

Falling test electric dipole in the Schwarzschild geometry

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INTRODUCTION & AIM

- **Matter and antimatter** were produced symmetrically at the Big Bang.
- However, the observable **universe is dominated by matter**.
- Recent experiments at CERN indicate that both **matter and antimatter fall downward** in a gravitational field.

This raises the question of whether electric charge can influence gravitational motion.

➤ We investigate this problem using a **test electric dipole in Schwarzschild geometry**.

Why dipole?

Dipole and tidal forces are of the same order, r^{-3} , and the simplest antimatter can be visualized as an H-atom and an anti-H-atom, i.e., a dipole-anti-dipole system.

METHOD

Step 1. Starting with the Schwarzschild black hole line element

$$ds^2 = \left(1 - \frac{2m}{r}\right) dt^2 - \frac{dr^2}{\left(1 - \frac{2m}{r}\right)} - r^2 d\theta^2 - r^2 \sin^2 \theta d\varphi^2$$

Step 2. Using the Lagrangian method to derive geodesic equations

$$L = \frac{1}{2} g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu + A_\mu \dot{x}^\mu$$

Step 3. Calculating the acceleration of a massive particle.

Step 4. Plotting acceleration vs distance for different cases.

Step 5. Using the conversion factor between geometrized units ($G = c = 1$) and standard (SI) units, to obtain the acceleration of the particle anti-particle in ordinary units.

Test potential vector of a point charge q	Test electric field of a dipole with dipole moment P
$A_t = \frac{q}{r}$	$A_t = \frac{P \cos \theta}{r^2}$

General geodesic equations for a test electric dipole

$$\left(1 - \frac{2m}{r}\right) \dot{t} + \frac{P \cos \theta}{r^2} = E$$

$$-r^2 \sin \theta \cos \theta \dot{\varphi}^2 - \frac{P \sin \theta}{r^2} \dot{t} + \frac{d}{d\tau} (r^2 \dot{\theta}) = 0$$

$$r^2 \sin^2 \theta \dot{\varphi} = l$$

$$\left(1 - \frac{2m}{r}\right) \dot{t}^2 - \frac{\dot{r}^2}{1 - \frac{2m}{r}} - r^2 \dot{\theta}^2 - r^2 \sin^2 \theta \dot{\varphi}^2 = 1$$

Dot: proper time derivative.

m: central mass.

P: dipole moment.

E & l: energy and angular momentum constants.

- ❖ When the angle $\theta = 0, \pi$, meaning the dipole anti-dipole is along $\pm z$ axis.

RESULTS & DISCUSSION

Test Point Charge acceleration:

❖ in geometrized units:

$$\frac{d^2 r}{dt^2} = \frac{q - m}{r^2} + \frac{8m^2 - 12mq + 3q^2}{r^3} + O\left(\frac{1}{r^4}\right)$$

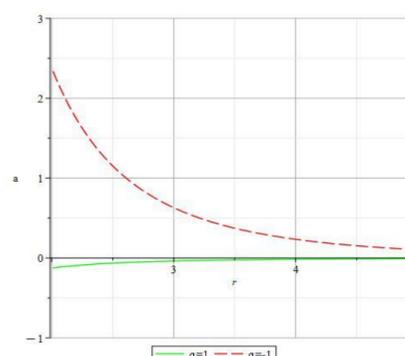


FIG 1a. Point charge acceleration vs r for m=1

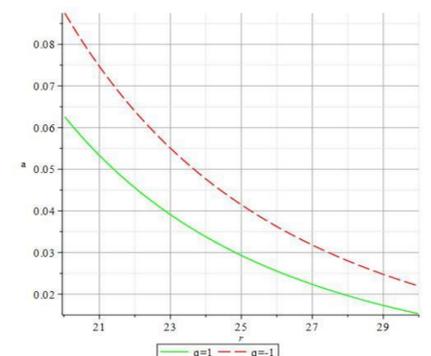


FIG 1b. Point charge acceleration vs r for m=10

Test Dipole acceleration:

❖ in geometrized units:

$$\frac{d^2 r}{dt^2} = -\frac{m}{r^2} + \frac{8m^2}{r^3} + \frac{2P}{r^3} - \frac{18mP}{r^4} + O\left(\frac{1}{r^5}\right)$$

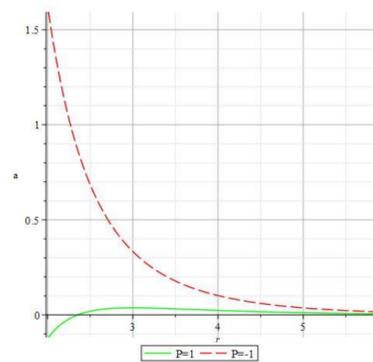


FIG 2a. Test electric dipole acceleration vs r for m=1

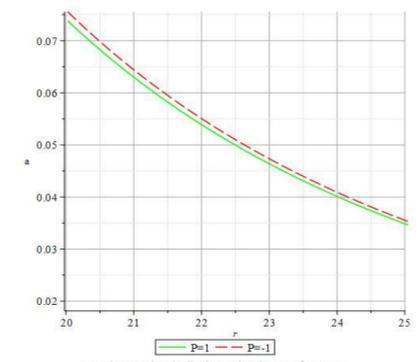


FIG 2b. Test electric dipole acceleration vs r for m=10

CONCLUSION

- **Dipole acceleration difference is negligible** compared to $g = -9.81 \frac{m}{s^2}$.
- **Amplifying the dipole effect is unrealistic** for an Earth-like planet, $m_o r_o \ll 1$.
- **Artificial satellite experiments** may produce a **detectable difference**.
- **Very small satellite mass** (vs. Earth) is **crucial for detectability**.
- **Successful measurements could reveal antimatter anti-acceleration**, helping explain the **absence of antimatter** in the observable universe.
- For **large r**, acceleration goes to zero however, for **average r**, difference is seen.

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