to Archimedes testament. And due to these figures Cicero has found Archimedes's grave later, 200 years after his death.





$V_{cyl} : V_{sph} = S_{cyl} : S_{sph} = \underline{\mathbf{3:2}}$

This Table demonstrates a known example of application of the quint 3/2 to construct a symmetrical sequence of 7 musical notes of the Pythagorean scale; a frequency ratio between any adjacent notes of this sequence is equal to the quint 3/2 (the designation of notes is given on Helmholtz system).

Musical note	fa (F)	do (C)	sol (G)	re (D^1)	la (A^1)	mi (E^2)	si (B^2)
Note frequency, Hz	87	130	196	293	440	660	990
Frequency Ratio to 293Hz (re (d^1))	$\mathbf{(3/2)^{-3}}$	$\mathbf{(3/2)^{-2}}$	$\mathbf{(3/2)^{-1}}$	$\mathbf{(3/2)^{0}}$	$\mathbf{(3/2)^{1}}$	$\mathbf{(3/2)^{2}}$	$\mathbf{(3/2)^{3}}$

Each quint genetic matrix **[3, 2; 2, 3]**$^{(n)}$ contains an individual sequence of (n+1) kinds of numbers which reproduces geometric progression, a coefficient of which is equal to the quint 3/2:

$$\begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix} \qquad \begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix}^{(2)} = \begin{vmatrix} 9 & 6 & 6 & 4 \\ 6 & 9 & 4 & 6 \\ 6 & 4 & 9 & 6 \\ 4 & 6 & 6 & 9 \end{vmatrix}$$

[3, 2; 2, 3]$^{(1)} \Rightarrow$ **3, 2**

[3, 2; 2, 3]$^{(2)} \Rightarrow$ **9, 6, 4**

[3, 2; 2, 3]$^{(3)} \Rightarrow$ **27, 18, 12, 8**

$$\begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix}^{(3)} =$$

27	18	18	12	18	12	12	8
18	27	12	18	12	18	8	12
18	12	27	18	12	8	18	12
12	18	18	27	8	12	12	18
18	12	12	8	27	18	18	12
12	18	8	12	18	27	12	18
12	8	18	12	18	12	27	18
8	12	12	18	12	18	18	27

....

[3, 2; 2, 3]$^{(6)} \Rightarrow$ **729, 486, 324, 216, 144, 96, 64**

....

In such way we have the “genetic” triangle:

3	9	27	81	243	….
2	6	18	54	162	….
	4	12	36	108	….
		8	24	72	….
			16	48	….
				32	….

But this genetic triangle was published 2000 (!) years ago by Nichomachus of Gerasa in his famous book “Introduction into arithmetic”. (J.Kappraff* and G.Adamson informed S.Petoukhov about this coincidence).

* - J. Kappraff, "The *Arithmetic* of Nicomachus of Gerasa and its Applications to Systems of Proportion", *Nexus Network Journal,* vol. 2, no. 4 (October 2000), http://www.nexusjournal.com/Kappraff.html

3	9	27	81	243	….
2	6	18	54	162	….
	4	12	36	108	….
		8	24	72	….
			16	48	….
				32	….

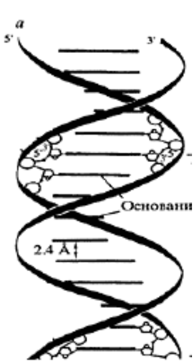
This "genetic" triangle was famous for centuries as the bases of the Pythagorean theory of musical harmony and aesthetics. In accordance with this triangle, the Parthenon and other great architectural objects were created because architecture was interpreted as the non-movement music, and the music was interpreted as the dynamic architecture.

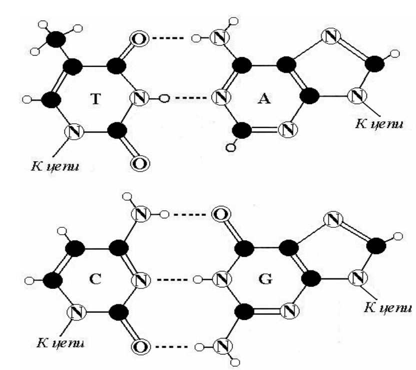
The numeric sequence from each genomatrix **[3, 2; 2, 3]$^{(n)}$** can be compared to a quint sequence of musical notes. **If one confronts the least number from a matrix with a musical note (fa (F)) then all series of numbers automatically corresponds with a series of musical notes**. For example, the sequence of numbers **27, 18, 12, 8** of **[3, 2; 2, 3]$^{(3)}$** correspond to the frequency sequence of notes **fa(F) - do(c) - sol(g) - re(d1).**

