

## Impact of Equation of State Parameters on the Complexity of Anisotropic Stellar Models

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### INTRODUCTION & AIM

Compact stars are highly dense stellar remnants whose matter density far exceeds that of ordinary stars, leading to extremely strong gravitational fields. This class of objects includes neutron stars, pulsars, and black holes. To investigate the internal structure and physical behavior of relativistic compact stars, an appropriate Equation of State (EOS) is assumed to describe the relationship between pressure and density of stellar matter. The equilibrium configuration of such stars can be obtained by solving the Tolman–Oppenheimer–Volkoff (TOV) equations for a given EOS. Alternatively, one may solve Einstein's field equations using various constructive approaches, such as assuming conformal motions, embedding class I conditions, conformally flat spacetimes, or specific ansatz for the metric potentials.

In recent years, the concept of complexity has emerged as an important tool in modeling superdense compact stars. Herrera[1] redefined complexity in the context of realistic relativistic stellar configurations and extended the idea to dynamical gravitating systems. A configuration is said to have zero complexity when pressure anisotropy and density inhomogeneity either vanish identically or balance each other exactly. Further studies have shown that complexity may depend on deviations from spherical symmetry[2]. On the other hand, it has also been demonstrated that, under Vaidya–Tikekar (VT) geometry, superdense stellar models satisfying the complexity-free condition can be constructed[3].

A significant method for obtaining exact solutions for compact stars was introduced by Vaidya and Tikekar[4], who proposed a specific form of the metric potential characterized by curvature parameters. An important feature of realistic compact star models is anisotropy, defined as the difference between radial and tangential pressures inside the star. Anisotropy plays a crucial role in determining stability and structural properties of compact objects[5]. The influence of anisotropy and methods for generating static anisotropic solutions have been extensively studied in the literature[6]. Earlier, under the Finch–Skea geometry [7], the influence of model parameters on the complexity factor was analyzed for four different equations of state. In the present study, we extend this analysis by considering three distinct equations of state [8,9,10] within the Vaidya–Tikekar framework. Our objective is to examine how the EOS parameters influence the complexity factor, pressure anisotropy, and density inhomogeneity in superdense compact star models.

### METHOD

Assuming spherical symmetry for the relativistic superdense star, we consider the metric of the following form to describe the geometry inside star

$$ds^2 = -e^{\nu(r)} dt^2 + e^{\lambda(r)} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

Assuming the matter distribution of the stellar interior is anisotropic in nature, the energy-momentum tensor is described by that of a perfect fluid, in the form

$$T_{ij} = (\rho + p)u_i u_j + p_t g_{ij} + (p_r - p_t)\chi_i \chi_j$$

where the matter variables  $\rho$  is the energy-density,  $p_r$  and  $p_t$ , respectively are radial and transverse fluid pressures.  $u_i$  is the 4-velocity of the fluid and  $\chi_i$  is a unit space-like 4-vector along the radial direction.

To obtain a solution for relativistic star we are using Vaidya Tikekar geometry, the metric is given by

$$e^{\lambda(r)} = \frac{1 - \frac{Kr^2}{L^2}}{1 - \frac{r^2}{L^2}} \quad \text{where, } K = \text{Departure from spherical curvature} \\ L = \text{dimension of length}$$

Now, Einstein's Field equations are as follows

$$\rho = \frac{1}{r^2}(1 - e^{-\lambda}) + \frac{\lambda'}{r} e^{-\lambda} \\ p_r = -\frac{1}{r^2}(1 - e^{-\lambda}) + \frac{\nu'}{r} e^{-\lambda} \\ p_t = \frac{e^{-\lambda}}{4} \left( 2\nu'' + \nu'^2 - \nu'\lambda' + \frac{2\nu'}{r} - \frac{2\lambda'}{r} \right)$$

The complexity of an anisotropic stellar system can be expressed as

$$YTF = 8\pi\Pi - \frac{4\pi}{r^3} \int_0^r r^3 \rho dr$$

Here Anisotropy is expressed as  $\Pi = p_r - p_t$

Density inhomogeneity can be calculated from the difference of YTF and anisotropy.

In terms of metric potentials YTF can also be written as,

$$YTF = \frac{e^{-\lambda}}{4r} [\nu\{r(\lambda - \nu) + 2\} - 2r\nu']$$

The interior of the star is assumed to consist of an anisotropic matter distribution. In addition to the physical mechanisms that generate anisotropy within the stellar fluid, density inhomogeneity is also present. When these two contributions do not balance each other, the system exhibits non-zero complexity.

To investigate the influence of Equation of State (EOS) parameters on the complexity of anisotropic compact stars, we analyze three different stellar models. All models are constructed within the common framework of the Vaidya–Tikekar (VT) interior spacetime geometry but differ in the choice of EOS.

**Case I: Linear Equation of State (Sharma et al., Annals of Physics, 414, 168079 (2020)).** Using the Vaidya–Tikekar metric ansatz, a compact star model was developed assuming a linear equation of state of the form:  $p_r = \alpha\rho - \beta$ , where  $\alpha$  and  $\beta$  are equation of state parameters. This model provides a simple yet physically significant description of dense stellar matter.

**Case II: Colour–Flavour–Locked (CFL) Equation of State (Das et al., Africa Mathematica, 37, 12 (2026)).** In this model, a compact stellar configuration was constructed under VT geometry using the Colour–Flavour–Locked (CFL) equation of state, given by:  $p_r = \alpha\rho + \beta\rho^{\frac{1}{2}} - \gamma$ , where  $\alpha, \beta$  and  $\gamma$  are equation of state parameters. This form is motivated by dense quark matter considerations.

