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Dynamical Behaviour of f(Q) Gravity in Bianchi-I Spacetime: Fixed Points, Anisotropy, and Cosmological Implications

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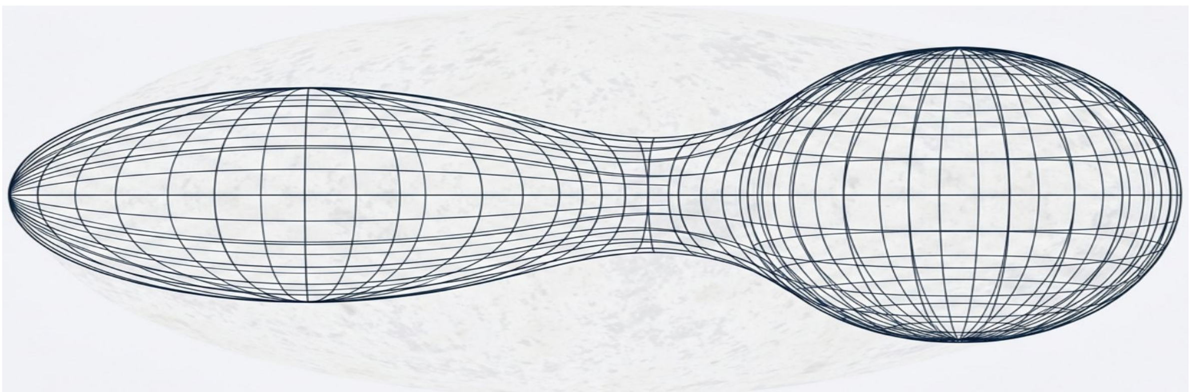
INTRODUCTION

Classical Mechanics to Special Relativity

Motivated by the results of 14 years long repeated experiments of Michaelson and Morley, which did state speed of light can not be relative : $c \pm v = c$; Einstein proposed his theory of special relativity in 1905. The main postulates :
(i) Every law should be unique/invariant with respect to reference frames, (ii) Speed of light should be constant.



General Relativity : $\mathbf{G}_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$

In 1915, Einstein completed the series of publications where he has proposed his general theory of relativity. Here the basic assumption was the corelation of the space time curvature and the matter /energy present in it. It was a modified version of previously postulated **Mach's principle**.

What is Q ?

- Q represents the non-metricity scalar.
- Q is derived from the non-metricity tensor, which quantifies the lack of metric compatibility in the teleparallel framework.
- Q represents the overall "curvature" or "non-metricity" of space-time.

Why non-metricity scalar Q is required ?

- In standard General Relativity (GR), gravity is described by the Ricci scalar R.
- In Teleparallel Gravity, gravity is described using torsion scalar T.
- In symmetric teleparallel gravity (on which $F(Q)$ gravity is based), curvature and torsion are both set to zero, so the only remaining structure is nonmetricity.

What is F(Q) gravity ?

- F(Q) gravity is a class of modified gravity theories based on metric-affine geometry, where the gravitational field is described not by curvature (as in General Relativity) or torsion (as in Teleparallel Gravity), but by nonmetricity.
- Nonmetricity refers to the failure of the connection to preserve the metric under parallel transport. In mathematical terms:

$$Q_{\alpha\mu\nu} = \nabla_\alpha g_{\mu\nu}$$

$$Q = -g^{\mu\nu}\left(P^{\lambda}_{\mu\nu} Q^{\mu\nu}_{\lambda}\right) .$$

This non-metricity scalar Q plays almost a similar role as the Ricci scalar R plays in GR.

Replacing Curvature with Non-Metricity (Q)

Editorial Physics

$$S = \int \sqrt{-g}\left[\frac{1}{2} f(Q) + \mathcal{L}_m\right] d^4x$$

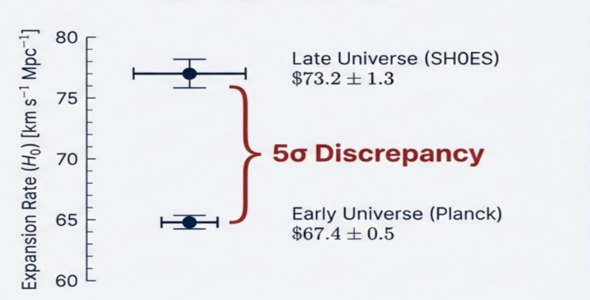
The Action Integral — Arbitrary function of Non-Metricity — Matter Lagrangian

General Relativity describes gravity via Curvature (R). Symmetric Teleparallel Gravity replaces this with **Non-Metricity (Q)**—the variation of vector length during parallel transport.

