

Spacetime Quasi-normal Mode Oscillations of Anisotropic Neutron Stars

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INTRODUCTION & AIM

- Neutron star (NS) asteroseismology (the study of neutron star perturbation) offers an opportunity to probe nuclear physics through stellar oscillations.
- Pressure anisotropy can arise from various physical mechanisms, including elasticity, viscosity, and magnetic fields.
- Previous work on fluid modes (f-modes and p-modes, etc.) has shown equation of state (EOS) independent frequency relation and instability for special configuration.
- We compute, for the first time, the spacetime oscillation modes (so-called w-modes) of anisotropic neutron stars.
- W-modes originate from the metric perturbations, so they cannot be computed under the Cowling approximation.
- Previous work on isotropic NS w-modes has found EOS insensitive frequency-compactness relation.
- In this work, we study how anisotropy influence w-mode frequency compared to isotropic stars, search for unstable w-modes, and search EOS-independent universal relations.

METHOD

○ Anisotropy model

The anisotropy model we use is the modified Horvat model that describes the radial and tangential pressure difference:

$$\sigma = p_r - p_t = \beta \mu^2 p_r$$

Where $\mu = 2m/r$, and m is the enclosed mass in r and β is the dimensionless anisotropy parameter.

○ Einstein Equation

Anisotropy enters the stress-energy tensor by

$$T_{\mu\nu} = \rho u_\mu u_\nu + p_r (g_{\mu\nu} + u_\mu u_\nu) + \sigma \Omega_{\mu\nu}, \quad \Omega_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu - k_\mu k_\nu$$

Where \vec{k} is the unit radial vector perpendicular to \vec{u} , the four velocity.

○ Perturbation Framework

Consider the metric perturbation and fluid displacement vector in form

$$\delta g_{\mu\nu} = \begin{pmatrix} e^\nu(r)H_0(r) & i\omega r H_1(r) & 0 & 0 \\ i\omega r H_1(r) & e^\lambda H_2(r) & 0 & 0 \\ 0 & 0 & r^2 K(r) & 0 \\ 0 & 0 & 0 & r^2 \sin^2 \theta K(r) \end{pmatrix} r^\ell Y_{\ell m}(\theta, \phi) e^{i\omega t}$$

and

$$\zeta^r = \frac{W(r)}{r e^{\lambda(r)/2}} Y_{\ell m}(\theta, \phi) e^{i\omega t}, \zeta^\theta = \frac{V(r)}{r^2} \partial_\theta Y_{\ell m}(\theta, \phi) e^{i\omega t}, \zeta^\phi = \frac{V(r)}{r^2 \sin^2 \theta} \partial_\phi Y_{\ell m}(\theta, \phi) e^{i\omega t}.$$

with a given oscillation frequency, ω . By defining

$$\sum_{\ell, m} r^\ell X Y_{\ell m} e^{i\omega t} = -e^{\nu/2} \Delta p_r$$

We can relate W and V to X. By rewriting the Einstein equations in the above variables, we can write the propagation equation for a given frequency in a matrix form (some of the equations shows $H_0=H_2$ and H_1 can be written in H_0 and K):

$$\vec{Y}' = \mathbf{M}\vec{Y}, \quad \vec{Y} = (H_0, K, W, X)$$

With that, we can solve the perturbation variables throughout the star.

○ Initial Conditions

We require regularity of perturbation functions at the center of the star, and vanishing pressure at the surface of the star:

