

Exploring Black Hole Dynamics in massive scalar–Gauss–Bonnet Gravity through Numerical Relativity

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Scalar Gauss-Bonnet Gravity

Scalar-Gauss-Bonnet Gravity (sGB) is a widely studied and motivated extension of GR, as it arises naturally in the low-energy limit of string theory. Its action reads:

$$S = \int d^4x \sqrt{-g} \left[R - \frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi - V(\phi) + \lambda^2 f(\phi) G_{\text{GB}} \right], \quad (1)$$

where R is the Ricci scalar, ϕ a real scalar field, $G_{\text{GB}} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}$ the Gauss–Bonnet invariant, $f(\phi)$ the coupling function, λ the Gauss–Bonnet coupling constant, and $V(\phi) = \frac{1}{2}m_\phi^2\phi^2$ the massive scalar potential.

The corresponding **equations of motion** are:

$$\begin{aligned} G_{\mu\nu} + \Gamma_{\mu\nu}(\phi, f(\phi), R, \dots) &= T_{\mu\nu}^\phi(\phi, f(\phi)), \\ \square\phi &= V'(\phi) - \lambda^2 f'(\phi) G_{\text{GB}}, \end{aligned} \quad (2)$$

where $T_{\mu\nu}^\phi$ is the total stress–energy tensor, including the scalar field, while $\Gamma_{\mu\nu}$ encodes the additional geometric factors, as well as contributions induced by the scalar–Gauss–Bonnet coupling $f(\phi)$. The choice of the coupling function $f(\phi)$ gives different mechanisms of scalar hair formation:

- **Shift-symmetric** $f(\phi) \propto \phi$, **Dilatonic** $f(\phi) \propto e^\phi$: scalar hair are dynamically formed during evolution.
- **Quadratic** $f(\phi) \propto \phi^2$: possible spontaneous scalarization.

Figure 1 shows different snapshots of a scalarizing BH. The initial configuration assuming a single, static BH with null scalar field is evolved in shift-symmetric sGB theory.

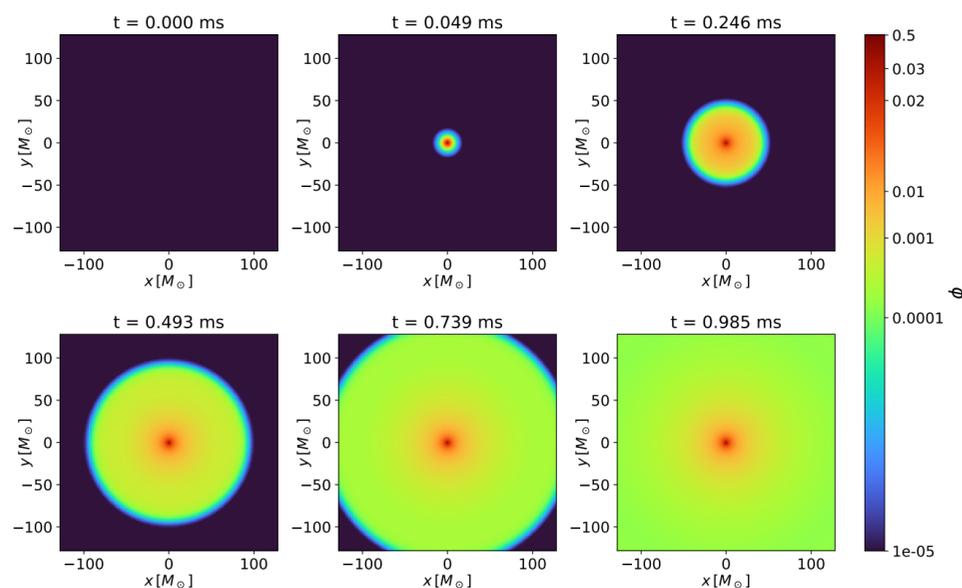


Figure 1. Snapshots at different times of a 2D slice of the scalar field developing around a static black hole in shift-symmetric sGB theory.

Numerical Setup

The equations of motion (Eq. 2) have to be solved using **numerical relativity** (NR) techniques.

