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Universal Wavefunction and Entropy in Minisuperspace Quantum Gravity

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INTRODUCTION & AIM

Although semiclassically time can be recovered via the WKB approximation, the problem of time in quantum gravity remains open. One promising approach is to use entropic quantities as an internal arrow of time, defined through correlations between geometry and matter degrees of freedom[1,2].

The aim of this work is to numerically solve the Wheeler–DeWitt (WdW) equation in a minisuperspace model and analyze matter–geometry entanglement in the resulting universal wavefunction. We employ a spectral Galerkin method and obtain results that suggest that geometry-matter correlations contribute nontrivially to reduced density matrix and entanglement entropy

METHODS

We consider a closed Friedmann–Lemaître–Robertson–Walker universe with scale factor a, minimally coupled to a homogeneous massive scalar field χ, in presence of a positive cosmological constant Λ and with interaction constant λ. In units $\frac{2G}{3\pi}=1$, $\hbar$=1, and choosing operator ordering p=1, the Wheeler–DeWitt equation takes the form[3]:

$$\left(\frac{1}{a}\frac{\partial^2}{\partial a^2}+\frac{1}{a^2}\frac{\partial}{\partial a}-\frac{1}{a^3}\frac{\partial^2}{\partial \chi^2}-ka+\frac{\Lambda a^3}{3}+\frac{1}{2}\lambda a^3\chi^2\right)\psi(a,\chi)=0$$

Direct finite-difference methods are numerically unstable due to the rapidly growing potential term. This difficulty can be addressed using a spectral Galerkin method, in which the wavefunction is expanded in an a-dependent harmonic oscillator basis:

$$\psi(a,\chi)=\sum_{m}^{N\to\infty}\sigma_m(a)\varphi_m(\chi;a)$$

After substitution to WdW equation χ-sector reduces to a harmonic oscillator with a-dependent frequency $\omega(a)=\sqrt{2\lambda}a^2$, and mass $m_0=\frac{1}{2}$

$$\left(\frac{\partial^2}{\partial\chi^2}-\lambda a^6\chi^2\right)\varphi_n(\chi;a)=E_n(a)\varphi_n(\chi;a)$$

Here $E(a)=\sqrt{2\lambda}a^3\left(m+\frac{1}{2}\right)$ and solutions given by Hermite-like functions

$$\varphi_m(\chi;a)=\frac{a^{\frac{3}{4}}}{\sqrt{2^m m!}}\sqrt[8]{\frac{\lambda}{2\pi^2}}\,e^{-\frac{1}{8}\sqrt{2\lambda}a^3\chi^2}H_m\left(a^{\frac{3}{2}}\sqrt[4]{\frac{\lambda}{2}}\chi\right)$$

After substituting this to the WdW equation and simplifying derivatives with recursion relations one may integrate full equation and use the fact that hermite functions are orthogonal to remove the sum. Multiplication by φ_n and integration along χ gives final equation for $\sigma(a)$

$$\frac{\partial^2\sigma_m(a)}{\partial a^2}+\frac{3}{4a^3}\left[\frac{\partial\sigma_{n-2}(a)}{\partial a}\sqrt{n^2-n}-\frac{\partial\sigma_{n+2}(a)}{\partial a}\sqrt{(n+2)(n+1)}\right]$$
$$=\frac{9}{64a^2}\left[\sqrt{(n-3)(n-2)(n-1)n}\,\sigma_{n-4}(a)-[(n+2)(n+1)+(n^2-n)]\sigma_n(a)\right.$$
$$\left.+\sqrt{(n+4)(n+3)(n+2)(n+1)}\,\sigma_{n+4}(a)\right]-\left(\frac{E_n(a)}{a^2}-ka^2+\frac{\Lambda a^4}{3}-\frac{1}{a}\frac{\partial}{\partial a}\right)\sigma_n(a)$$

This equation may be solved using standard numerical methods such as RK4, or by spectral methods.

RESULTS & DISCUSSION

Applying the RK4 algorithm to the coupled σ-equations yields solutions dominated by the ground mode n=0. Plots fig 1. and fig 2. contain normalized function $\sigma_n(a)$ for boundary conditions $\sigma_{n=0,n=1}(a_{min}=0.3)=1/N$, $\sigma_{n>1}(a_{min})=0$ and cosmological constant $\Lambda=0.1$ and interaction constant $\lambda=1$. Fig.3 represents full wavefunction