- Second-order field equations (Avoids Ostrogradsky instability)
- Explains Dark Energy without a Cosmological Constant
- Vanishing Curvature and Torsion

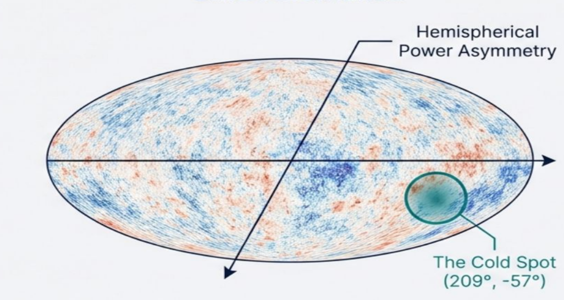
The Cracks in the Standard Model

The Hubble Tension



The universe appears to expand faster locally than predicted by early physics.

CMB Anomalies



Unexplained lopsidedness and cold spots challenge the assumption of perfect isotropy.

Hypothesis: The early universe was not isotropic. We need a theory that allows for initial anisotropy that naturally heals over time.

The Laboratory: Bianchi-I Anisotropy

To stress-test $f(Q)$ gravity, we place it in a universe that expands at different rates in different directions.

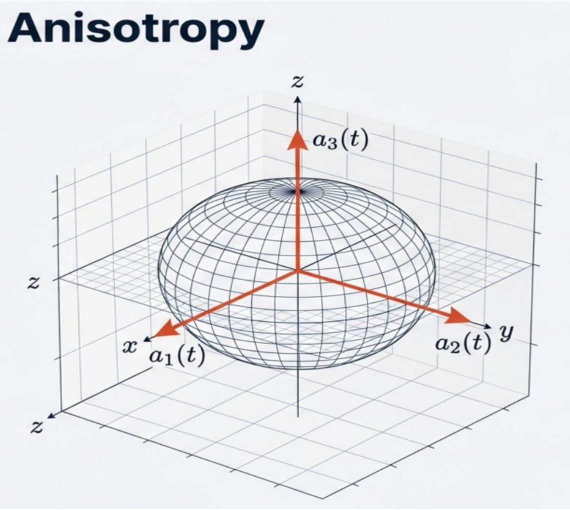
Metric Equation:

$$ds^2 = -dt^2 + \sum_{i=1}^{3} a_i^2(t)(dx^i)^2$$

σ: The deviation from perfect spherical expansion.

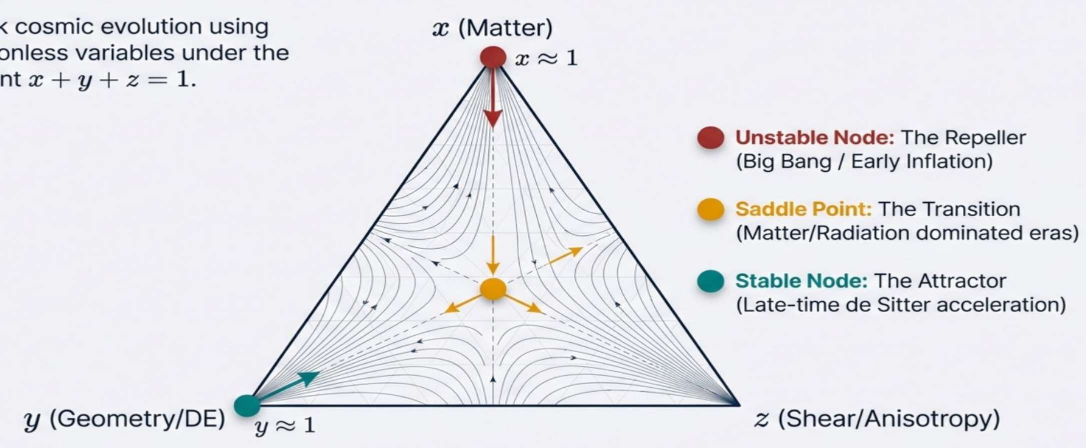
- If $\sigma^2 = 0$ → Standard Isotropic Universe (FLRW)
- If $\sigma^2 > 0$ → Anisotropic Universe

