

# Semileptonic decays of $B_c$ involving vector mesons in self-consistent covariant light-front quark model

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## Introduction

The form factors of semileptonic  $B_c \rightarrow V$  meson decays are calculated in the framework of covariant light-front quark model. We have investigated the self-consistency in these form factors by studying type-I scheme. We find that the form factors obtained by covariant light-front approach within type-I correspondence scheme are inconsistent. But this can be resolved by employing generalized correspondence type-II scheme, which advocates an additional replacement  $M \rightarrow M_0$  relative to the traditional type-I scheme. Further, we predicted total branching fractions of various semileptonic decays of  $B_c$  mesons on the basis of these form factors and helicity formalism.

## Self-consistent Covariant Light-Front Quark Model

**Inconsistency Puzzle :**  $[Q]_{CLF}^{\lambda=0} \neq [Q]_{CLF}^{\lambda=\pm}$

**Type-I correspondence:**

$$\sqrt{2N_c} \frac{\chi_V(x_1, \mathbf{k}_\perp^{(c)})}{x_2} \rightarrow \frac{\psi_V(x_1, \mathbf{k}_\perp^{(c)})}{\sqrt{x_1 x_2 \hat{M}_0^{(c)}}}, \quad D_{V,con}^{(c)} \rightarrow D_{V,LF}^{(c)},$$

where  $\hat{M}_0^{(c)} \equiv \sqrt{M_0^{(c)} - (m_1^{(c)} - m_2)^2}$ .

$D_{V,con}^{(c)} = M^{(c)} + m_1^{(c)} + m_2$  and  $D_{V,LF}^{(c)} = M_0^{(c)} + m_1^{(c)} + m_2$ .

$$M_0^{(c)} = \sqrt{\frac{m_1^{(c)2} + k_\perp^{(c)2}}{x_1} + \frac{m_2^2 + k_\perp^{(c)2}}{x_2}} \quad \text{with } \mathbf{k}_\perp^{(c)} = \mathbf{k}'_\perp - x_2 \mathbf{q}_\perp.$$

A suitable choice for the radial wave function is the phenomenological Gaussian-type wave function, i.e.,

$$\psi(x_1, \mathbf{k}'_\perp) = 4 \frac{\pi^{\frac{3}{4}}}{\beta^{\frac{3}{2}}} \sqrt{\frac{\partial k'_z}{\partial x_1}} \exp \left[ -\frac{k_z'^2 + k_\perp'^2}{2\beta^2} \right]$$

**Type-II correspondence:**

$$\sqrt{2N_c} \frac{\chi_V(x_1, \mathbf{k}_\perp^{(c)})}{x_2} \rightarrow \frac{\psi_V(x_1, \mathbf{k}_\perp^{(c)})}{\sqrt{x_1 x_2 \hat{M}_0^{(c)}}}, \quad M^{(c)} \rightarrow M_0^{(c)}.$$

## $B_c$ to $V$ Form Factors

$$\langle V(P'', \varepsilon'') | V_\mu | B_c(P') \rangle = \varepsilon_{\mu\nu\alpha\beta} \varepsilon''^\nu * {}^V P^\alpha q^\beta g(q^2),$$

$$\langle V(P'', \varepsilon'') | A_\mu | B_c(P') \rangle = -i \{ \varepsilon'' * f(q^2) + \varepsilon'' * P \cdot P_\mu a_+(q^2) + q_\mu a_-(q^2) \}$$

The form factors  $g(q^2)$ ,  $f(q^2)$ ,  $a_+(q^2)$ ,  $a_-(q^2)$  are related to the commonly used BSW form factors  $V(q^2)$ ,  $A_0(q^2)$ ,  $A_1(q^2)$ ,  $A_2(q^2)$ ,  $A_3(q^2)$ :

$$\begin{aligned} V^{B_c V}(q^2) &= - (M_{B_c} + M_V) g(q^2) \\ A_1^{B_c V}(q^2) &= - \frac{f(q^2)}{M_{B_c} + M_V} \\ A_2^{B_c V}(q^2) &= (M_{B_c} + M_V) a_+(q^2) \\ A_3^{B_c V}(q^2) - A_0^{B_c V}(q^2) &= \frac{q^2}{2M_V} a_-(q^2) \end{aligned}$$

## $q^2$ dependence

The form factor is expressed as:

$$F(q^2) = \frac{1}{1 - \frac{q^2}{M_{pole}^2}} \sum_{k=0}^K a'_k [z(q^2) - z(0)]^k$$

where  $a'_k$  are real coefficients and  $z(q^2) \equiv z(q^2, t_0)$  is the function

$$z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

$$t_+ = (M_{B_c} + M_V)^2 \text{ and } t_0 = (M_{B_c} + M_V)(\sqrt{M_{B_c}^2} - \sqrt{M_V^2})^2.$$