27	18	18	12	18	12	12	8
18	27	12	18	12	18	8	12
18	12	27	18	12	8	18	12
12	18	18	27	8	12	12	18
18	12	12	8	27	18	18	12
12	18	8	12	18	27	12	18
12	8	18	12	18	12	27	18
8	12	12	18	12	18	18	27

Genomatrix **[3, 2; 2, 3]$^{(6)}$** contains a sequence of 7 numbers, which correspond to the whole quint sequence of 7 notes: **fa(F) - do(c) - sol(g) - re(d1) - la (a1) - mi (e2) - si (b2).**

Such musical analogies take place not only in the case of the hydrogen bonds but also for a few other parameters of molecules DNA, for example, for number of atoms in rings of nitrogenous bases. The quantity of non-hydrogen atoms in molecular rings of pyrimidines (C and T) is equal to 6 and the quantity of non-hydrogen atoms in molecular rings of purines (A and G) is equal to 9. Their quint ratio 9:6=3:2 can be considered as a fundament for appropriate quint genomatrices and for "atomic" genetic melody of the nitrogenous bases and triplets along DNA. Two filaments of DNA have different – "complementary" - kinds of such atomic genetic music.





The family of the **golden genetic matrices** $([3, 2; 2, 3]^{(n)})^{1/2}$ = $[\varphi^1, \varphi^{-1}; \varphi^{-1}, \varphi^1]^{(n)}$ defines another numeric triangle, because a set of entries in each matrix represents a fragment of a geometrical progression with the **coefficient** φ^2 (square of the golden section):

$$\begin{vmatrix} \varphi & \varphi^{-1} \\ \varphi^{-1} & \varphi \end{vmatrix}; \qquad \begin{vmatrix} \varphi^{2} & \varphi^{0} & \varphi^{0} & \varphi^{-2} \\ \varphi^{0} & \varphi^{2} & \varphi^{-2} & \varphi^{0} \\ \varphi^{0} & \varphi^{-2} & \varphi^{2} & \varphi^{0} \\ \varphi^{-2} & \varphi^{0} & \varphi^{0} & \varphi^{2} \end{vmatrix}$$

$$\begin{array}{lllll} \varphi^{1} & \varphi^{2} & \varphi^{3} & \varphi^{4} \ldots. \\ \varphi^{-1} & \varphi^{0} & \varphi^{1} & \varphi^{2} \ldots. \\ & \varphi^{-2} & \varphi^{-1} & \varphi^{0} \ldots. \\ & & \varphi^{-3} & \varphi^{-2} \ldots. \\ & & & \varphi^{-4} \ldots \end{array}$$

φ^{3}	φ^{1}	φ^{1}	φ^{-1}	φ^{1}	φ^{-1}	φ^{-1}	φ^{-3}
φ^{1}	φ^{3}	φ^{-1}	φ^{1}	φ^{-1}	φ^{1}	φ^{-3}	φ^{-1}
φ^{1}	φ^{-1}	φ^{3}	φ^{1}	φ^{-1}	φ^{-3}	φ^{1}	φ^{-1}
φ^{-1}	φ^{1}	φ^{1}	φ^{3}	φ^{-3}	φ^{-1}	φ^{-1}	φ^{1}
φ^{1}	φ^{-1}	φ^{-1}	φ^{-3}	φ^{3}	φ^{1}	φ^{1}	φ^{-1}
φ^{-1}	φ^{1}	φ^{-3}	φ^{-1}	φ^{1}	φ^{3}	φ^{-1}	φ^{1}
φ^{-1}	φ^{-3}	φ^{1}	φ^{-1}	φ^{1}	φ^{-1}	φ^{3}	φ^{1}
φ^{-3}	φ^{-1}	φ^{-1}	φ^{1}	φ^{-1}	φ^{1}	φ^{1}	φ^{3}

ANALOGIES BETWEEN THE TRIANGLES

Significant analogies exist between this «golden» triangle and the quint triangle of Nichomachus of Gerasa. The quint triangle is connected with three main mathematical ratios of three quantities "a", "b", "c", which were used in the Pythagorean theory of musical harmony and aesthetics of proportions [J.Kappraff, 2000]:

- the arithmetic mean: $c = (a+в)/2$;
- the geometric mean: $c = (a*в)^{0.5}$;
- the harmonic mean: $c = 2*a*в/(a+в)$.

