

## Fractional Approach to Dynamic Analysis of Coupled Circular Plates with Hereditary Effects

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### INTRODUCTION & AIM

### METHOD

- Multilayer systems with viscoelastic creep layers appear in aerospace, civil engineering, and microsystems, where time-dependent deformation significantly affects long-term stability.
- Classical integer-order models cannot fully capture hereditary (memory-dependent) behaviour.
- Fractional calculus provides a more accurate framework for modelling viscoelastic damping and creep.

#### Aim of the study

- Develop a general analytical model for vibrations of an M-plate system connected by hereditary viscoelastic interlayers.
- Derive closed-form expressions for natural frequencies, mode shapes, and time-dependent responses.
- Analyse how fractional order ( $\alpha$ ) and kinetic stiffness influence dynamic behaviour.

#### Model

- System:  $M$  circular plates coupled by viscoelastic creep layers.
- Governing equations: partial integro-differential equations from hereditary viscoelasticity and D'Alembert's principle.
- Creep layers modelled via fractional derivatives ( $0 < \alpha \leq 1$ ).

#### Reduction

- Separation of variables using Bernoulli's method
- Reduction to ordinary integro-differential equations
- Solution via Laplace transforms
- Spectral properties obtained from determinant  $\Delta_{nm}(p) = 0$

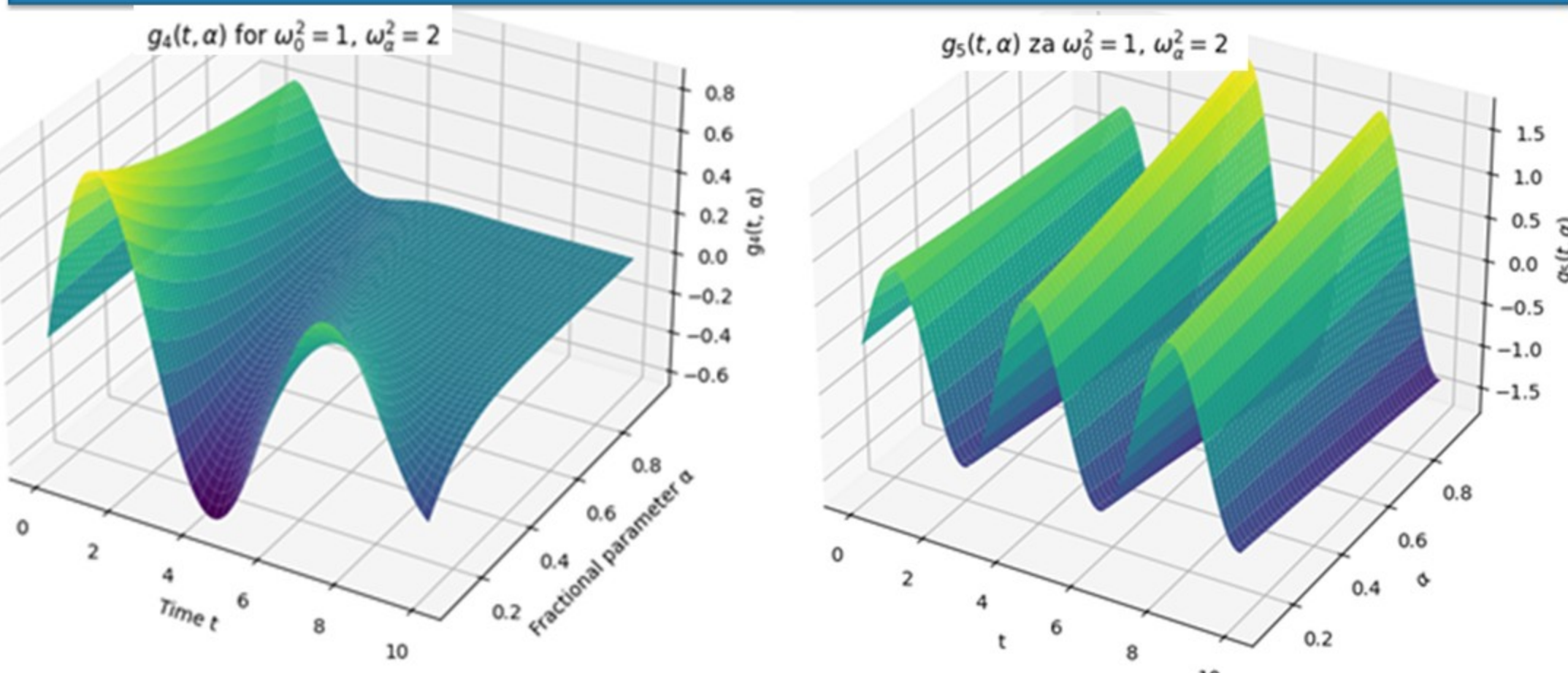
#### Key Outputs

Explicit analytical expressions for:

- Natural frequencies
- Eigenfunctions
- Fractional time-response functions including hereditary damping
- Identification of multi-frequency regimes in M-plate systems.

