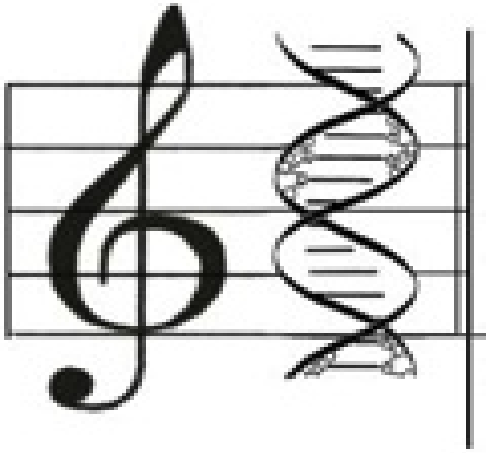
Two-interval musical scales and binary structures in computer science and biology



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Introduction

From ancient times, understanding the phenomenon of music and building musical structures were associated with mathematics. This report analyses relation of music with binary structures. Principles of binary opposition permeate living matter at different levels of its organization.

For example:

- The binary number system, widely used in computer calculations and informatics;
- The complementary pairs of nitrogenous bases of DNA molecules of heredity;
- Organization of muscular movement on the base of muscle pairs of flexor-extensor;
- Pairs of male-female, which give life to new generations, etc.

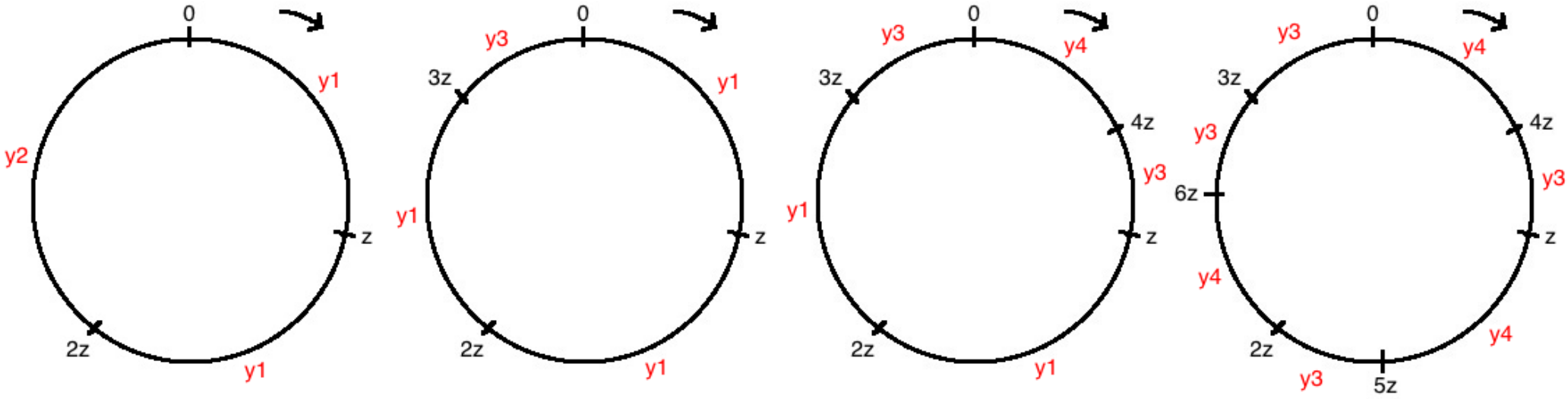
Two-interval musical scales

In the field of musical culture, the binary principle is realized, in particular, in the existence of two-interval musical scales.

This report focuses on the analysis of development of two-interval musical scales on the base of the well-known algorithm of Pythagoras. For the author, the initial types of such musical scales were the known two-interval musical scales: the Pythagorean musical scale and so called “genetic scales” which base of known data about molecular parametrs of molecular-genetic code and "Golden Section" = 1,618...

The algorithm of Pythagoras for constructing musical scales

Pythagoras constructed music scale as delaying the progression of the same interval (quint) from the base sound in space octave repeatability. It can be depicted as building equal intervals arcs on the circle. Where the circle symbolizes the octave.



