Accretion Induced Mass Evolution of a Schwarzschild Black Hole Under Biswas-Roy-Biswas Redshift Parameterized Dark Energy: A Differential Ages Constraint Analysis

Sukanya Dutta*
Department of Mathematics, The University of Burdwan, Golapbag Academic Complex, Burdwan-713104

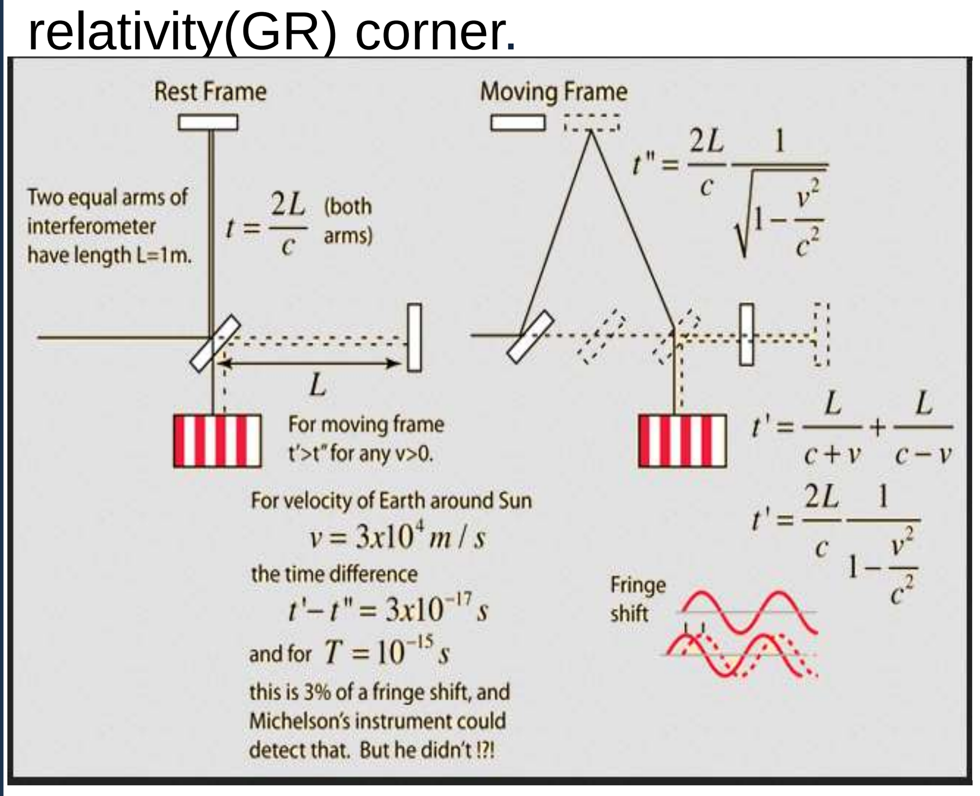
INTRODUCTION & AIM

The Topic covers the accretion of a particular kind of dark energy which is following a definite kind of redshift parametrization model named Biswas-Roy-Biswas (BRB) and the dark energy which we have chosen is suppose to accrete on Schwarzschild black hole in Einstein gravity. The model emphasizes low-redshift dynamics and offers significant flexibility in constraining observational data from SNeIa, BAO and CMB.

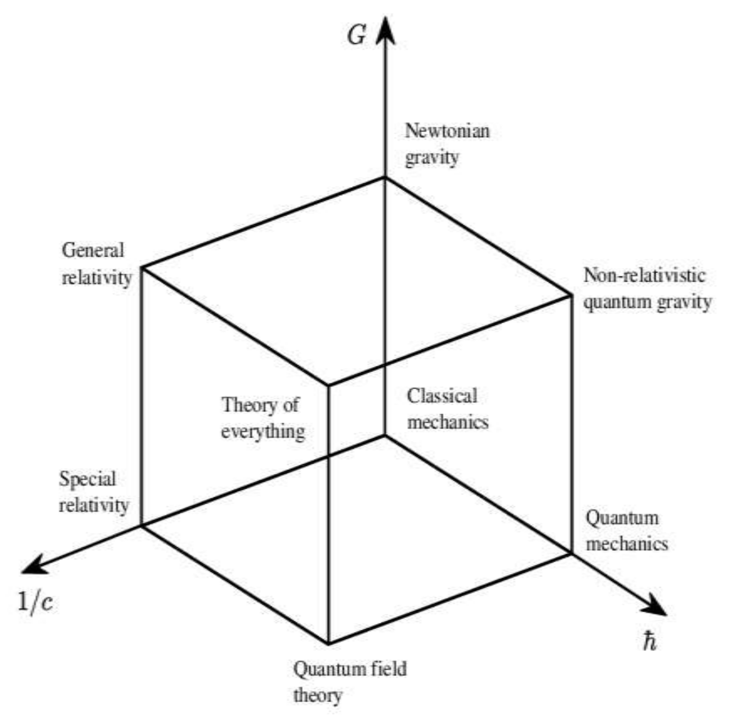
Classical Mechanics to General Relativity : Black Holes