Research Question:
Does $f(Q)$ gravity force $\sigma \to 0$ over time?



Mapping the Phase Space

We track cosmic evolution using dimensionless variables under the constraint $x + y + z = 1$.



METHOD

Define the dimensionless variables as, [7]

$$x = \frac{\kappa\rho}{6 f_Q H^2}, \quad y = -\frac{f}{12 f_Q H^2} \quad \text{and} \quad z = \frac{\sigma^2}{6H^2} .$$

Constrained by the relation

$$x + y + z = 1 .$$

Now, the dynamical system equations can be constructed by taking the derivatives of the dimensionless variables x, y and z with respect to the variable $N = \ln a$, given as,

$$x' = -3x(1+\omega) + 6x\left\{\frac{(y-1-x\omega)(1+\Gamma-\Gamma z-2z)}{(\Gamma+2)(z-1)}\right\} - \frac{6xz}{(\Gamma+2)(z-1)}$$

$$y' = 3\Gamma\left(\frac{z+y-1-x\omega}{\Gamma+2}\right) + 6y\left[\frac{(y-1-x\omega)(1+\Gamma-\Gamma z-2z)}{(\Gamma+2)(z-1)}\right] - \frac{6yz}{(\Gamma+2)(z-1)} \quad \text{and}$$

$$z' = 6z(x\omega - y)$$

$$\frac{\dot H}{H^2} = 3\left\{\frac{(2z+z\Gamma-\Gamma)(-z-x-x\omega)+2z}{(\Gamma+2)(z-1)}\right\}$$

$$\omega_{\text{eff}} = -1 - \frac{2}{3}\frac{\dot H}{H^2}, \quad \Gamma = \frac{f_Q}{Q f_{QQ}}$$

$$\frac{Q f_Q}{f} = \frac{(1-z)}{2(1-x-z)}$$

Suppose a vector-valued function is given by,

$$f(x_1, x_2) = \begin{bmatrix} f_1(x_1,x_2) \\ f_2(x_1,x_2) \end{bmatrix} .$$

The Jacobian matrix linearizes the system around an equilibrium point and is defined as (considering 2D system):

$$J\left(\frac{f_1, f_2}{x_1, x_2}\right) = \left[\frac{\partial f_i}{\partial x_j}\right] = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix} ,$$

where $i, j = 1, 2$.

For the functions x' and z', the Jacobian matrix takes the form

$$J\left(\frac{x', z'}{x, z}\right) = \begin{bmatrix} \frac{\partial x'}{\partial x} & \frac{\partial x'}{\partial z} \\ \frac{\partial z'}{\partial x} & \frac{\partial z'}{\partial z} \end{bmatrix} .$$

Model I : $f(Q) = mQ^n$

The model $f(Q) = mQ^n$ is known as a power law model with the parameters m which sets the strength of the gravitational computing and n which determines how non-linearly the gravity theory depends on the scalar Q. At $n = 1$, this model reduces to $f(Q) = mQ$ which is equivalent to GR if $m = 1$. For specific powers $n > 1$, this model can mimic inflationary potentials which incorporating scalar fields like the inflation in standard models.