Inside the quint triangle, these ratios are represented for each internal number and its neighboring pairs of numbers. For example, number 18 is the arithmetic mean for numbers 9 and 27 located above it: $(9+27)/2=18$. The same number 18 is the geometric mean for numbers 6 and 54 located on the sides $(6*54)^{0.5}=18$. The same number 18 is the harmonic mean for numbers 12 and 36 located under it: $2*12*36/(12+36)=18$.

3	9	27	81	243	….
2	6	18	54	162	….
	4	12	36	108	….
		8	24	72	….
			16	48	….
				32	….

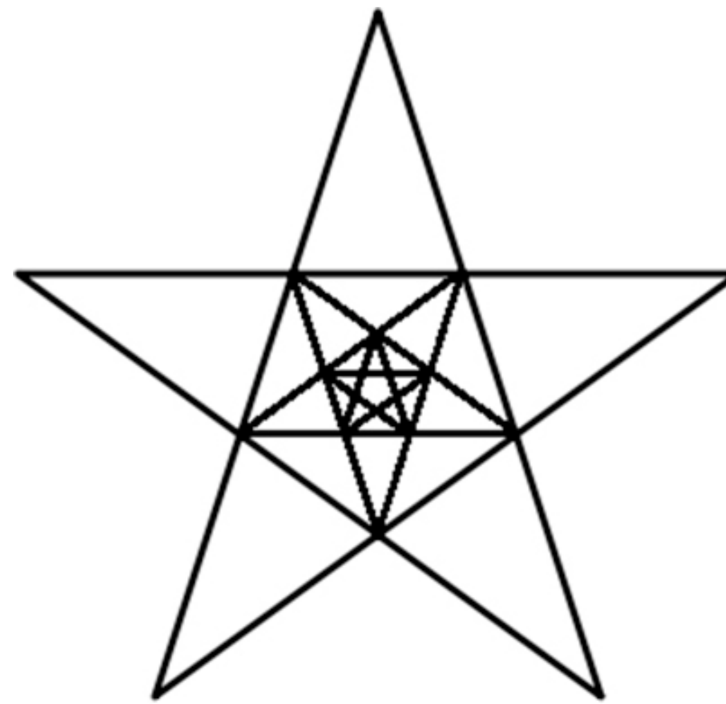
But inside the «golden» triangle, all its numbers from the same places are also connected with these basic ratios of the Pythagorean theory of harmony. For example, number φ^{-1} is double the arithmetic mean for numbers φ^{-2} и φ^{-3}: $\varphi^{-2}+\varphi^{-3}=\varphi^{-1}$. The same number φ^{-1} is the geometric mean for φ^{0} и φ^{-2}: $(\varphi^{0}+\varphi^{-2})^{0.5}=\varphi^{-1}$. The same number φ^{-1} is harmonic mean for φ^{1} и φ^{0}: $\varphi^{1}*\varphi^{0}/(\varphi^{1}+\varphi^{0})=\varphi^{-1}$.

$$
\begin{array}{lllll}
\boldsymbol{\varphi^{1}} & \boldsymbol{\varphi^{2}} & \boldsymbol{\varphi^{3}} & \boldsymbol{\varphi^{4}} & \ldots. \\
\boldsymbol{\varphi^{-1}} & \boldsymbol{\varphi^{0}} & \boldsymbol{\varphi^{1}} & \boldsymbol{\varphi^{2}} & \ldots. \\
 & \boldsymbol{\varphi^{-2}} & \boldsymbol{\varphi^{-1}} & \boldsymbol{\varphi^{0}} & \ldots. \\
 & & \boldsymbol{\varphi^{-3}} & \boldsymbol{\varphi^{-2}} & \ldots. \\
 & & & \boldsymbol{\varphi^{-4}} & \ldots
\end{array}
$$

These results testify additionally that the "golden" triangle can be the basis of a new system of musical harmony and aesthetics of proportion by analogy with the quint triangle of Nichomachus of Gerasa.

The coefficient φ^2 of geometric progressions in the “golden” triangle, which was received from the tensor families of genetic matrices $([3, 2; 2, 3]^{(n)})^{1/2}$, exists also in a chain of regular 5-stars (pentagrams), which are embedded in each other, as the constant ratio of scaling the adjacent stars.

φ^1	φ^2	φ^3	φ^4	....
φ^{-1}	φ^0	φ^1	φ^2	....
	φ^{-2}	φ^{-1}	φ^0	....
		φ^{-3}	φ^{-2}	....
			φ^{-4}	...