**Case III: Polyotropic Equation of State (Baskey et al., New Astronomy, 108, 102164 (2024)).** Exact solutions were obtained in the VT background spacetime by assuming a polytropic equation of state:  $p_r = \alpha\rho^{1+\frac{1}{n}} + \beta$  where  $\alpha$  and  $\beta$  are the real constants and 'n' is the polytropic index. In the present study, we restrict ourselves to the case n=1.

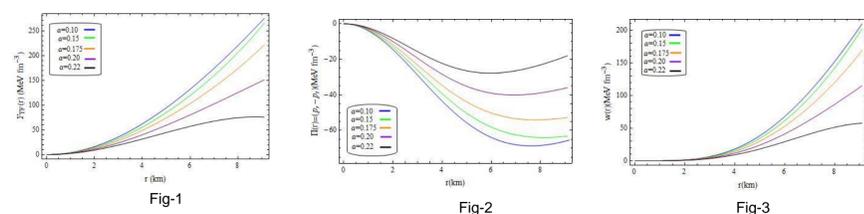
### RESULTS & DISCUSSION

In this work, we examine the dependence of the complexity factor, pressure anisotropy, and density inhomogeneity for three distinct cases corresponding to different equations of state within the Vaidya–Tikekar geometry. To analyze the influence of EOS parameters on the complexity of anisotropic stellar models, we consider a specific compact star configuration with mass  $M = 1.58 \times 1.475 \text{ km}$ , and radius  $R = 9.1 \text{ km}$ . For a consistent comparison among the three models based on different EOS forms, we fix the curvature parameter at  $k = -10$  for all cases.

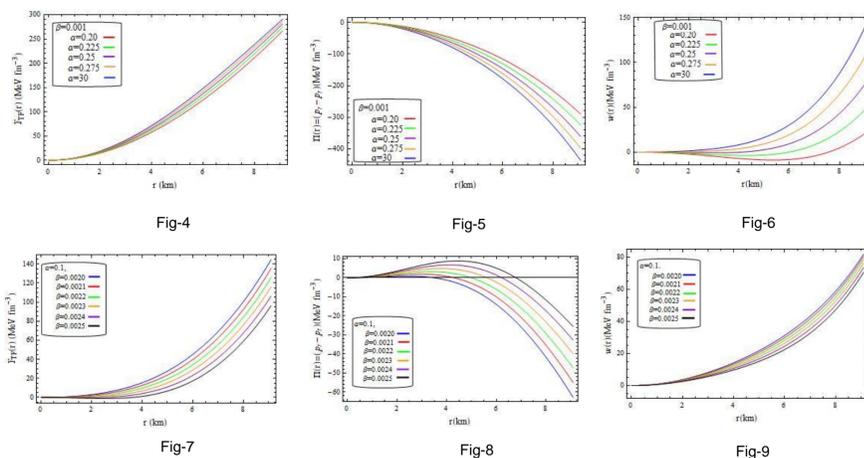
**CASE-I:** In Fig. 1, the complexity factor is plotted against the radial coordinate for different values of the model parameter ( $\alpha$ ). The graph indicates that the complexity factor decreases as the value of the model parameter increases.

In Fig. 2, pressure anisotropy is shown for various values of the model parameter. The results reveal that anisotropy increases with increasing values of the parameter.

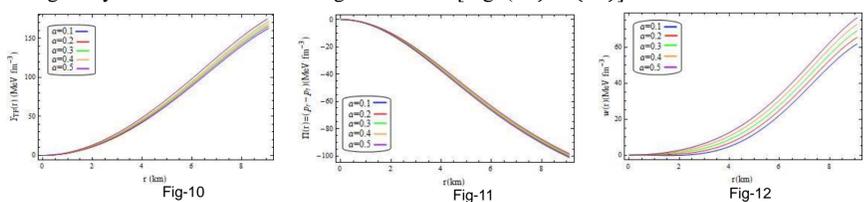
In Fig. 3, density inhomogeneity is plotted as a function of the radial coordinate for different values of the model parameter. It is observed that density inhomogeneity decreases as the model parameter increases.



**CASE-II:** The Colour-flavoured-locked equation of state involves three different model parameters as  $\alpha, \beta$  and  $\gamma$ . First, by fixing the parameter  $\beta$  and varying  $\alpha$ , we observe that both the complexity factor and density inhomogeneity increase with increasing  $\alpha$ , while the pressure anisotropy decreases as  $\alpha$  increases [Fig. (4)–(6)]. Next, by keeping  $\alpha$  fixed and varying  $\beta$ , we find that both the complexity factor and density inhomogeneity decrease with increasing values of  $\beta$  [Fig. (7) – (9)].



**CASE-III:** In the Polytropic equation of state Under the Vaidya–Tikekar framework with a polytropic equation of state, the plot of the complexity factor versus the radial coordinate shows that complexity increases with increasing values of the model parameter  $\alpha$ . Furthermore, the results indicate that pressure anisotropy decreases as the model parameter increases, whereas density inhomogeneity increases with increasing values of  $\alpha$  [Fig. (10) – (12)].



### CONCLUSION

Under the Vaidya–Tikekar background geometry, the behavior of complexity varies significantly with the choice of equation of state. For the **linear equation of state**, both the complexity factor and density inhomogeneity decrease as the model parameter increases. In the case of the **CFL equation of state**, the variation depends on the parameter considered. When the first EOS parameter is increased, the complexity factor and density inhomogeneity increase, while pressure anisotropy decreases. However, for the second EOS parameter, both complexity and density inhomogeneity decrease, whereas anisotropy increases. For the **polytropic equation of state**, the results indicate that both the complexity factor and density inhomogeneity increase with increasing values of the model parameter.

### FUTURE WORK / REFERENCES

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