Now, the Q derivatives of the model-I can be obtained as,

$$f_Q = mnQ^{n-1} \quad \text{and} \quad f_{QQ} = mn(n-1)Q^{n-2}$$

$$z = \frac{2n(x-1)+1}{1-2n} = 1 - \frac{2nx}{2n-1}$$

$$\Gamma = \frac{1}{n-1}$$

$$x' = 3x\left\{\frac{2x(1+\omega)}{(2+\frac{1}{n-1})(z-1)} + 2(x+z+x\omega) - 1 - \omega\right\} \quad \text{and}$$

$$z' = 6z(x\omega + x + z - 1)$$

$$\frac{\dot H}{H^2} = \frac{3z\{z-1-2n(z-1)\} - 3x(\omega+1)\{(2n-1)z-1\}}{(2n-1)(z-1)}$$

$$N(h(x)) = h'(x)\cdot x' - z' = h'(x)\cdot 3x\left[\frac{2x(1+\omega)}{(2+\frac{1}{n-1})\{h(x)-1\}} + 2\{x+h(x)+x\omega\} - 1 - \omega\right] - [6h(x)\{x\omega+x+h(x)-1\}] .$$

Model-II : $f(Q) = exp\{nQ\}$

$$x' = \frac{6x\{x^2(\omega+1) - 2x(\omega+1) + x(2\omega+3)z + (z-1)(2z-\omega-1)\}}{x+2z-2} \quad \text{and} \quad \Gamma = \frac{f_Q}{Qf_{QQ}} = \frac{1}{nQ} = \frac{2(1-x-z)}{1-z}$$

$$z' = 6z(x\omega + x + z - 1) .$$

$$\frac{\dot H}{H^2} = -\frac{3\left[x^2(\omega+1) + x\{(2\omega+3)z - 1 - \omega\} + 2(z-1)z\right]}{x+2z-2}$$

$$N(h(x)) = h'(x)\cdot x' - z' = h'(x)\left[\frac{6x\{x^2(\omega+1) - 2x(\omega+1) + x(2\omega+3)h(x) + (h(x)-1)(-\omega+2h(x)-1)\}}{x+2h(x)-2}\right] - [6h(x)\{x\omega+x+h(x)-1\}]$$

Model-III : $f(Q) = \alpha Q^2 + vQ^2\log(Q)$

$$x' = 3x\left[\frac{2x(\omega+1)\{16x+13(z-1)\}}{(z-1)\{40x+33(z-1)\}} + 2(x\omega+x+z) - \omega - 1\right] \quad \text{and} \quad \Gamma = \frac{8x+7z-7}{16x+13(z-1)}$$

$$z' = 6z(x\omega + x + z - 1) .$$

$$\frac{\dot H}{H^2} = -\frac{3\left[8x^2(\omega+1)(5z-1) + x(z-1)\{(33\omega+73)z - 7(\omega+1)\} + 33(z-1)^2 z\right]}{(z-1)\{40x+33(z-1)\}}$$

$$N(h(x)) = h'(x)\cdot x' - z' = h'(x)\cdot 3x\left[\frac{2x(\omega+1)\{16x+13(h(x)-1)\}}{(h(x)-1)\{40x+33(h(x)-1)\}} + 2\{x\omega+x+h(x)\} - \omega - 1\right] - [6h(x)\{x\omega+x+h(x)-1\}]$$

Model-IV : $f(Q) = \eta Q_0\sqrt{\frac{Q}{Q_0}}\log\left(\frac{\lambda Q_0}{Q}\right)$

$$x' = 3x\left\{\frac{x(\omega+1)}{3-4x-3z} + 2(x\omega+x+z) - \omega - 1\right\} \quad \Gamma = \frac{8(x+z-1)}{1-z}$$

$$z' = 6z(x\omega + x + z - 1) \quad \text{and} \quad \frac{\dot H}{H^2} = -\frac{3\left(4x^2(\omega+1) - 4x(\omega+1) + x(3\omega+7)z + 3(z-1)z\right)}{4x+3z-3}$$

$$N(h(x)) = h'(x)\cdot x' - z' = h'(x)\cdot 3x\left[\frac{x(\omega+1)}{3-4x-3h(x)} + 2\{x\omega+x+h(x)\} - \omega - 1\right] - [6h(x)\{x\omega+x+h(x)-1\}]$$

RESULTS & DISCUSSION

In this section, we calculate the stability of the different $f(Q)$ gravity models for the Bianchi-I in the connection Γ_1. To perform this analysis in the presence of the matter component and will plot the phase portraits for every model.