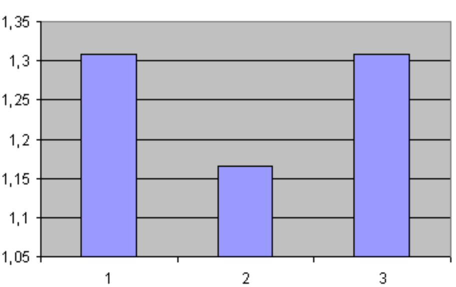


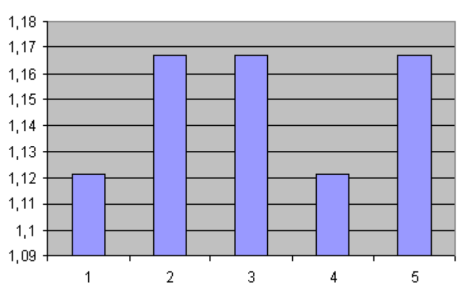
The same ratio φ^2 exists in phyllotaxis laws because it defines an "ideal angle of phyllotaxis" $360^0/\varphi^2$. By this reason, these genetic scales can be named also as "phyllotaxis scales". But finally they have been named as "Fibonacci-stage scales" because of their close connection with Fibonacci numbers: $F_{n+2}=F_n+F_{n+1}$: 1, 2, 3, 5, 8, 13, 21, 34, ...

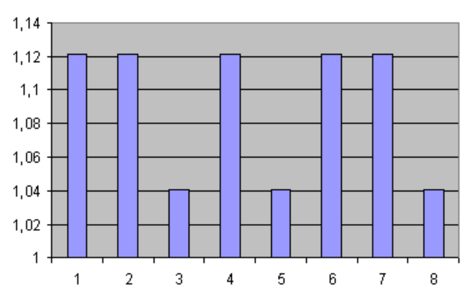
$$
\begin{array}{cccccc}
\varphi^{1} & \varphi^{2} & \varphi^{3} & \varphi^{4} & \ldots. \\
\varphi^{-1} & \varphi^{0} & \varphi^{1} & \varphi^{2} & \ldots. \\
 & \varphi^{-2} & \varphi^{-1} & \varphi^{0} & \ldots. \\
 & & \varphi^{-3} & \varphi^{-2} & \ldots. \\
 & & & \varphi^{-4} & \ldots
\end{array}
$$

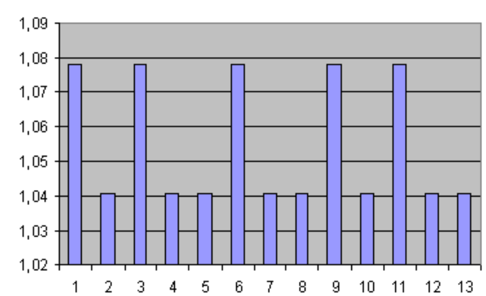
This "golden" triangle is connected with a system of mathematical scales: a hierarchical system of "Fibonacci-stage scales" (see details in the book: Petoukhov S.V. "The Matrix Genetics, Algebras of the Genetic Code, Noise-immunity", Moscow, 2008, 316p., in Russian, for open reading on the website http://petoukhov.com/).

When we have constructed algorithmically these scales (by analogy with a known algorithmic construction of the Pythagorean scale), it was unexpectedly revealed that number of stages in each of these scales is equal to Fibonacci numbers: we have received scales with 3-, 5-, 8-, 13-, 21-, 34-,… stages. Numbers of small and big intervals in each scale are also automatically equal to Fibonacci numbers.

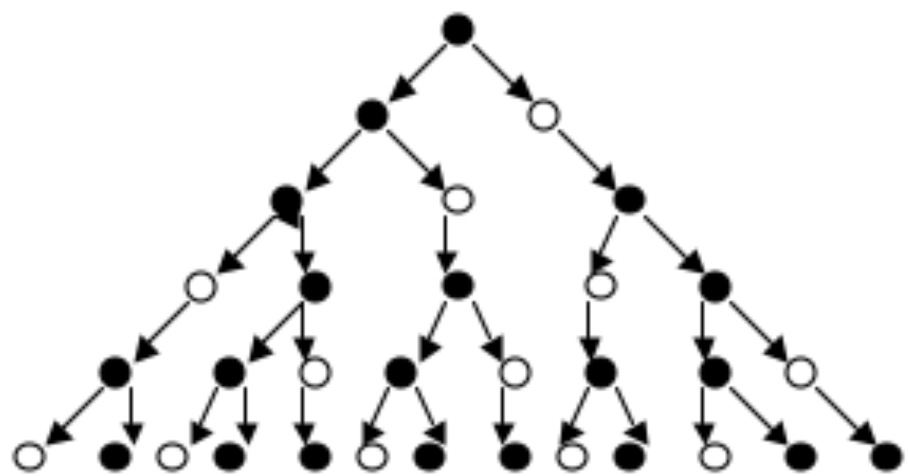








The more Fibonacci stages in a scale, the more Fibonacci quantities of small and big intervals exist in it in accordance with the following algorithmic tree (black circles mean big intervals, white circles – small intervals). This tree coincides with the famous Fibonacci tree in his mathematical problem about a reproduction of rabbits.



