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Modeling the QCD equation of state throughout the thermal deconfining phase from hadronic matter to a Quark-Gluon Plasma

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ABSTRACT

This work investigates the thermal deconfining phase transition from a hadronic gas (HG) composed of massive pions to a quark–gluon plasma (QGP) described by the Polyakov–Nambu–Jona-Lasinio (PNJL) model with two light quark flavors. The PNJL model extends the Nambu–Jona-Lasinio (NJL) approach, including chiral symmetry dynamics and the Polyakov loop, allowing a more realistic description of the QGP phase. A temperature-dependent switching function (SF) is used to ensure a smooth crossover between the hadronic and QGP phases. We analyze the temperature dependence of key thermodynamic quantities, including the pressure, energy density, entropy density, and the speed of sound squared, and study the influence of pion and quark masses on the equation of state (EOS). This analysis provides a better understanding of the interplay between particle properties and the thermodynamic properties of the system, helping to provide a more comprehensive and accurate picture of the thermal behavior of hadronic matter at zero chemical potential, including its response to changes in temperature and energy. This is essential for interpreting heavy-ion collision results and simulating early-universe conditions. Our analysis results are systematically compared with available lattice QCD data, providing valuable insights and allowing us to assess the accuracy of the model and its ability to describe strongly interacting matter at high temperature.

INTRODUCTION

According to lattice QCD investigations, the transition from hadronic matter to the QGP, in Quantum Chromodynamics (QCD) at high temperature and/or baryon density, has long been considered as an important challenge in particle and nuclear physics [1]. These simulations point to a smooth crossover at high temperature and low chemical potential [2], however their application to finite chemical potential is significantly hindered by the sign problem [3]. For this reason, effective theoretical frameworks, such as the Polyakov–Nambu–Jona-Lasinio (PNJL) model, are commonly employed to investigate the QCD phase diagram [4]. Moreover, the equation of state models the transition from hadronic matter to the QGP, by using a switching function to smoothly interpolate between the two phases in agreement with theoretical predictions and heavy-ion collision data [5].

In this work, we investigate the thermodynamic behavior of strongly interacting matter in a mixed HG-QGP system essentially by calculating key thermodynamic quantities, using infinite matter EOS, for a HG phase containing massive pions and a QGP phase described by the PNJL model with two flavors *(u, d)*. The EOS are constructed following a procedure similar to that in Ref. [6]. We examine their behavior with varying temperature, at zero chemical potential, with massless and massive particles in the two phases.

DESCRIPTION OF THE HADRONIC AND QGP PHASE

To carry out our study, we rely on the partition function (PF) of the HG phase. We consider an ideal gas of pions (π^+,π^-,π^0), which are the lightest mesons, with a mass: $m_\pi = 138MeV$ and the relevant PF is:

$$Z_{HG} = exp\left(\frac{d_\pi V_{HG}}{2\pi^2 T}\int_0^\infty \frac{k^4 dk}{\sqrt{k^2+m_\pi^2}\,(e^{\frac{1}{T}\sqrt{k^2+m_\pi^2}}-1)}\right) \tag{1}$$

where k is the momentum, V is the volume, and d_π the pion degeneracy factor.
The QGP phase is described within the two-flavor PNJL model. The corresponding thermodynamic potential is given by [4]:

$$\Omega = 2g_s\sum_{f=u,d}\sigma_f^2 - 6\sum_f\int_\lambda^\Lambda \frac{d^3p}{(2\pi)^3}E_f\Theta\left(\Lambda-|\vec{p}|\right) - 2\sum_f T\int_\lambda^\infty \frac{d^3p}{(2\pi)^3}\ln\left[1+3\left(\Phi+\bar{\Phi}e^{-(E_f-\mu_f)/T}\right)e^{-(E_f-\mu_f)/T}+e^{-3(E_f-\mu_f)/T}\right]$$
$$- 2\sum_f T\int_\lambda^\infty \frac{d^3p}{(2\pi)^3}\ln\left[1+3\left(\bar{\Phi}+\Phi e^{-(E_f+\mu_f)/T}\right)e^{-(E_f+\mu_f)/T}+e^{-3(E_f+\mu_f)/T}\right]+u\left(\Phi,\bar{\Phi},T\right) \tag{2}$$

