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A Novel Numerical Method for Solving Fractal-Fractional Differential Equations with Exponential Memories

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INTRODUCTION & AIM

Fractal–fractional [2] differential equations (FFDEs) provide an effective framework for modeling complex systems that exhibit memory effects, nonlocal behavior, and fractal characteristics. These models are widely used in applications such as financial systems, biological processes, and chaotic dynamics. However, due to the presence of nonlinearities and exponential memory kernels, obtaining accurate and efficient numerical solutions remains a significant challenge.

To address these challenges, this work proposes a new numerical approach that combines an iterative technique with a predictor–corrector scheme for solving fractal–fractional differential equations with exponential decay kernels.

Aim of the Study:

- To develop a new numerical method for solving fractal–fractional differential equations with exponential decay kernels.
- To integrate the Daftardar–Gejji and Jafari iterative method (DJM) [3] with a predictor–corrector scheme for improved performance.
- To enhance convergence properties and computational efficiency without using linearization or discretization.
- To establish theoretical results including stability and convergence of the proposed method.
- To validate the method through applications to financial systems, cancer models, memristor systems, and chaotic attractors.

METHOD

Problem Statement :

We consider fractal–fractional differential equations of the form:

$$D^{\alpha,\beta}u(t) = f(\mathrm{t}, \mathrm{u}(\mathrm{t})), 0 < \alpha, \beta \le 1$$

where:

α : fractional order β : fractal dimension $M(\alpha)$: normalization function

Proposed Numerical Method :

➢ **Predictor Step**

$$u_{n+1}^{P_1} = u_n + \beta(t_n)^{\beta-1} f(t_n, u_n)\left[\frac{h\alpha}{2M(\alpha)} - \frac{1-\alpha}{M(\alpha)}\right]$$

$$u_{n+1}^{P_2} = \beta(t_{n+1})^{\beta-1} f(t_{n+1}, u_{n+1}^{P_1})\left[\frac{h\alpha}{2M(\alpha)} + \frac{1-\alpha}{M(\alpha)}\right]$$

➢ **Corrector Step**

$$u_{n+1}^{C} = u_{n+1}^{P_1} + \left(\frac{\alpha\beta h}{2M(\alpha)} + \frac{\beta(1-\alpha)}{M(\alpha)}\right)(t_{n+1})^{\beta-1} f(t_{n+1}, u_{n+1}^{P_1} + u_{n+1}^{P_2})$$

Algorithm Steps

- Initialize u_0 ,step size h, parameters α, β
- Compute predictor $u_{n+1}^{P_1}$
- Compute refined predictor $u_{n+1}^{P_2}$
- Apply corrector formula
- Update solution: $u_{n+1} = u_{n+1}^{C}$
- Repeat for all time steps

Fractal–Fractional Financial Model

The fractal–fractional financial model [1] describes nonlinear, memory-dependent, and chaotic financial systems. It incorporates memory effects (fractional order) and time-scale irregularities (fractal dimension) for realistic modeling.

The system is formulated using fractal–fractional differential equations with a exponential kernel.

Numerical results show that the method effectively captures chaotic behavior, memory effects, and complex financial dynamics.

RESULTS & DISCUSSION

Example 1. Consider the following fractal–fractional differential equation with exponential memory:

$$^{FFE}D_t^{\alpha,\beta}x(t) = -t + 9,$$

with initial condition:

$$x(0) = 0.1$$

Numerical solution Via Proposed Predictor–corrector scheme Simulations carried out using MATLAB.

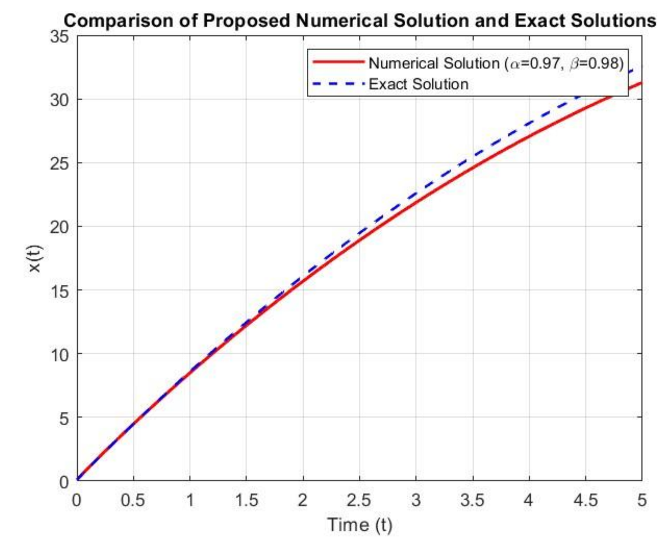

Figure 1. Numerical solution of the fractal–fractional differential equation in the exponential kernel sense with parameters α=0.97 and β=0.98 The numerical results are compared with the exact solution.