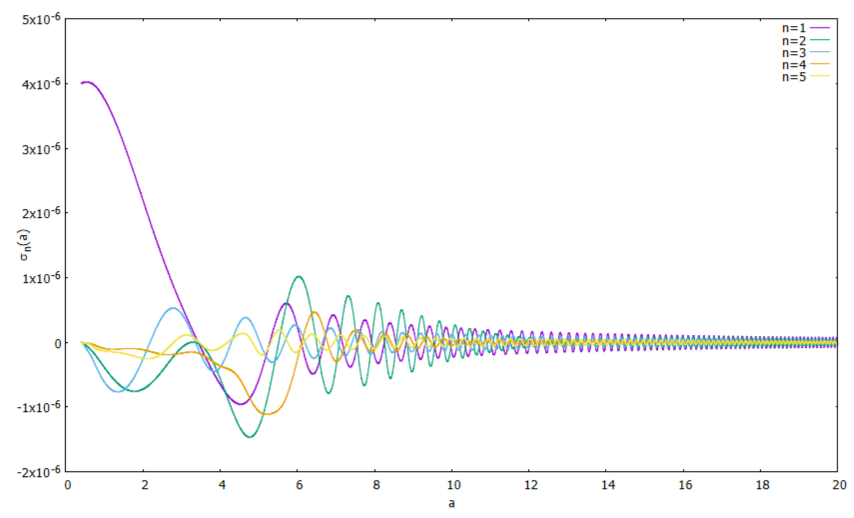

Figure 1. $\sigma_n(a)$ for n=1,..5

In chosen range of a solution satisfies $\sigma_{n=0}(a)\gg\sigma_{n\neq0}(a)$. Higher modes start with slight oscillation but they remain supressed.