Crit. Point	Existence	Eigenvalues	Type of crit. point	Stability	C_T^2	Cosmology
$x \to 0$	$\forall\omega$	-6 and -6	Hyperbolic ($\forall\omega$)	Stable node	$\frac{1}{2n-1}$ for $n > 0$	$a(t) = a_0 e^{H_0 t}$ (i.e., de Sitter)
$x \to \frac{2n-1}{2n}$	$n \neq 0$, $\forall\omega$	$\frac{3}{n}\{(2n-1)\omega - 1\}$ and $3(\omega+1)$	Hyperbolic ($\omega \neq -1, \frac{1}{2n-1}$)	Stable node for ($\omega < -1$)		$a(t) \propto t^{-\frac{2n}{3(\omega+1)}}$
				Saddle point for ($\omega > -1$, $\omega < \frac{1}{2n-1}$)		
				Unstable node for ($\omega > -1$, $\omega > \frac{1}{2n-1}$)		
			Non-hyperbolic ($\omega = -1, \frac{1}{2n-1}$)	Central manifold for ($\omega = -1, \frac{1}{2n-1}$)		

Table 1: Critical points with existence conditions, eigen values, type of critical points, stability, cosmology pattern and value of C_T^2 for model-I: $f(Q) = mQ^n$.

Thus for $n > \frac{1}{2}$, $c_s^2 > 0$ and $m > 0$, the no-ghost condition holds if $Q > 0$.

$$\ddot h_{ij} + \left(3H + (n-1)\frac{\dot Q}{Q}\right)\dot h_{ij} + \frac{k^2}{a^2}h_{ij} = 0 .$$

While working with the tensor perturbation, the tensor modes propagate with the speed :

$$C_T^2 = \frac{f_Q}{f_Q + 2Qf_{QQ}} = \frac{1}{2n-1} .$$

The expression for the tensor propagation speed,

$$c_T^2 = \frac{f_Q}{f_Q + 2Qf_{QQ}}$$

$$\frac{\Delta T}{T}(\hat n) \approx -\int_{t_{ls}}^{t_0} \sigma_{ij} n^i n^j dt$$

Crit. Point	Existence	Eigenvalues	Type of crit. point	Stability	C_T^2	Cosmology
$x \to 0$	$\forall\omega$	$-3(\omega+1)$ and -6	Hyperbolic ($\omega \neq -1$)	Stable node for $\omega > -1$	$\frac{1}{1+2nQ}$ with $2nQ << 1$	$a(t) = a_0 e^{H_0 t}$ (i.e., de Sitter)
				Saddle Point for $\omega < -1$		
			Non-hyperbolic ($\omega = -1$)	Center Manifold for $\omega = -1$		
$x \to 1$	$\forall\omega$	0 and 6ω	Non-hyperbolic ($\forall\omega$)	Fully degenerate for $\omega = 0$		$a(t) = a_0 e^{H_0 t}$ (i.e., de Sitter)
				unstable for $\omega > 0$		
				Center Manifold for $\omega < 0$		

Table 2: Critical points with existence conditions, eigenvalues, type of critical points, stability, cosmology pattern and value of C_T^2 for model-II: $f(Q) = exp\{nQ\}$.

The propagation speed of scalar perturbation is,

$$C_s^2 = \frac{1 + n\frac{Q}{H}}{1 + 2nQ} .$$

The stability occurs when $C_s^2 > 0$, if $n > 0$ and the no ghost condition holds if $n > 0$.

