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Localization of the Elko Field on the Bloch Branes

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INTRODUCTION & AIM

Braneworld models provide new perspectives on long-standing problems in high-energy physics and cosmology. They can unify gravity with other forces by embedding our 4D universe as a brane in higher-dimensional spacetime. These models, inspired by string theory and extra-dimensional scenarios like the Randall-Sundrum setup, address fundamental issues such as the hierarchy problem [1], cosmological constant problems [2] Casimir force [3].

Thick branes generated by scalar fields offer smooth, realistic structures (no delta-function singularities).

The **Bloch brane** is a particularly interesting thick brane arising from two interacting real scalar fields (ϕ and χ) with an interaction parameter r → leads to internal structure and split energy density peaks for small r. [4]

Elko spinors are non-standard spin-1/2 fermions (mass dimension 1, self-conjugate under charge conjugation) that obey the Klein-Gordon equation rather than Dirac equation and promising dark matter candidates.[5]

Matter field localization in braneworld models is crucial because it ensures that standard model particles (fermions, gauge bosons) are effectively confined to the 4D brane, reproducing our observed four-dimensional physics while the extra dimension remains hidden.[6]

In this work, we investigate the localization of a five-dimensional Elko field on the Bloch brane generated by two scalar fields. We first analyze the free Elko case and demonstrate that, in analogy with other thick brane models, the free zero mode cannot be localized. We then introduce a Yukawa-like coupling between the Elko field and the background scalar configuration in the five-dimensional action.

Bloch brane

In this section, we review a loch thick brane setup constructed by gravity and two bulk scalar fields in the curved space time. The metric is given by

$$d\,s^2 = e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2 \tag{1}$$

We consider the action [4]

$$S = \int d^5x\sqrt{-g}\left[\frac{1}{4}R - \frac{1}{2}\partial_M\phi\partial^N\phi - \frac{1}{2}\partial_M\chi\partial^N\chi - V(\phi,\psi)\right] \tag{2}$$

In the above expression, e^{2A} is the warp factor and $A = A(y)$ is the warp function. We suppose that the scalar fields ϕ, χ and the warp factor only depend on the extra coordinate y. In this case the equations of motion reduce to the simpler form

$$A'' = -\frac{2}{3}(\phi'^2 + \chi'^2) \tag{3}$$

$$A'^2 = \frac{1}{6}(\phi'^2 + \chi'^2) - \frac{1}{3}V(\phi,\chi) \tag{4}$$

$$\phi'' + 4A'\phi' = \frac{\partial V(\phi,\chi)}{\partial\phi} \tag{5}$$

$$\chi'' + 4A'\chi' = \frac{\partial V(\phi,\chi)}{\partial\chi} \tag{6}$$

where the prime represents derivative with respect to y. To solve Eqs (3)-(8), we can consider an approach includes the first-order formalism to the braneworld scenario initially introduced in Ref. [7]. The method consists of a super-potential function $W(\phi,\chi)$ related to thick brane solutions $\phi' = \frac{1}{2}\frac{\partial W}{\partial \phi}$, $\chi' = \frac{1}{2}\frac{\partial W}{\partial \chi}$ and warp factor $\acute{A} = -\frac{1}{3}\frac{\partial W}{\partial A}$. By considering the following form of the super potential $W(\phi,\chi)$,

$$W(\phi,\chi) = 2\phi - \frac{2}{3}\phi^3 - 2r\phi\chi^2 \tag{7}$$

the two-field solutions and the warp factor are obtained as

$$\phi(y) = \tanh(2ry) \tag{8}$$

$$\chi(y) = \sqrt{\frac{1-2r}{r}}\,\mathrm{sech}(2ry) \tag{9}$$

We see that the width of these solutions depends on 1/r, so it increases for decreasing r.

We notice that the limit r → 1/2 changes the two-field solution to the one-field solution.

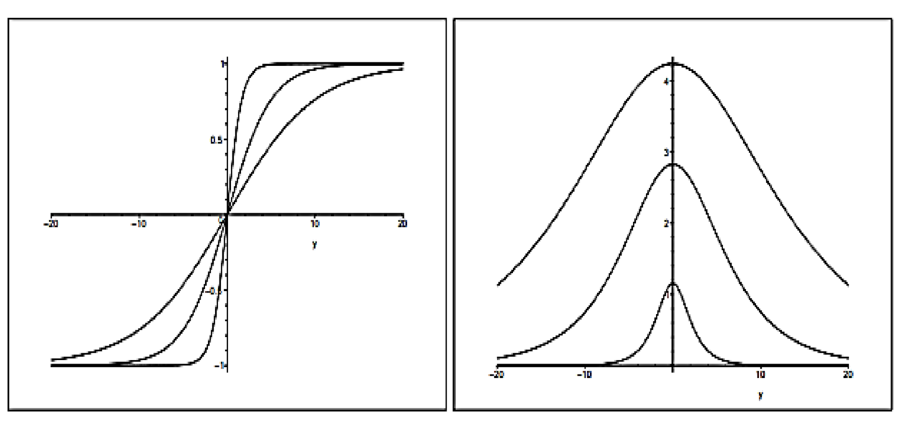

Figure 1: Plots of the solutions $\phi(y)$ (right) and $\chi(y)$ (left) for $r = 0.05, 0.1$ and 0.3 respectively. Here and in the other figures the thickness of the lines increases with increasing r.