CLASSICAL MECHANICS => STR => GENERAL RELATIVITY

Classical Mechanics/ Galilean Transformations fails to show Maxwell's equations as invariants. This, together with **Michelson and Morley**'s inference, required to shift our understanding from the origin of the **cube of Physics** to the Special theory of relativity(STR) corner. STR, soon combined with the Newton's theory of gravity, gave birth to General relativity(GR) corner. *[GW151226, GW190814]*



Michelson and Morley's Experiment



cube of Physics

Black Hole as Compact Object: Schwarzschild's Black Hole Solution

$$ds^2 = -c^2\left(1-\frac{2GM}{c^2 r}\right)dt^2+\frac{dr^2}{\left(1-\frac{2GM}{c^2 r}\right)}+r^2\left(d\theta^2+\sin^2\theta\, d\varphi^2\right)$$

Reviews of Accretion Studies Around Black Holes

Accretion Around a Compact object : First ever studied by F. Michel.
Viscous accretion : Incorporated by Shakura and Sunyaev as a multiplication of unknown parameter, α_{ss}, density(ρ) and sound speed square(c_s^2). *[Astrophys & Sp.Sc., 15, (1972)153.]*
Pseudo-Newtonian Potentials: To avoid the extreme non linearity caused by general relativity theory PNF is introduced. PNF depending on the distance from the black hole and its rotation. *[ApJ 581(2002)427]*

METHODOLOGY

Proposition of Λ : $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G_N}{c^4} T_{\mu\nu}$

- $g_{\mu\nu}$: The nature of curvation of corresponding space time.
- R : Ricci scalar,
- $R_{\mu\nu}$: Ricci tensor,
- Λ : Cosmological constant,
- $\mathbf{G}_{\mu\nu}$: The Einstein tensor,
- G_N : Newton's gravitational constant,
- $T_{\mu\nu}$: Stress Energy tensor.

*[Reviews of Modern Physics **61**,1–23 (1989)]*

Stress Energy Tensor & Vacuuum Energy : The form of perfect fluid is $T_{\mu\nu} = \left(\rho + \frac{p}{c^2}\right) u_\mu u_\nu + p g_{\mu\nu}$. $T^{vacuum}_{\mu\nu} = p^{vac} g_{\mu\nu} = -\rho c^2 g_{\mu\nu}$. EFE turns $G_{\mu\nu} = \frac{8\pi G_N}{c^4}\left(T^{matter}_{\mu\nu} + T^{vacuum}_{\mu\nu}\right) = \frac{8\pi G_N}{c^4}\left(T^{matter}_{\mu\nu} - \rho^{vacuum} c^2 g_{\mu\nu}\right)$. This shows Cosmological constant and vacuum energy shares identical behaviours related as $\rho^{vacuum} = \frac{\Lambda c^2}{8\pi G_N}$. *[Albert Einstein, 844-847, 1915]*

Quantitative Calculation of Vacuum Energy : Mismatch

Vacuum Energy density : As Observation says $\Rightarrow \rho^{vacuum} c^2 = \frac{(2.24\times10^{-3} eV)^4}{(\hbar c)^3}$.
Vacuum Energy density : As Theory predicts

$$\frac{\rho^{vacuum} c^2}{\rho^{vacuum}_{GM} c^2} = \frac{(2.24\times10^{-3} eV)^4}{(1.22\times10^{28} eV)^4} = 1.13\times10^{-123}.$$