Fibonacci-stage scales form an hierarchy, where **each scale is embodied in all scales with higher Fibonacci numbers**. For example, all sound frequencies of 5-stage scale belong also to the 13-stage scale, the 21-stage scale, etc.

This hierachy resembles Russian matreshka:

Table shows an example of frequencies (Hz) of the hierarchal structure of the Fibonacci-stage scales with 2-, 3-, 5-, 8-, 13-, 21-stages.

№21	256	265	276	287	298	310	322	334	336	348	362	376	390	392	406	422	440	455	474	493	510	512
№13	256		276	287		310	322		336		362	376		392		422	440		474	493		512
№8	256			287			322		336			376		392			440			493		512
№5	256			287					336					392			440					512
№3	256								336					392								512
№2	256													392								512

These mathematical scales, which were constructed by analogy with Pythagorean musical scale, can be considered as new musical scales, which open new opportunities for composers. Examples of musical compositions in these genetic scales are represented in the concert of genetic music on 4 July in Vienna from the Moscow P. I. Tchaikovsky Conservatory - http://summit.is4is.org/wp-content/uploads/2015/06/personal-program2.pdf .

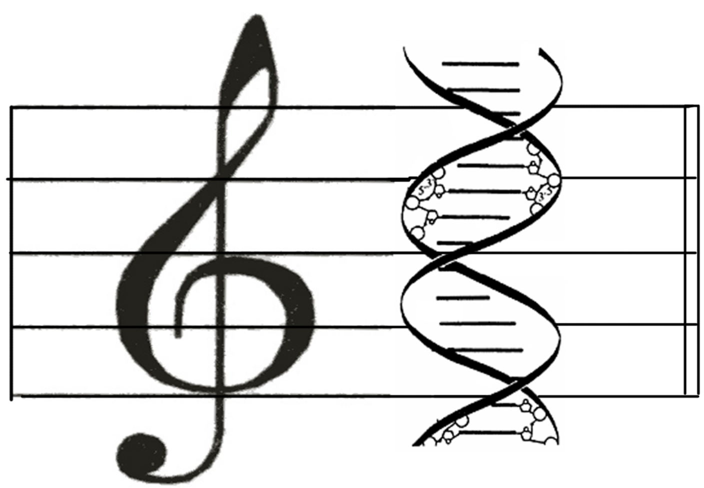
Described facts are related with a problem of genetic bases of aesthetics and inborn feeling of harmony. The famous physicist Nobel prize winner Richard Feynman noted about feeling of musical harmony: “*Whether far we stand from Pythagor in understanding of why only some sounds are pleasant for hearing? The general theory of aesthetics, apparently, has been moved forward not significantly since Pythagorean times*” [Feynman’s lectures of physics, v. 4].

From the viewpoint of musical harmony in structures of molecular-genetic system, outstanding composers seem to be researchers of harmony in the organization of living substance.

Many composers declared about a mysterious connection of music with the golden section early. In our opinion, this connection has genetic bases.

Science and culture seem to be connected each other more closely than one could demonstrate it till now.

In our opinion, music is not only the tool for a call of emotions and pleasures, but also one of the principles of the organization and language of living substance.



The aesthetic aspects of genetic music can be connected with informational aspects, which provide an effect of recognition of a kindred language during listening genetic music. This effect of recognition can be provided by biological algorithms of signal processing inside organisms. For example, in the case of genetic music coming from the outside world, our organism can recognize those ratios, on which its own genetic system is built, and organism responds positively to this manifestation of a structural kinship between the outside world and its own genetic physiology. This positive reaction resembles two persons talking in the same language (if they talk in different languages, mutual understanding doesn't arise though they can speak more and more loudly).

The Moscow P.I.Tchaikovsky Conservatory has created recently a special “Center for Interdisciplinary Researches of Musical Creativity”. One of tasks of this center is studying the "genetic musical scales” from different viewpoints including new opportunities for composers and for musical therapy. Head of scientific department of Moscow Conservatory Prof. K.Zenkin is now here as the chairman and the organizer of this Section “Music, Information and Symmetries”.

The next lecture of a composer and physicist Ivan Soshinskiy will continue this topic. He will also present compositions of genetic music on the concert today evening.