Theorem

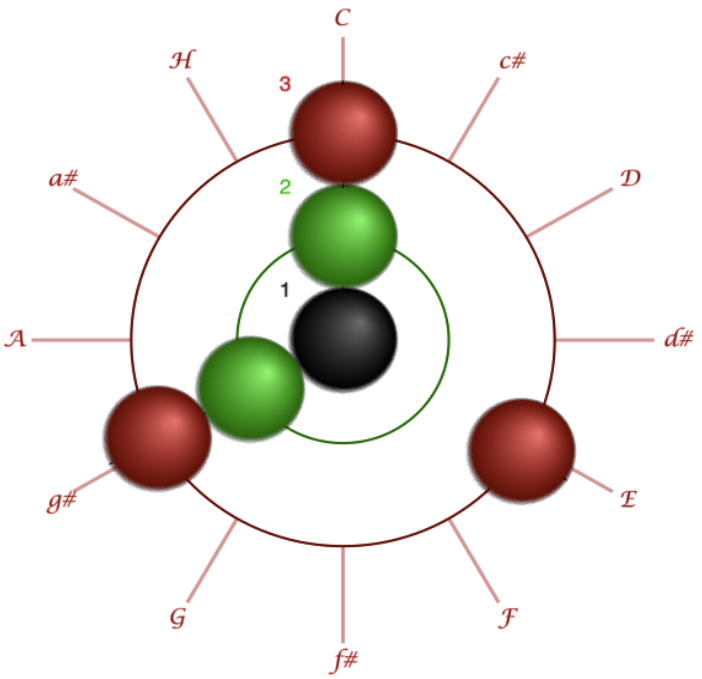
The author proved the theorem according to which only three kinds of scales can be created on the base of the algorithm of Pythagoras:

- one-interval (rare);
- two-interval (are relatively regularly);
- and three-interval (mostly).

The graphical method for the analysis of musical scales

The author has created a convenient graphical method for the analysis of musical scales, generated by the algorithm of Pythagoras for different values of its parameters. He has solved the problem of automation and visual analysis of such algorithmic processes of creation of two-interval scales.

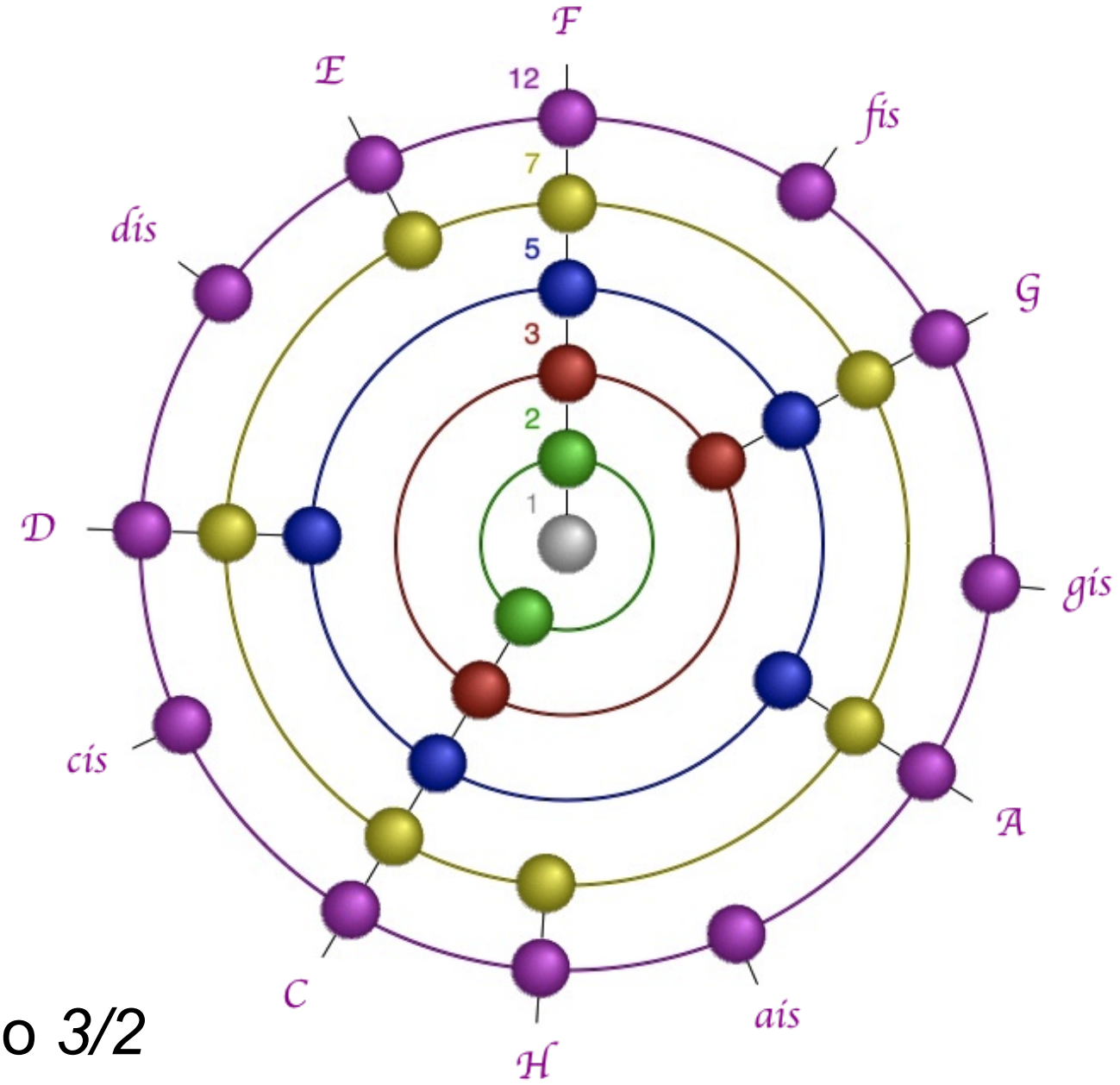
The solution to this problem is based on the writing of a specialized computer program and a visual representation of the family of two-interval scales with a different number of their stages in the form of concentric circles, such as the following:



Comments to visualize

Smooth increase in the length of the arc leads to the effect of rotation when constructing two-interval musical scales. The number and relative positions of steps in one of the series of this family of musical scales are shown on each of the concentric circles.

Image of Pythagorean system



quint ratio 3/2

Analysis of diagram

1 stage (gray). Center is the main tone, in our case it is sound F.

2 stages (green). The sounds F and C determine the quarto-quint rate. Sound C is the first not equal overtone of sound F.

3 stages (red). Sounds CFG are three main steps and basics of classical harmonic relationship Tonic - Subdominant - Dominant.

5 stages (blue). Sounds CDFGA make pentatonic scale spread in folk and Eastern music.

7 stages (yellow). Sounds CDEFGAH - diatonic (white gamma-key), which correspond to the seven old modes.

12 stages (purple). Twelve sounds make chromatic scale of classical academic music. Appeared on this circle sharp sounds cis-dis-fis-gis-ais are closer to the sound its resolution up to the next chromatic sound. And these five notes make the pentatonic scale.

Historical basis

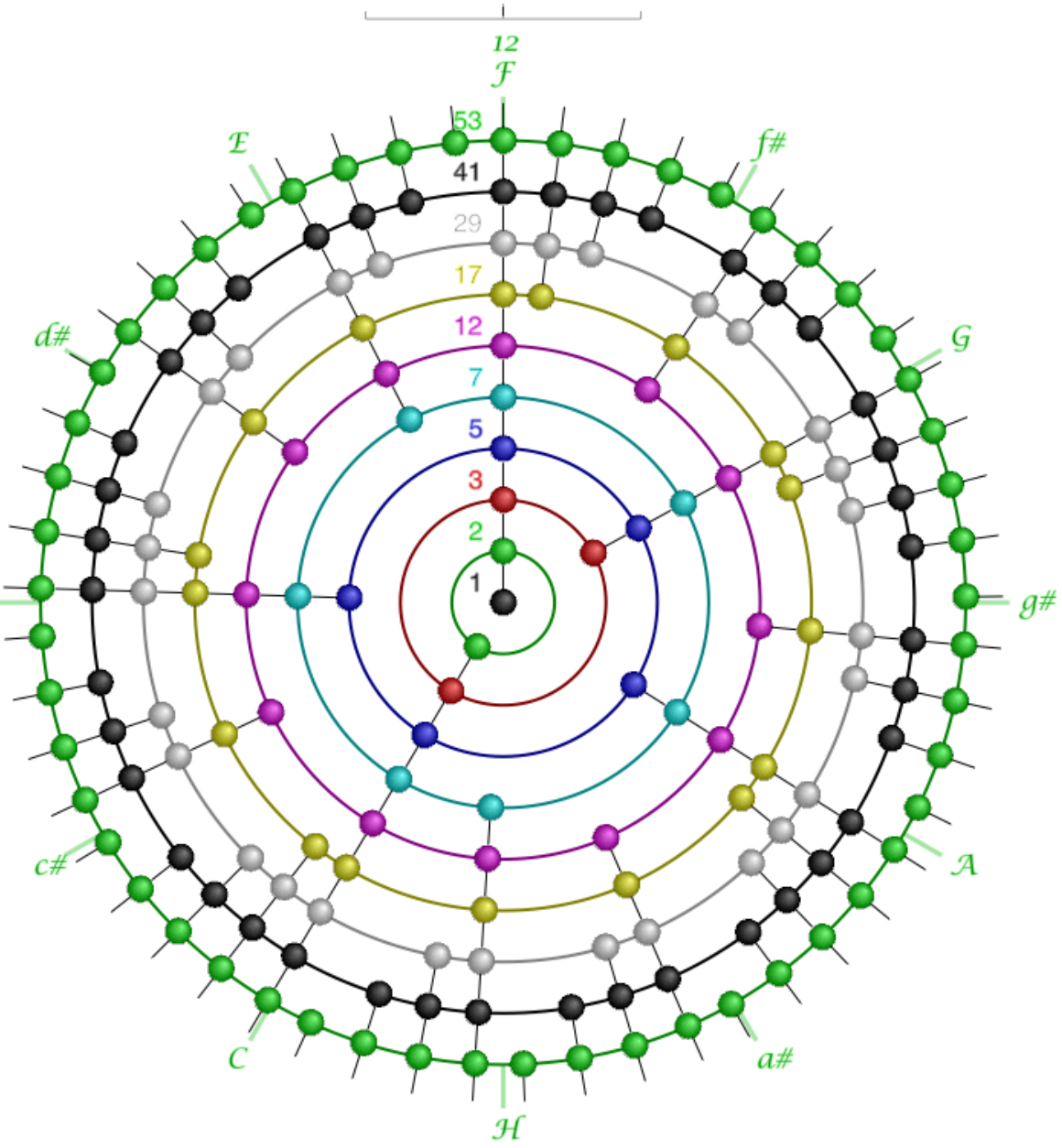
All of these scales (3, 5, 7, 12-stages scales) are resistant steps of musical-historical development, in contrast to non two-interval 4, 6, 8, 9, 10, 11-stages scales, which almost never occur in the musical culture. Thereby confirming that two-interval scale plays are of special importance in the musical perception of a human.