*[Weinberg S., Rev. Mod. Phys. **61**, 1–23 (1989).]*

Hence Cosmic Coincidence Problem for Λ

Needed to search for Dynamical Dark Energy Models

Biswas-Roy-Biswas(BRB) Dark Energy model has chosen

REQUIRED DATA

For the energy density function of the Biswas-Roy-Biswas(BRB) DE model we recall an ansatz for the functional form of $\rho_{c\phi}$ as

$$\frac{1}{\rho_\phi}\frac{d\rho_\phi}{da} = -3\left[\frac{\lambda_1}{1+ak_1} + \frac{\lambda_2(1-a)}{(1+ak_2)^2}\right],$$

where λ_1, λ_2, k_1 and k_2 are constants. Integrating, we get

$$\rho_\phi = A\frac{(1+ak_2)^{\frac{3\lambda_2}{k_2^2}}}{(1+ak_1)^{\frac{3\lambda_1}{k_1}}} exp\left\{\frac{3\lambda_2(1+k_2)}{k_2^2(1+ak_2)}\right\},$$

where $A = \rho_{\phi0}\frac{(1+k_1)^{\frac{3\lambda_1}{k_1}}}{(1+k_2)^{\frac{3\lambda_2}{k_2^2}}} exp\left\{-\frac{3\lambda_2}{k_2^2}\right\}$ and $\rho_{\phi0}$ is the present time (at $z=0$) of the scalar field density. *[P. Biswas, P. Roy, and R. Biswas, Astrophys. Space Sci., vol. 365, no. 7, p. 117, 2020.]*

Best fit values of BRB model parameters from different recentmost datasets

Tools	χ^2	λ_1	λ_2	k_1	k_2
$H(z)-z$	4655.117874	$0.97378^{+0.8667}_{-0.8638}$	$0.9978^{+0.5274}_{-0.5604}$	$1.0670^{+1.0114}_{-1.0206}$	$0.8287^{+0.7565}_{-0.7446}$
$H(z)-z+BAO$	5416.228601	$0.9239^{+0.8649}_{-0.8678}$	$1.0093^{+0.8649}_{-0.8678}$	$1.1089^{+1.0358}_{-1.0360}$	$0.8299^{+0.7577}_{-0.7446}$
$H(z)-z+BAO+CMB$	14610.907613	$1.0761^{+0.8362}_{-0.8295}$	$0.7359^{+0.6107}_{-0.6422}$	$1.1519^{+1.0356}_{-1.0321}$	$0.8513^{+0.7292}_{-0.7208}$

[AstroJournal,D.J.e.Eisenstein vol. 633,p560–574,Nov.2005, R. Jimenez, vol. 573,p. 37–42, jul 2002]

RESULTS WITH PLOTS

The equation of mass in terms of redshift function z for BRB Parameterization

$$M = \frac{M_0}{1+\frac{4\pi\zeta_2^{BRB}M_0}{3}\int_{\rho_0}^{\rho}\frac{d\left(\rho_{\phi0}\frac{(1+k_1)^{\frac{3\lambda_1}{k_1}}}{(1+k_2)^{\frac{3\lambda_2}{k_2^2}}}\exp\left\{-\frac{3\lambda_2}{k_2^2}\right\}\frac{(1+ak_2)^{\frac{3\lambda_2}{k_2^2}}}{(1+ak_1)^{\frac{3\lambda_1}{k_1}}}\exp\left\{\frac{3\lambda_2(1+k_2)}{k_2^2(1+ak_2)}\right\}\right)}{H_0\sqrt{\Omega_{rad0}a^{-4}+\Omega_{dm0}a^{-3}+\Omega_{\phi0}\beta\frac{(1+ak_2)^{\frac{3\lambda_2}{k_2^2}}}{(1+ak_1)^{\frac{3\lambda_1}{k_1}}}\exp\left\{\frac{3\lambda_2(1+k_2)}{k_2^2(1+ak_2)}\right\}+\Omega_{k0}(1+z)^2}}}$$

where $\Omega_{rad0}, \Omega_{dm0}, \Omega_{\phi0}, \Omega_{k0}$ are the present day dimensionless densities. Redshift z =($\frac{1}{a}$ -1), H_0 is the Hubble parameter at redshift z=0, M_0(M (z = 0)) present day mass of BH, constant ζ_2^{BRB} is calculated numerically as the function of density over a large range onto the solution of Schwarzschild BH and $\beta = \frac{(1+k_1)^{\frac{3\lambda_1}{k_1}}}{(1+k_2)^{\frac{3\lambda_2}{k_2^2}}}\exp\left(-\frac{3\lambda_2}{k_2^2}\right)$.