### RESULTS & DISCUSSION

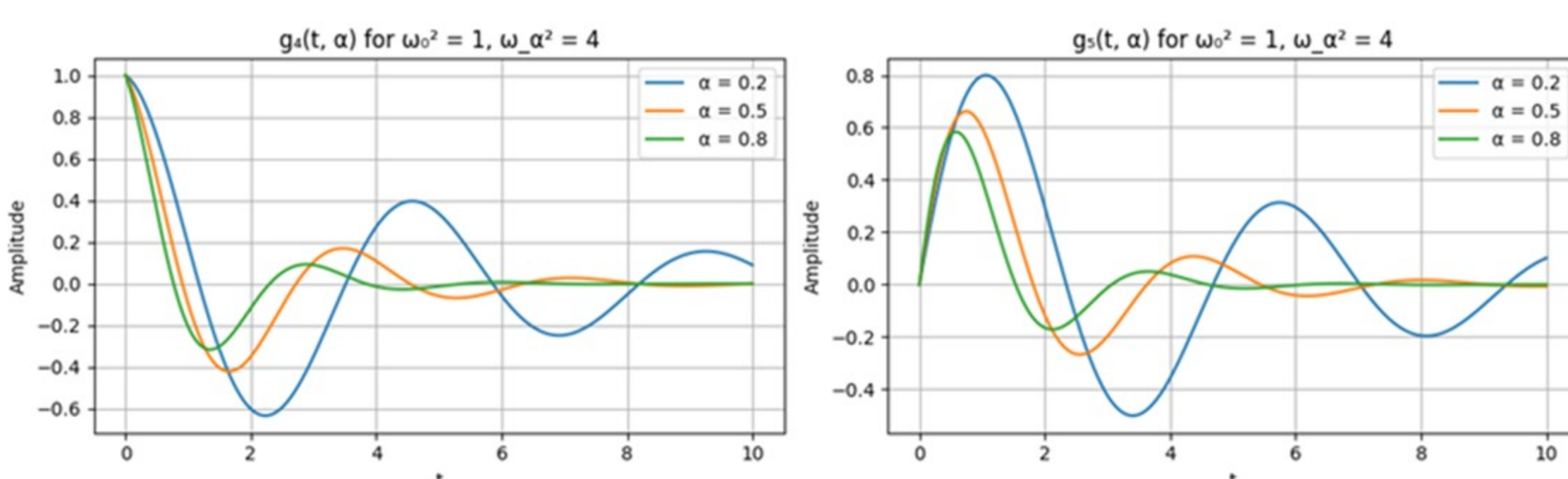
### CONCLUSION



1. Multi-frequency vibration regimes
  - Coupling through hereditary layers produces  $M$  distinct natural frequencies per mode family.
  - Modal families remain temporally uncoupled, enabling precise identification of resonance conditions.

#### 2. Effect of fractional parameter $\alpha$

- Low  $\alpha$  ( $\approx 0.2$ )  $\rightarrow$  weak hereditary damping  $\rightarrow$  sustained oscillations.
- High  $\alpha$  ( $\approx 0.8$ )  $\rightarrow$  strong memory effects  $\rightarrow$  rapid amplitude decay.
- Fractional order acts as a tuneable damping control.



- Developed a general analytical fractional-order model for hereditary vibrations in multilayer circular plate systems.
- Demonstrated multi-frequency regimes, temporal modal uncoupling, and tuneable damping via fractional order.
- Showed how fractional damping and coupling stiffness jointly influence oscillation amplitude, decay rate, and energy transfer.
- Framework provides a powerful tool for optimizing multilayer structures under long-term loading.

### FUTURE WORK / REFERENCES

- Include non-ideal creep laws and anisotropic interlayers
- Extend to nonlinear oscillations and thermal effects
- Integrate with numerical FEM models for complex geometries
- Apply in design of vibration isolators, microdevices, sandwich structures

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