Example 2. The financial chaotic system is given by: [1]

$$ {}_0^{C}D_t^{\alpha,\beta}x(t) = z(t) + (y(t) - \nu)x(t)$$

$$ {}_0^{C}D_t^{\alpha,\beta}y(t) = 1 - \mu y(t) - x(t)^2$$

$$ {}_0^{C}D_t^{\alpha,\beta}z(t) = -x(t) - \gamma z(t)$$

Initial Conditions $x(0) = 1, y(0) = 2, z(0) = 0.9$

Parameters $\mu = 0.2, \nu = 0.9$

$\alpha \in (0,1]$: fractional order, $\beta \in (0,1]$: fractal dimension.

Methodology

Fractal–fractional derivative (Caputo type with exponential kernel)

Numerical solution Via Proposed Predictor–corrector scheme Simulations carried out using MATLAB.

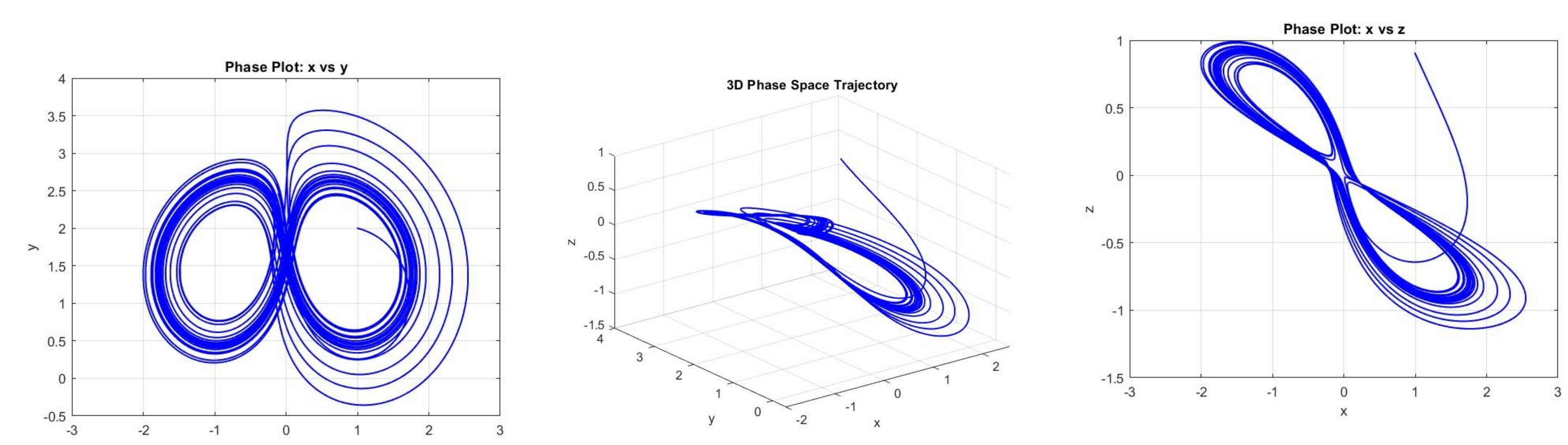

Figure 2. Fractal–fractional financial model's dynamic behavior with exponential law kernel for α=0.98, β = 0.98, ν=0.9 , μ=0.2, γ =1.5, $x(0) = 1, y(0) = 2$ *and* $z(0) = 0.9$

CONCLUSION

- The fractal–fractional financial model effectively captures chaotic dynamics, memory effects, and nonlinear behavior, providing a more realistic representation of financial systems.
- The proposed numerical scheme ensures high accuracy and stability, and the results demonstrate strong potential for applications in financial analysis and forecasting.

FUTURE WORK

Future Work :

- Extension of the Financial chaotic model to stochastic and real data driven system
- Development of advanced numerical scheme for higher accuracy and effiency

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