Development of new Pythagorean tunings

$k = 0.58496$
$p = 1.5$

The next higher stages of two-interval Pythagorean scales are 17-, 29-, 41-, 53-stages scales. These scales have a special micro-chromatic structure. The author has written a special computer driver for the sounding music in arbitrary micro-chromatic scales. For example, we can listen to the 29-stage scale (gray cycle).

Play



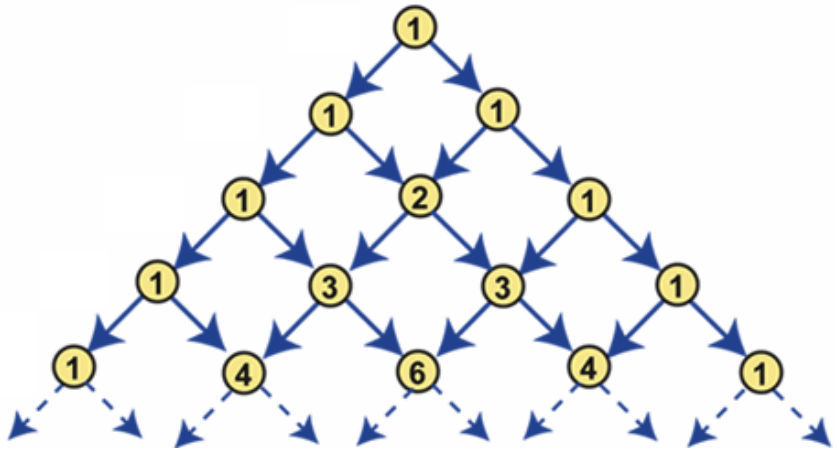
The inverse problem

For the algorithm of Pythagoras the inverse problem has been also solved: knowing the sequence of values of the number of stages inside two-interval scales, which are nested each into other, how one can determine the appropriate multiplying factor.

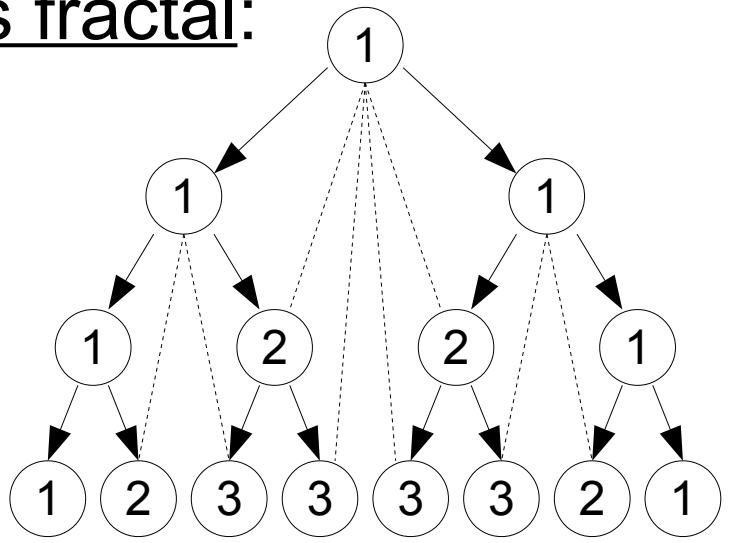
"Pascal's fractal"