$$\vec{Y}' = 0 (r = 0), X = 0 (r = R)$$

RESULTS & DISCUSSION

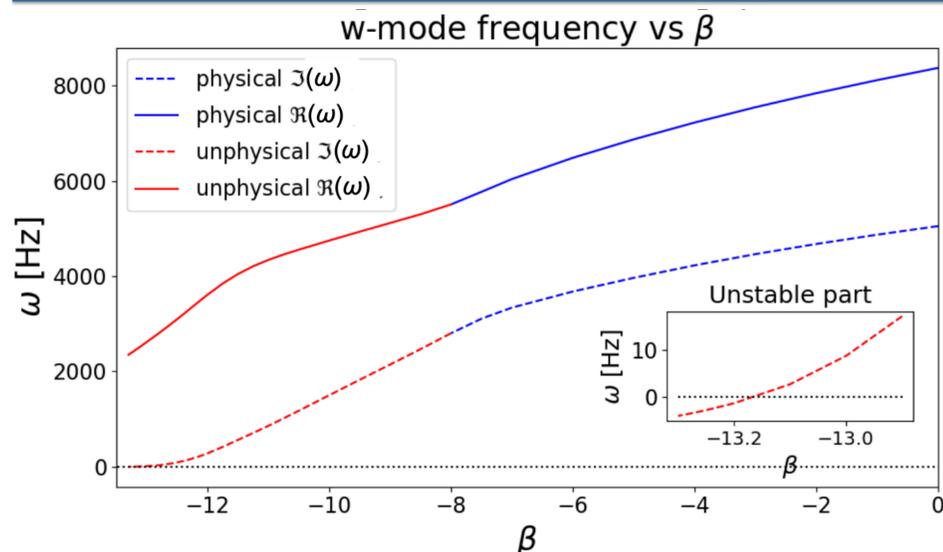


Figure 1. Frequencies of w-modes against the anisotropy parameter β for NSs with MS1 EOS and central density of $\rho_c = 6 \times 10^{14} \text{g/cm}^3$. Physical and unphysical branch are in blue and red.

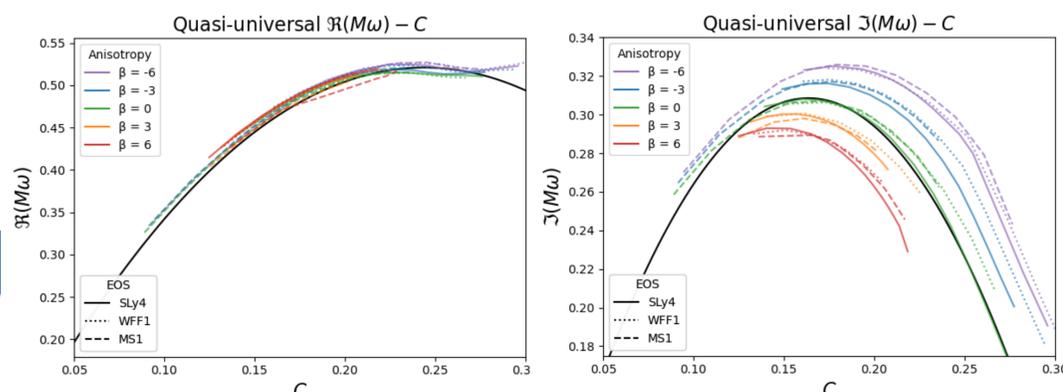


Figure 2. The real (left) and imaginary (right) parts of the scaled frequency for w-modes as a function of compactness, C , for anisotropy parameters and EOSs. The black curve is the fit for isotropic NSs found by Tsui and Leung.

❖ By searching for the oscillation frequency in a range of central density, equation of state (EOS), and anisotropy, we found EOS-insensitive relation for the scaled frequency (mass times frequency), $M\omega$, of w-mode.

❖ We found no instability for physical NS (positive energy and pressure everywhere, and causality). We found unstable first polar w-mode for unphysical star

CONCLUSION

- ✓ W-mode frequencies increase as the anisotropy parameter β increase for fixed central density.
- ✓ $\Re(M\omega)$ to C relation is insensitive to both EOS and the anisotropy β
- ✓ $\Im(M\omega)$ to C relation depends sensitively on β while it remains almost EOS-universal for a fixed β .
- ✓ Future study for physically motivated anisotropy, such as the ones for elastic NSs or those modeled based on liquid crystal.

FUTURE WORK / REFERENCES

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