$$A(y) = \frac{1}{9r}\left[(1-3r)\tanh(2ry)^2 - (2)\ln(\cosh(2ry)\right] \tag{10}$$

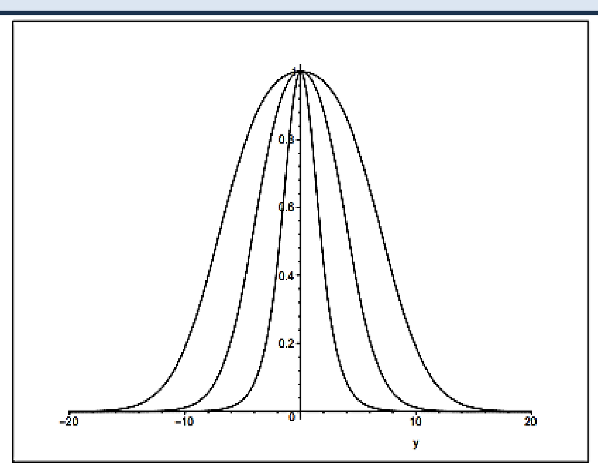

Figure 2: Plots of the solutions of the warp factor $\exp[2A(y)]$ for the values $r = 0.05, 0.1$ and 0.3.

The **warp factor** shapes the potential that governs the zero-mode wavefunction profile: a sufficiently rapid decay of the warp factor can trap massless or light modes near the brane

Localization of five-dimensional Elko spinor

Elko is a new matter field which can be used to investigate some cosmological problems such as the horizon problem, the dark energy problem. Let us consider the action of an Elko spinor field λ which is coupled to gravity as [5]

$$S = \int d^5x\sqrt{-g}\left[-\frac{1}{4}\left(D_M\lambda D^M\bar{\lambda} + D_M\bar{\lambda}D^M\lambda\right) + \bar{\lambda}\gamma^M F(y)\gamma^5\lambda\right] \tag{11}$$

where F(y) is a general scalar function of the extra dimensional coordinate. The covariant derivatives are also given by $D_M\lambda = \partial_M\lambda + \Omega_M\lambda$ where the non-vanishing components of the spin connection is $\Omega_M = \frac{1}{2}\acute{A}(z)\gamma_\mu\gamma_5$. It can be usuful to apply the following conformal metric that is

$$d^2s = e^{2A(z)}\left(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2\right) \tag{12}$$

Where we have used $dz = e^{-A(y)}dy$. The equation of motion for the Elko spinor field can be expressed as

$$\acute{\dot{\alpha}}_n(z) - \left[V_0(z) - m_n^2 + im_n\dot{A}(z)\right]\alpha_n(z) = 0 \tag{13}$$

Where the effective potential of Elko field is given by

$$V_0(z) = \frac{13}{4}\dot{A}^2 + \frac{3}{2}\ddot{A} + 2e^A F(z)\,, \tag{14}$$

Where the dot shows the derivative with respect to the parameter z. It also be note that in order to getting the above equation, we have used the Elko decomposition $\lambda = \lambda_+ + \lambda_-$ where $\lambda_\pm = e^{-\frac{3}{2}A}\sum_n \alpha_n(z)\left[\varsigma_\pm^n(x) + \tau_\pm^n(x)\right]$. *The* $\varsigma_\pm^n(x)$, $\tau_\pm^n(x)$ are two linear independent four-dimensional (4D) Elko spinor fields which satisfy four-dimensional Klein Gordon equation.

- **Free** Elko fields zero mode;

We can study free massless Elko fields on the loch brane by setting $m_n = 0$ and $F(z) = 0$ in Eqs (13) and (14) leading to following shapes of effective potentials and zero mode.