The tensor modes propagate with the speed is,

$$C_T^2 = \frac{1}{1 + 2nQ}$$

$$\frac{\sigma_{ij}}{H}\bigg|_{t_0} = \frac{\sigma_{ij,*}}{H_0} a^{-3}(t_0) e^{-n(Q_0 - Q_*)}$$

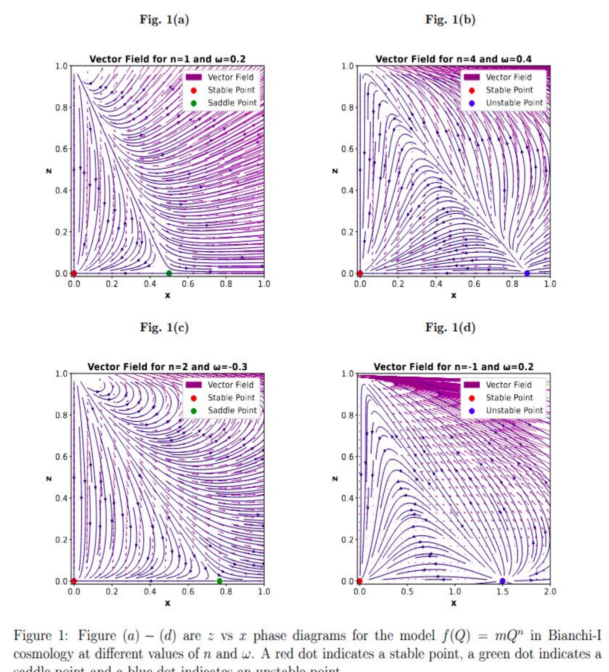


Figure 1: Figure (a) – (d) are z vs x phase diagrams for the model $f(Q) = mQ^n$ in Bianchi-I cosmology at different values of n and ω. A red dot indicates a stable point, a green dot indicates a saddle point and a blue dot indicates an unstable point.

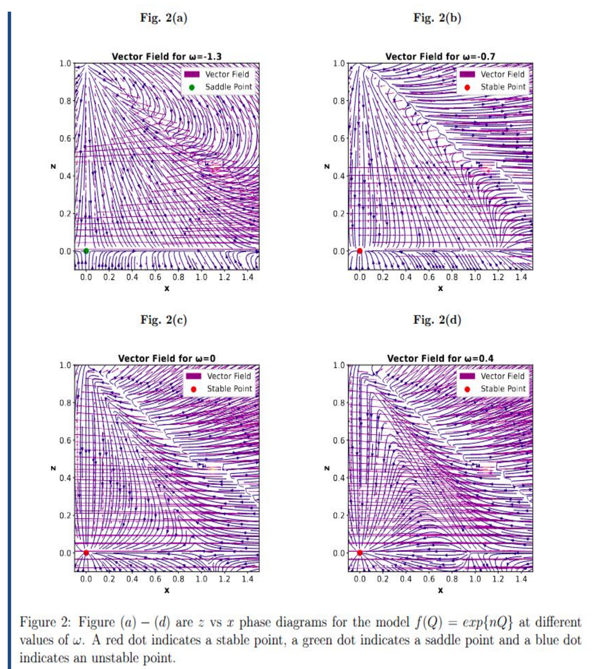


Figure 2: Figure (a) – (d) are z vs x phase diagrams for the model $f(Q) = exp\{nQ\}$ at different values of ω. A red dot indicates a stable point, a green dot indicates a saddle point and a blue dot indicates an unstable point.

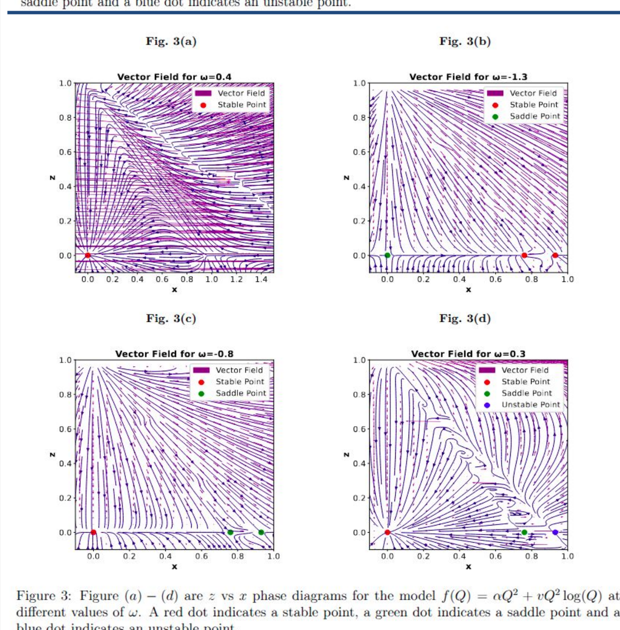


Figure 3: Figure (a) – (d) are z vs x phase diagrams for the model $f(Q) = \alpha Q^2 + vQ^2\log(Q)$ at different values of ω. A red dot indicates a stable point, a green dot indicates a saddle point and a blue dot indicates an unstable point.