Fig. 1(a) : For H(z)-z data Analysis **Fig. 1(b) :** For H(z)-z+BAO Analysis **Fig. 1(c) :** For H(z)-z+BAO+CMB Analysis **Fig. 1(d) :** $\log(\frac{M}{M_0})$ vs redshift plot

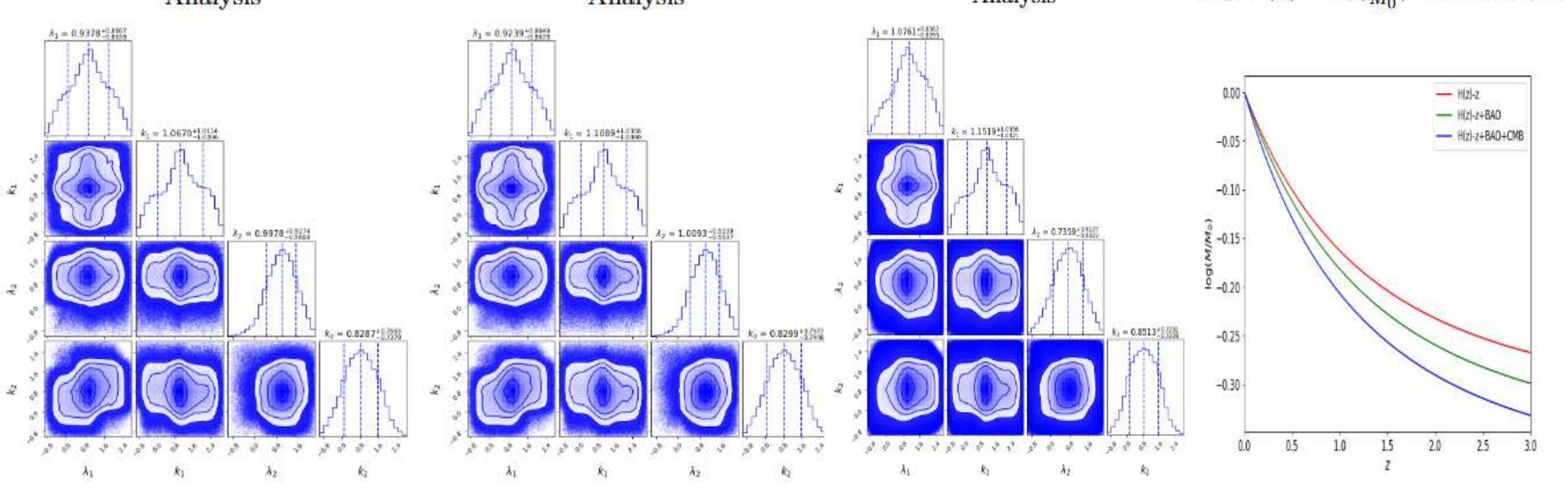


Figure 1: Fig. 1(a)-1(c) represent confidence contours in λ_1-k_1 , $\lambda_1-\lambda_2$, λ_1-k_2 , $k_1-\lambda_2$, $k_2-\lambda_2$ and k_1-k_2 planes and individual distributions of free parameters λ_1 , k_1 , λ_2 and k_2. 1(d) represents the variation of the $\log\left\{\frac{M}{M_0}\right\}$ due to the accretion of BRB type DE onto Schwarzchild BH.

CONCLUSION

- In Fig.1(d) we plot $log_{10}(\frac{M^{BRB}}{M_0^{BRB}})$ vs. z for growing mass and the mass to increase in past.
- Mass of the BH is found to gain almost 55% of its present time mass in the era from 3 redshift to the zero redshift and the increasing trend is consistent with standard hierarchical growth.
- Constrained by observational H(z) data, the parameters indicate that DE remains close to Λ but may evolve slightly with time, potentially leading to a future deceleration phase.
- Model offering flexible yet observationally consistent description of late-time cosmic acceleration.

FUTURE SCOPES

Universe exhibits a preferred DE evolution, corresponding to a definite dynamical attractor which is surrounded by a plateau of quasi-degenerate states that yield nearly identical cosmic expansion histories. Structure reflects the insensitivity of current cosmological observations to subtle dynamical variations in the DE sector, a wellknown limitation in reconstructing fine features of the late-time acceleration. **We should work to overcome this limitation in future.**