In connection with the decision of this problem the author has developed a theory of the "Pascal's fractal": it is a geometric tree of numerical structure with its recursive organization, in which each number is formed as the sum of the two numbers above it (similar to the Pascal's triangle).

Pascal's triangle:

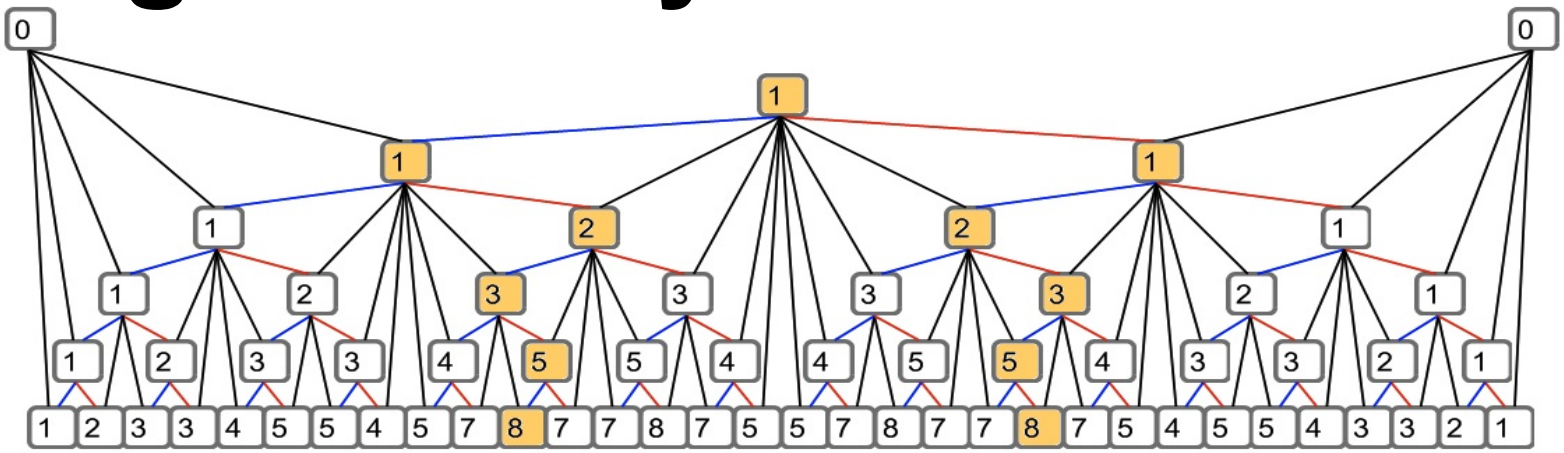


Pascal's fractal:



1, 2, 3, 3, 4, 5, 5, 4, 4, 5, 5, 4, 3, 3, 2, 1

Method of determining of multiplying factor by Pascal's Fractal



We compute the ratio: $$k=\sum_{i=1}^{m}\frac{(-1)^{s_i}}{n_{i-1}\cdot n_i}=1-\frac{1}{n_1\cdot n_2}+\frac{1}{n_2\cdot n_3}+\frac{1}{n_3\cdot n_4}+\ldots$$

For example:

If we want have 1, 1, 2, 3, 5, 8, 13, ... - stage scale then:

$$\phi=1-\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}-\frac{1}{3\cdot 5}+\frac{1}{5\cdot 8}-\frac{1}{8\cdot 13}+\ldots\approx 0.618\ldots$$

this is approximately equal to the «Golden section».

Results

- The computer program for construction and visualization of two-interval stages musical scales for arbitrary coefficients has been developed.

- We have solved the inverse problem that is to find a coefficient using «Pascal's fractal».

- This program has allowed the analyze of the Pythagorean system with the results, which corroborate the importance of the two-interval structures in history of culture.

Conclusion

Music is widely used in today's global connections among people and nations. Development of methods and means of musical culture through in-depth understanding of the fundamentals of musical scales can contribute positive effects of musical influences on society and its members, including possibilities of music therapy.

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Thank you!