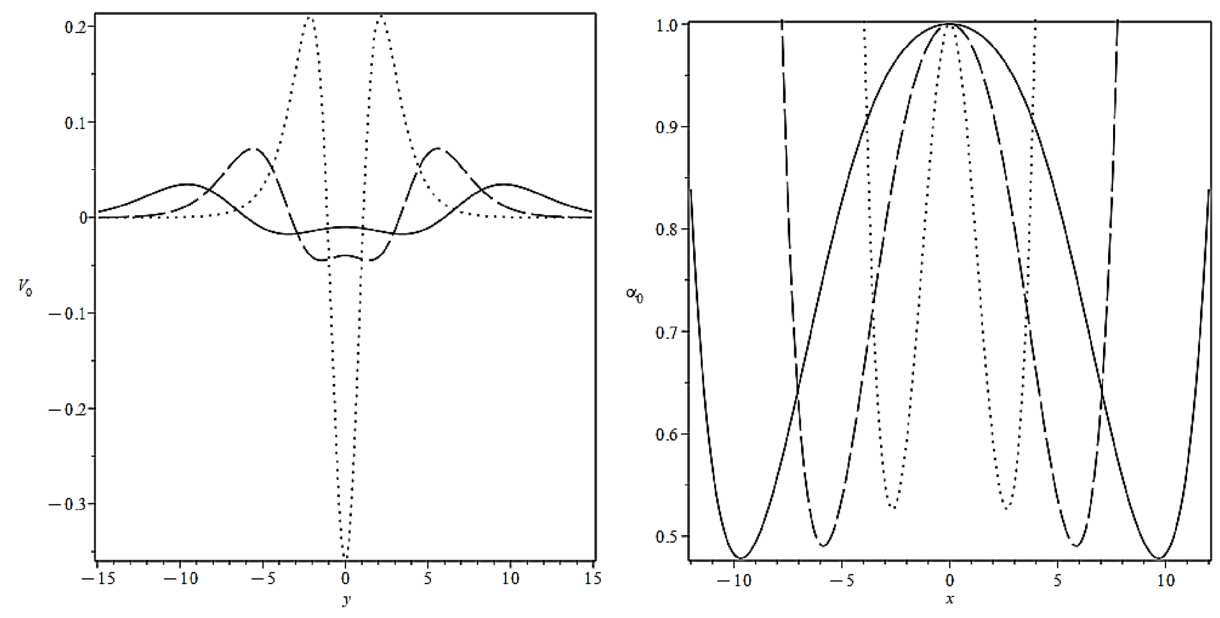

Figure3: Plots of the solutions α_0 (right) and V_0 (left) for r = 0.05 (solid line), 0.1(dashed) and 0.3 (dotted) respectively.

- **Elko** fields with Yukawa-like couplings

We show that massless Elko field can be localized on the Bloch brane if we consider a non-zero Yukawa like coupling as

$$F(y) = d_1\mathrm{sech}(2ry)^6 + d_2\mathrm{sech}(2ry)^4 + d_3\mathrm{sech}(2ry)^2 + d_4. \tag{15}$$

The d_1, d_2, d_3, d_4 are constant parameters related to r and positive ξ. This form of the function F(y) causes a zero mode by the form of $\mathrm{sech}(2ry)^\xi$ and the effective potential as shown in figure 4.

The depth of this potential well plays a critical role in trapping matter: a sufficiently deep well ensures the existence of bound states, particularly the zero mode, which represents the four-dimensional massless particle confined to the brane.

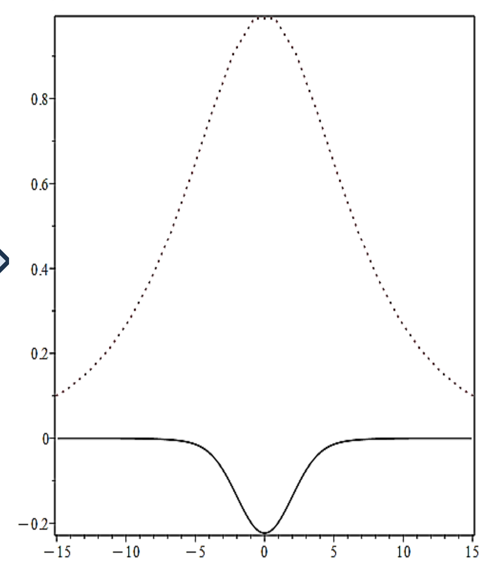

Figure 4: Plots of the solutions α_0 (dotted) and V_0 (solid) for $r = 0.1, \xi = 1$

CONCLUSION

In this work, we investigated the localization of five-dimensional Elko spinor fields on the Bloch brane. We showed that free (massless) Elko fields fail to localize, as the effective Schrödinger-like potential does not support a normalizable zero. By introducing a non-minimal Yukawa-like coupling to the background scalars, we modified the effective potential, creating a sufficiently deep attractive well that traps a normalizable zero mode under explicit conditions on the coupling strength and brane parameters.

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