For a given flavor (f=u,d), the corresponding quasiparticle energy is: $E_f = \sqrt{p^2+M_f^2}$. The constituent quark masses M_f are expressed as,

$$M_f = m_f - 2g_s\sigma_f \tag{3}$$

The Polyakov-loop effective potential is

$$\frac{u\left(\Phi,\bar{\Phi},T\right)}{T^4} = -\frac{b_2}{2}\Phi\bar{\Phi} - \frac{b_3}{6}\left(\Phi^3+\bar{\Phi}^3\right)+\frac{b_4}{4}\left(\Phi\bar{\Phi}\right)^2 - K\ln(J) \tag{4}$$

with

$$b_2 = a_0 + a_1\left(T_0/T\right)+a_2\left(T_0/T\right)^2+a_3\left(T_0/T\right)^3 \tag{5}$$

$$J = \left(9/6\pi^2\right)\left[1-6\Phi\bar{\Phi}+4\left(\Phi^3+\bar{\Phi}^3\right)+3\left(\Phi\bar{\Phi}\right)^2\right] \tag{6}$$

b_3 and b_4 are constants, as well as a_0, a_1, a_2, and a_4. K is a dimensionless parameter. In this work, we adopt the value $K = 0.02$.The parameter values used for the PNJL model are [4] : $m_{ud} = 5.5MeV, \Lambda = 651MeV,$

$g_s\Lambda^2 = 4.27, a_0 = 6.75, a_1 = -1.95, a_2 = 2.625, a_3 = -7.44, b_3 = 0.75, b_4 = 7.5, T_0 = 190MeV.$

SWITCHING FROM HADRONIC GAS TO QGP

In this section we start by constructing a pressure P which includes a HG piece, a QGP piece, by means of a switching function *S [5, 6]*:

$$P(T,\mu) = S(T,\mu)P_{QGP}(T,\mu)+[1-S(T,\mu)]P_{HG}(T,\mu) \tag{7}$$

Thus, to calculate the pressure of the HG phase, we use the thermodynamic relation [7]:

$$P_{HG} = T d\ln Z_{HG}/dV_{HG} \tag{8}$$

For the QGP phase, we use [8]:

$$P_{QGP} = P_{pnjl} = -\left(\Omega(T)-\Omega(T=0)\right) \tag{9}$$

and the switching function is given as [5]:

$$S = \frac{1}{2}+\frac{2}{\pi}\arctan\left(\frac{\eta_2}{\eta_1+\sqrt{\eta_1^2+\eta_2^2}}\right) \tag{10}$$

with

$$\eta_1 \equiv \frac{1}{2}\left[1+\tanh\left(\frac{a\left(b-\left|\frac{\psi}{\psi_c}\right|\right)}{\left|\frac{\psi}{\psi_c}\right|\left(1-\left|\frac{\psi}{\psi_c}\right|\right)}\right)\right] \quad \eta_2 \equiv \tan\left[\frac{\pi}{2^\theta}-\frac{\pi}{2}\right] \quad \theta = \left(\left(T^2+\mu^2\right)\left[\left(\cos\psi/\mu_0\right)^2+\left(\sin\psi/\mu_0\right)^2\right]\right)^{-r/2} \tag{11}$$

Here, $\psi = \arctan\left(\frac{\mu}{T}\right)$ and $\psi_c = \arctan\left(\frac{\mu_c}{T_c}\right)$.

The model parameters chosen for S are: $\mu_0 = 1250\,MeV, T_0 = 190\,MeV, T_c = 190\,MeV, r = 4, a = 1, b = 0.8.$

The quantities of interest in our work are the energy and entropy densities, expressed respectively as [7]

$$\varepsilon = T\partial P/\partial T - P \tag{14}$$

$$s = \partial P/\partial T \tag{15}$$

Another important physical quantity is the squared speed of sound, which is defined as [7]