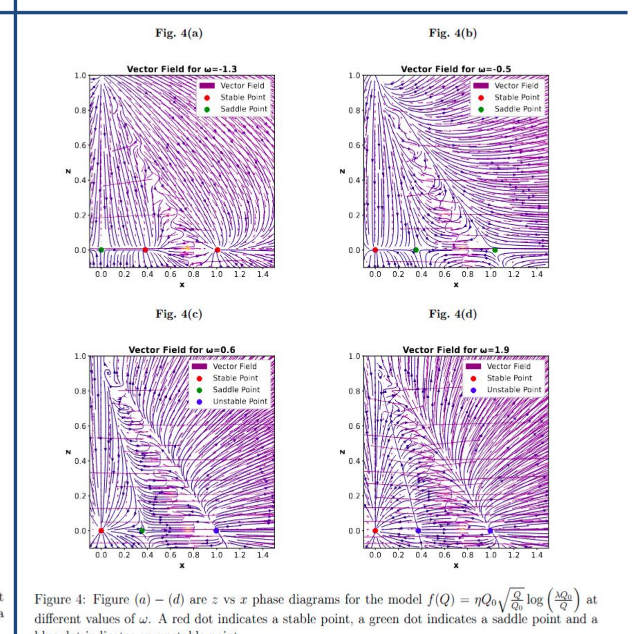
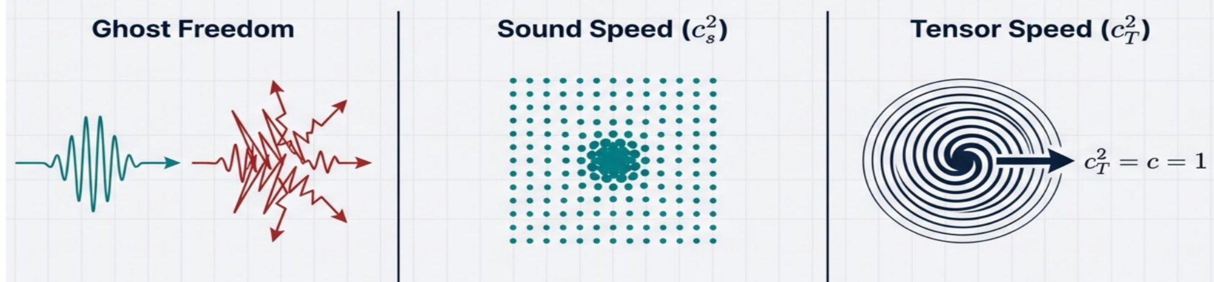


Figure 4: Figure (a) – (d) are z vs x phase diagrams for the model $f(Q) = \eta Q_0\sqrt{\frac{Q}{Q_0}}\log\left(\frac{\lambda Q_0}{Q}\right)$ at different values of ω. A red dot indicates a stable point, a green dot indicates a saddle point and a blue dot indicates an unstable point.

Among the models considered, the exponential form exhibits the most robust theoretical behaviour. A broad region of parameter space satisfies all the consistency requirements, including positive sound speed, absence of ghosts, acceptable tensor speed, and a stable de Sitter like attractor with decaying anisotropy. This makes the exponential model particularly appealing from the viewpoint of theoretical viability.

Do the models break the laws of physics?

Ghost Freedom — **Sound Speed (c_s^2)** — **Tensor Speed (c_T^2)**



No negative energy states. Condition: $f_Q + 2Qf_{QQ} > 0$

Must be real and positive ($c_s^2 \geq 0$) to avoid gradient instabilities.

Must equal light speed ($c = 1$). Model IV Constraint: Requires $\log(\lambda Q_0/Q) \approx 2$.