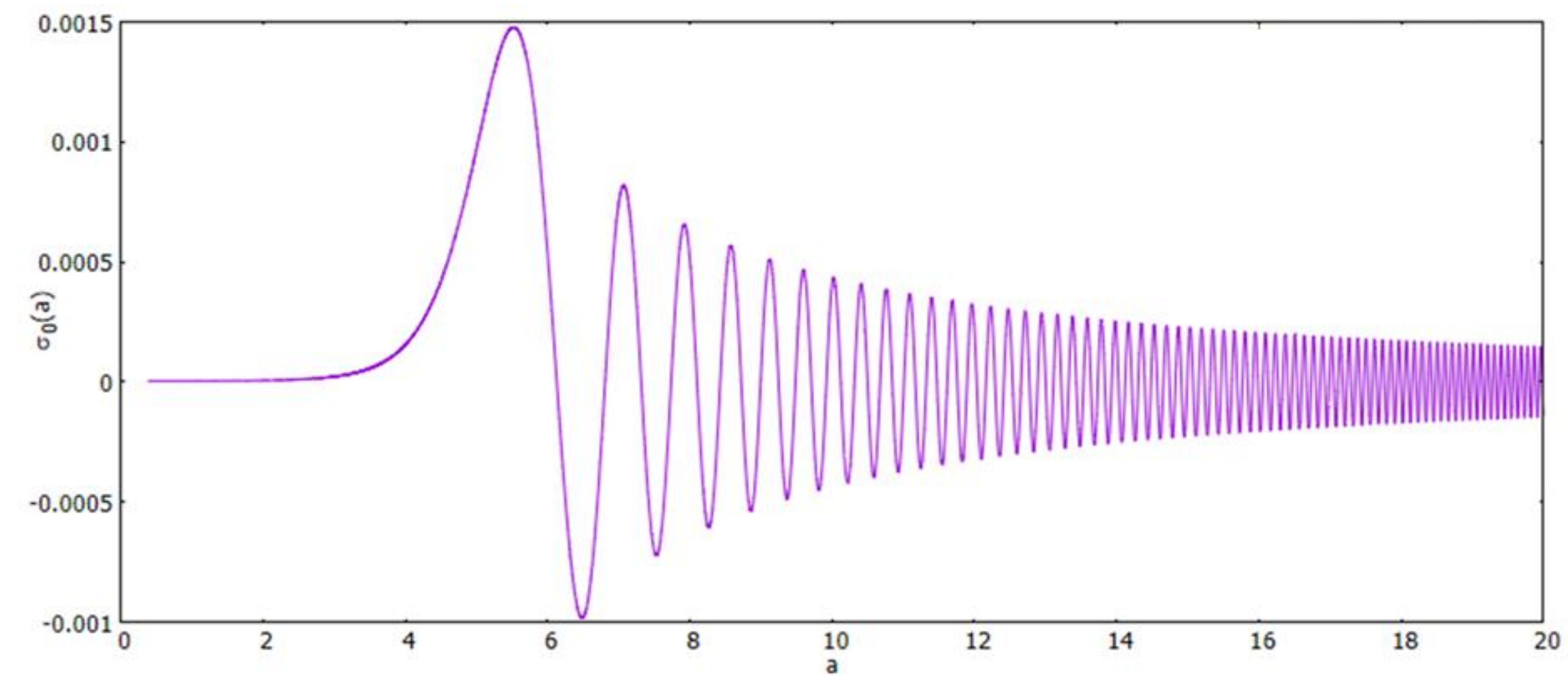

Figure 2. Ground mode $\sigma_0(a)$

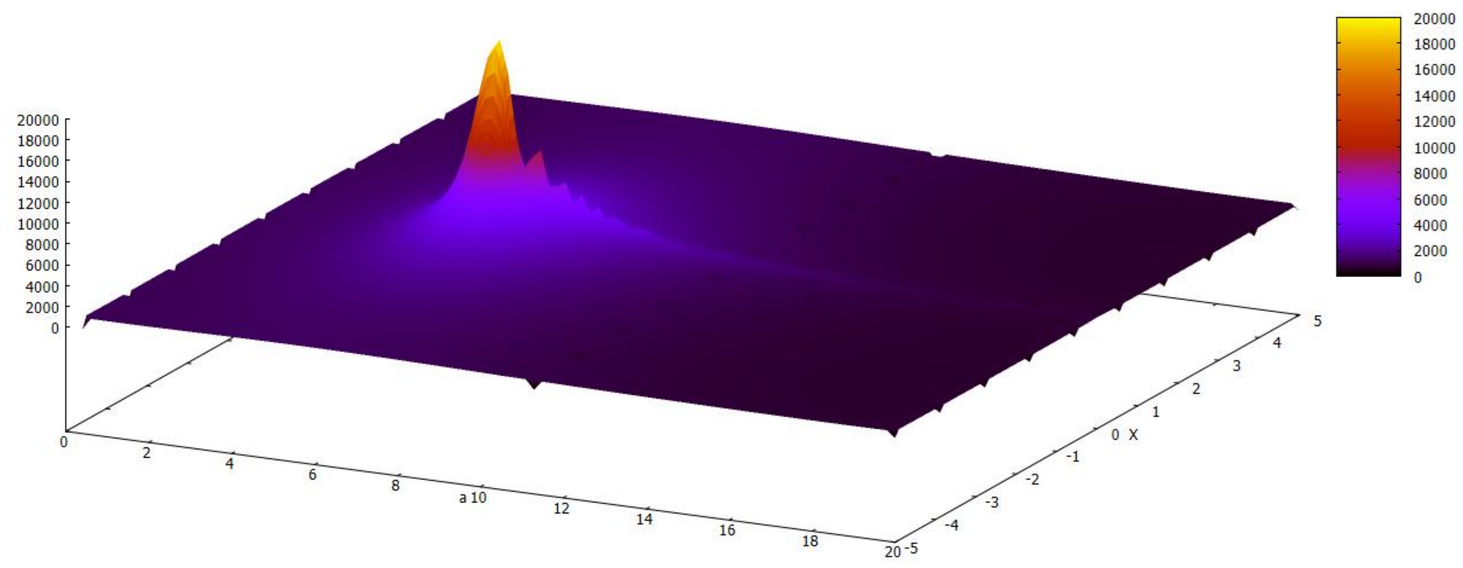

Figure 3. non-normalized $\psi\psi^*$ *for the same boundaries and constants as fig 1. and fig 2.*

To quantify matter–geometry entanglement, we may compute the reduced density matrix obtained by tracing over the scalar field:

$$\rho_{red}(a,a')=\sum_{n,m}^{N\to\infty}\sigma_m(a)\sigma_n^*(a')I_{n,m}(a,a')$$

$$I_{n,m}(a,a')=\int_{-\infty}^{\infty}\varphi_m(\chi;a)\varphi_n^*(\chi;a')\,d\chi$$

Because the harmonic oscillator basis depends on a, the overlap matrix $I_{n,m}(a,a')$ has non-zero $n\neq m$ elements and brings additional contribution to entanglement entropy.

$$S(\rho_{red})=-Tr(\rho_{red}\ln\rho_{red})$$

As a result $S(\rho_{red})$ contains 2 sources of entropy:

1. Excitations of modes $\sigma_{n>0}(a)$
2. Nondiagonal overlap terms proportional to $\int_{-\infty}^{\infty}\varphi_m(\chi;a)\varphi_n^*(\chi;a')\,d\chi$ which are a result of matter-geometry entanglement

CONCLUSION

We presented a spectral Galerkin method for solving the Wheeler–DeWitt equation in minisuperspace. The resulting universal wavefunction allows for a consistent analysis of geometry–matter entanglement.

The a-dependence of the matter basis induces nontrivial overlap terms in the reduced density matrix, reflecting squeezing-like correlations between geometry and scalar field degrees of freedom.

Further work will focus on a detailed analysis of the entanglement spectrum and its possible relation to emergent time and semiclassical dynamics.

REFERENCES

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