$$c_s^2 = \partial P/\partial\varepsilon \tag{16}$$

RESULTS & DISCUSSION

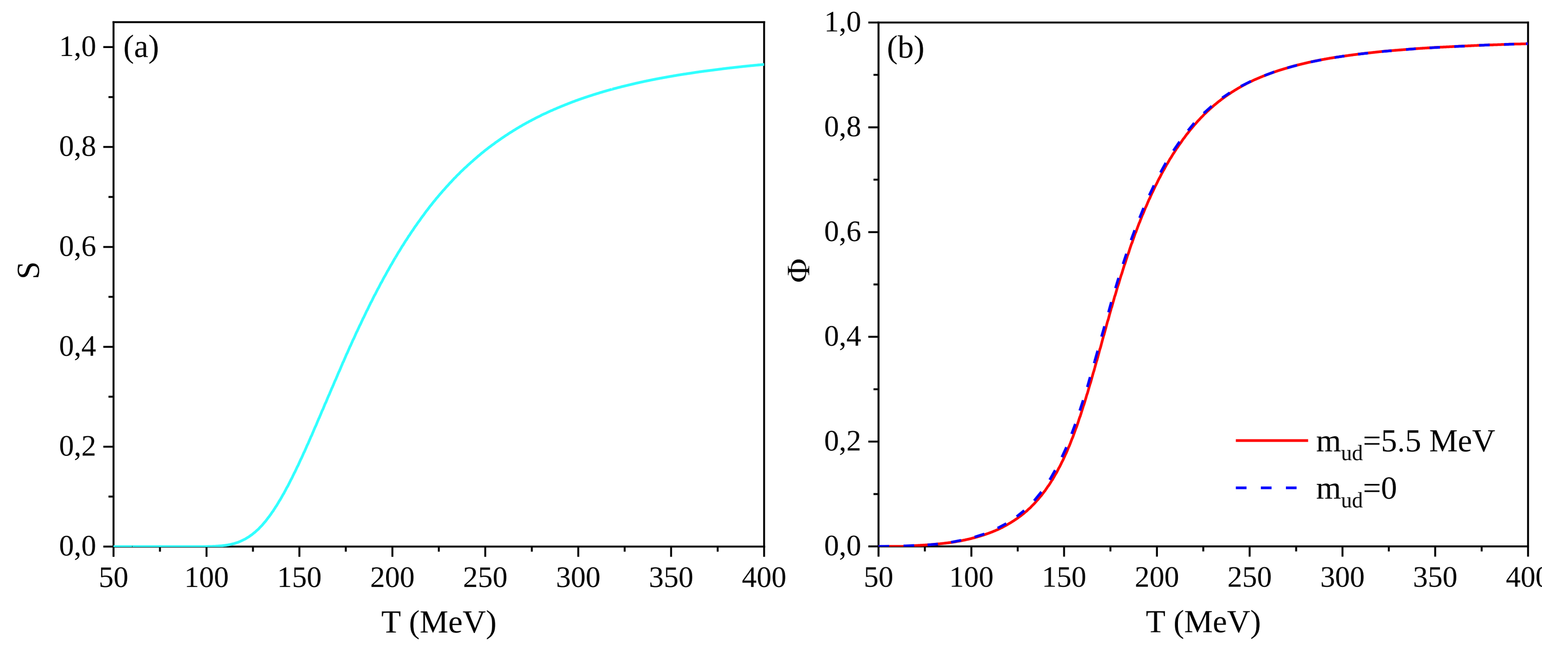

Fig. 1: (a) switching function S and (b) polyakov loop Φ as functions of temperature, at vanishing chemical potential *(μ=0)*.