- Standard GR evolutions employ the 3+1 ADM framework with **BSSN** or **CCZ4** formulations.
- In massive sGB, the scalar field requires modified schemes:
 1. **MGH** (Modified Generalized Harmonics) [1, 2],
 2. **mCCZ4** (modified CCZ4) [3], implemented in **GRFolres** [4] (built within the **GRChombo** [5] framework).

Quasinormal-modes in massive sGB

Perturbed black holes exhibit damped oscillations known as **quasinormal modes (QNMs)**. In massive sGB, late-time scalar radiation is expected to present long-lived **oscillatory tails**, with characteristic period [6]:

$$T \simeq \frac{2\pi}{m_\phi}. \quad (3)$$

To test this prediction, we evolve a static black hole with a small Gaussian scalar perturbation, ensuring linear dynamics while exciting the QNM spectrum (Fig. 2).

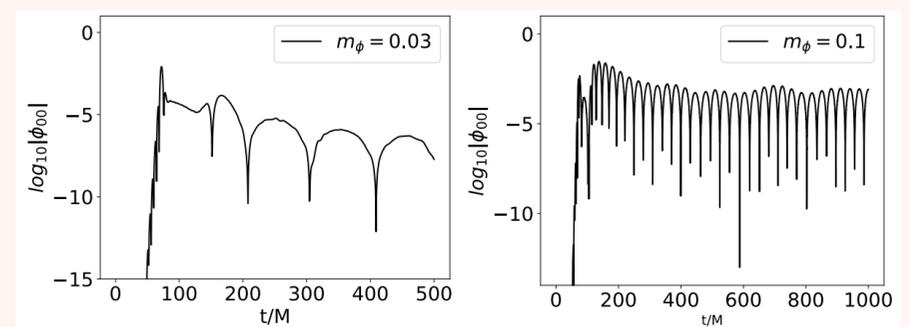


Figure 2. Time evolution of the scalar monopole mode ϕ_{00} extracted at $r_{\text{ext}} = 100 M_\odot$ for two values of the scalar field mass.

We perform a **Fast Fourier Transform (FFT)** of the signal and identify the dominant peak f_{peak} , with corresponding angular frequency $\omega_{\text{peak}} = 2\pi f_{\text{peak}}$, which for this kind of oscillatory signal yields: $\omega_{\text{peak}} \approx m_\phi$.

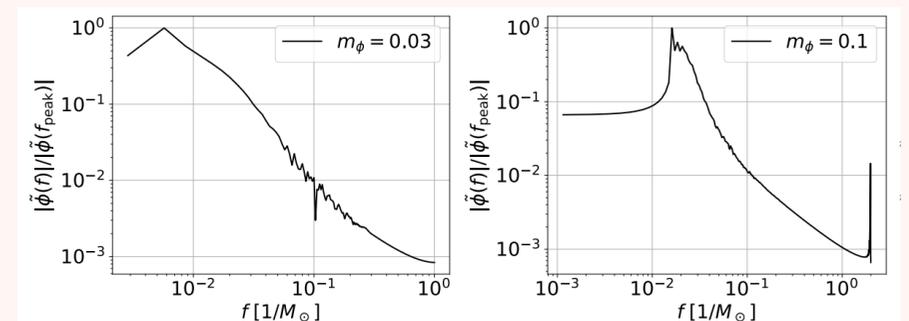


Figure 3. FFT spectra of the signal in Fig. 2 for two values of scalar field mass.

All spectra exhibit a clear dominant peak that shifts with the scalar mass, confirming that the oscillation frequency is directly set by m_ϕ . The extracted values are:

$$\begin{aligned} f_{\text{peak}} &= 5.723 \times 10^{-3} [1/M_\odot] \Rightarrow \omega_{\text{peak}} = 0.036 [1/M_\odot] \quad (m_\phi = 0.03), \\ f_{\text{peak}} &= 1.620 \times 10^{-2} [1/M_\odot] \Rightarrow \omega_{\text{peak}} = 0.101 [1/M_\odot] \quad (m_\phi = 0.1), \end{aligned}$$

References

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