## Semileptonic decays

The differential distribution of the  $B_c$  meson semileptonic decay to the final ground state vector meson  $V$  has the following expression:

$$\frac{d\Gamma(B_c \rightarrow V l \nu_l)}{dq^2} = \frac{G_F^2}{(2\pi)^3} |V_{q_1 q_2}|^2 \frac{q^2 \sqrt{\lambda}}{24 M_{B_c}^3} \left(1 - \frac{m_l^2}{q^2}\right)^2 H_{total}$$

$$\text{Total helicity structure: } H_{total} = (H_U + H_L) \left(1 + \frac{m_l^2}{2q^2}\right) + \frac{3m_l^2}{2q^2} H_S$$

$$\lambda = M_{B_c}^4 + M_V^4 + q^4 - 2(M_{B_c}^2 M_V^2 + M_V^2 q^2 + M_{B_c}^2 q^2)$$

$$H_U = |H_+|^2 + |H_-|^2; \quad H_L = |H_0|^2; \quad H_S = |H_t|^2$$

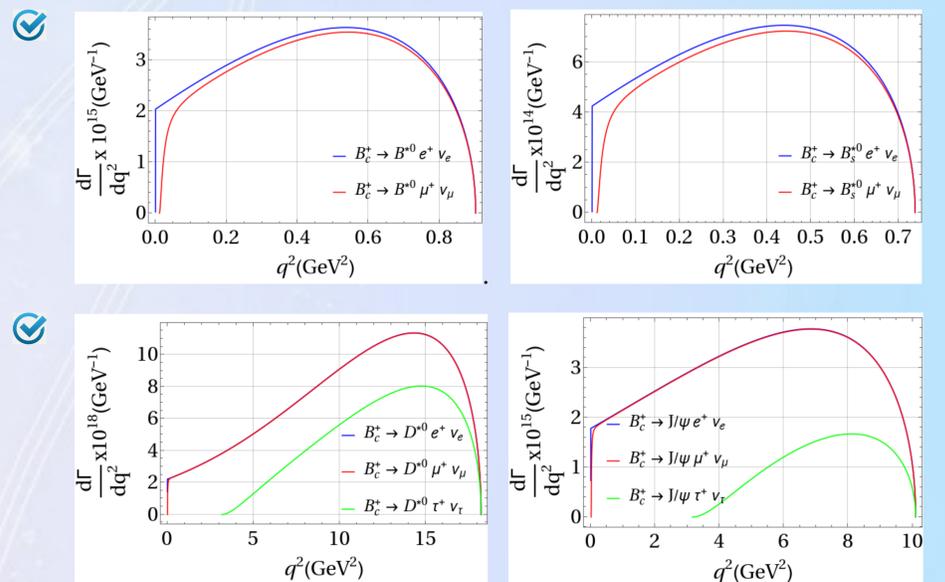
$$H_\pm(q^2) = (M_{B_c} + M_V) A_1(q^2) \mp \frac{\sqrt{\lambda}}{M_{B_c} + M_V} V(q^2)$$

$$H_0(q^2) = \frac{1}{2M_V \sqrt{q^2}} (M_{B_c} + M_V) (M_{B_c}^2 - M_V^2 - q^2) A_1(q^2) - \frac{\lambda}{M_{B_c} + M_V} A_2(q^2)$$

$$H_t(q^2) = \frac{\sqrt{\lambda}}{\sqrt{q^2}} A_0(q^2)$$



## Results



Numerical results for the Type-II scheme are larger by (50–60)% for bottom-conserving semileptonic decays, (57–78)% for  $B_c \rightarrow D^*$ , and around 20% for  $B_c \rightarrow J/\psi$  as compared to the branching ratios in the Type-I scheme.

Decay width ratios of bottom-conserving semileptonic decays involving pseudoscalar meson ( $B_s^0$  and  $B^0$ ) in final state for T2B match well with LQCD expectations.

	Ours	LQCD
$\frac{\Gamma(B_c^+ \rightarrow B_s^0 e^+ \nu_e)  V_{cd} ^2}{\Gamma(B_c^+ \rightarrow B^0 e^+ \nu_e)  V_{cs} ^2}$	$0.82^{+0.19+0.02}_{-0.20-0.00}$	$0.759 \pm 0.044$
$\frac{\Gamma(B_c^+ \rightarrow B_s^0 \mu^+ \nu_\mu)  V_{cd} ^2}{\Gamma(B_c^+ \rightarrow B^0 \mu^+ \nu_\mu)  V_{cs} ^2}$	$0.81^{+0.19+0.02}_{-0.20-0.00}$	$0.759 \pm 0.044$

Our **LFU ratio** involving  $b \rightarrow c \tau \nu_\tau$  for  $J/\psi$  in the final state match well with LQCD and other theoretical models; however, are smaller than the experimental measurement ( $\mathcal{R}_{J/\psi}^{LHCb} = 0.71 \pm 0.18 \pm 0.17$ ;  $\mathcal{R}_{J/\psi}^{CMS} = 0.49 \pm 0.26$ ).

	Ours	LQCD
$\mathcal{R}_{J/\psi} = \frac{\mathcal{B}(B_c^+ \rightarrow J/\psi \tau^+ \nu_\tau)}{\mathcal{B}(B_c^+ \rightarrow J/\psi e^+ \nu_e)}$	$0.25^{+0.05+0.02}_{-0.06-0.00}$	$0.2582 \pm 0.0038$

## References

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