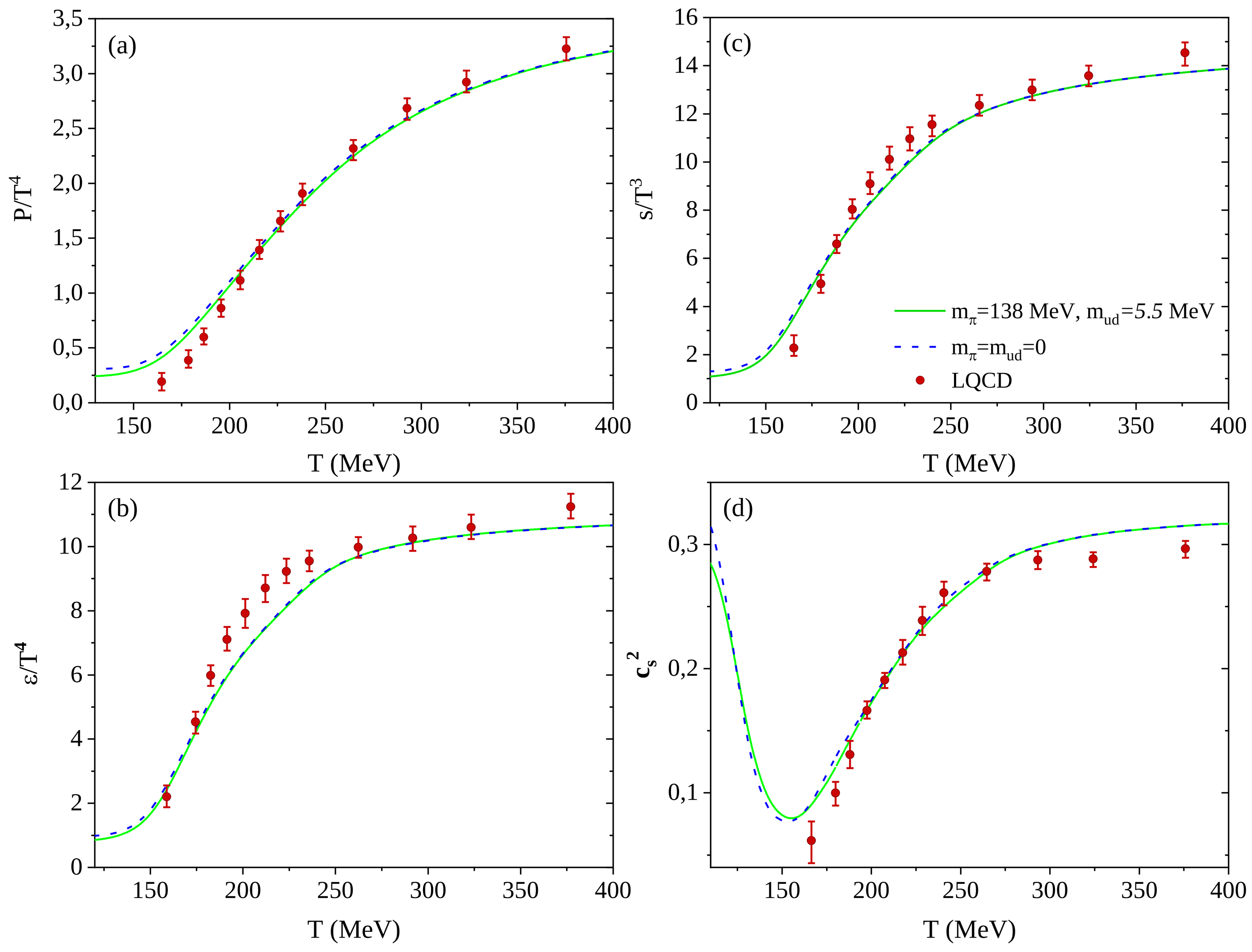

Fig. 2: Plots of the variations of (a) the pressure P normalized by T^4, (b) the energy density ε normalized by T^4, (c) the entropy density s normalized by T^3, and (d) the sound velocity squared, c_s^2 with temperature, at μ=0, in the two cases of massive particles (solid line) and massless particles (dashed line). The points are from lattice QCD (LQCD) data of Ref. [9].

Figure 1 illustrates the variations of both switching function S (panel (a)) and the polyakov loop Φ (panel (b)) with temperature T, at μ=0. It can clearly be noted that the effect of the quark mass is negligible in this range, since the difference between the two curves is almost insignificant. On the other hand, the switching function increases smoothly from 0 to 1, indicating a continuous crossover from the hadronic to the QGP phase. Figure 2 shows the variations of (a) the pressure normalized by T^4,(b) the energy density normalized by T^4,(c) the entropy density normalized by T^3, and (d) the sound velocity squared c_s^2, as a function of temperature T, at zero chemical potential μ =0, for both massive and massless cases. The results indicate that introducing a finite pion mass improves consistency with the lattice QCD data [9], and slightly slows down the transition speed compared to the massless case, while the effect of light quark masses remains weak and does not significantly alter the temperature dependence of the thermodynamic quantities.

CONCLUSION

In the present work, we found that the behavior of the thermodynamic quantities characterizing the deconfinement phase transition in an infinite volume in the presence of massive particles is similar to that obtained for massless particles, with only minor quantitative deviations in the crossover region, while the overall qualitative features and the smooth nature of the transition remain unchanged.

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