CONCLUSION

First gravity model chosen with non-metricity scalar is $f(Q) = mQ^n$. This mimics general relativity for $m = n = 1$. $n > 1$ is able to reproduce early inflation. For $0 < n < 1$, however, gravity gets weaken and can be interpreted as the effect of a repulsive force, i.e., the late-time cosmic acceleration is made after. This model can predict gravitational waves to propagate with the speed of light which is supported by GW170817 event.

In the second model, $f(Q) = e^{nQ}$ in Bianchi-I cosmology provides a minimal, purely geometric mechanism for explaining the universe's anisotropic origin and accelerated fate, all within a unified, covariant and second order field theory. $f(Q) = e^{nQ}$ grows rapidly, so does f_Q, leading to strong geometric feedback on the shear evolution. If matter is subdominant and $n > 0$, shear decays but slowly which implies anisotropic inflation. Linear perturbations grow away from this mode such that the system exits anisotropic phase naturally. Besides such unstable points, if we look towards stable nodes, we follow the shear equation $\dot\sigma + 3H\sigma \approx 0$ and to imply $\sigma \propto e^{-3Ht}$ and the universe ends in a fully isotropic accelerated expansion, consistent with CMB constraints. At late-time shear vanishes to restore isotropy.

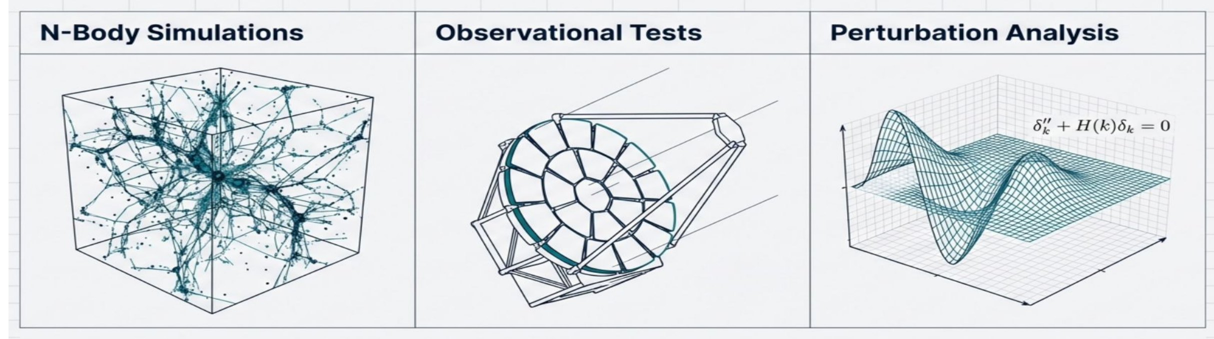
Model	Ghost-free	$c_s^2 \geq 0$	Late-time attractor	Viable region
Model I	Limited	Limited	Yes	Narrow ($n \simeq 1$)
Model II	Moderate	Moderate	Yes	Restricted
Model III	Restricted	Restricted	Conditional	Small
Model IV	Yes	Yes	Yes	Broad

A center manifold is also observed to form. This lies in the shear direction while other parameters are stable. Here shear neither grows nor decays first. Universe remains mildly anisotropic. Higher order corrections regulate whether the universe becomes isotropic later or not. Universe could hover near the manifold, mimicking near-isotropic expansion.

FUTURE WORK / REFERENCES

Future Prospects & Validation

N-Body Simulations — **Observational Tests** — **Perturbation Analysis**



Testing non-linear regimes. — Confronting models with upcoming survey data. — Full anisotropic perturbation treatment to confirm stability.

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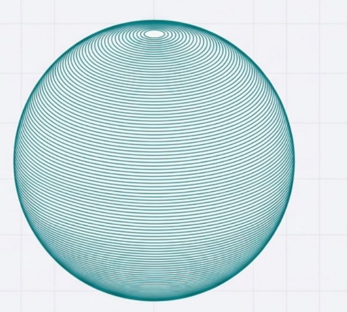
Gravity Heals the Universe



The universe may have started broken, but the dynamics of non-metricity drive it toward the ordered cosmos we inhabit.

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