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A Comprehensive Comparison of Lifetime Distributions Based on Goodness-of-Fit Analysis

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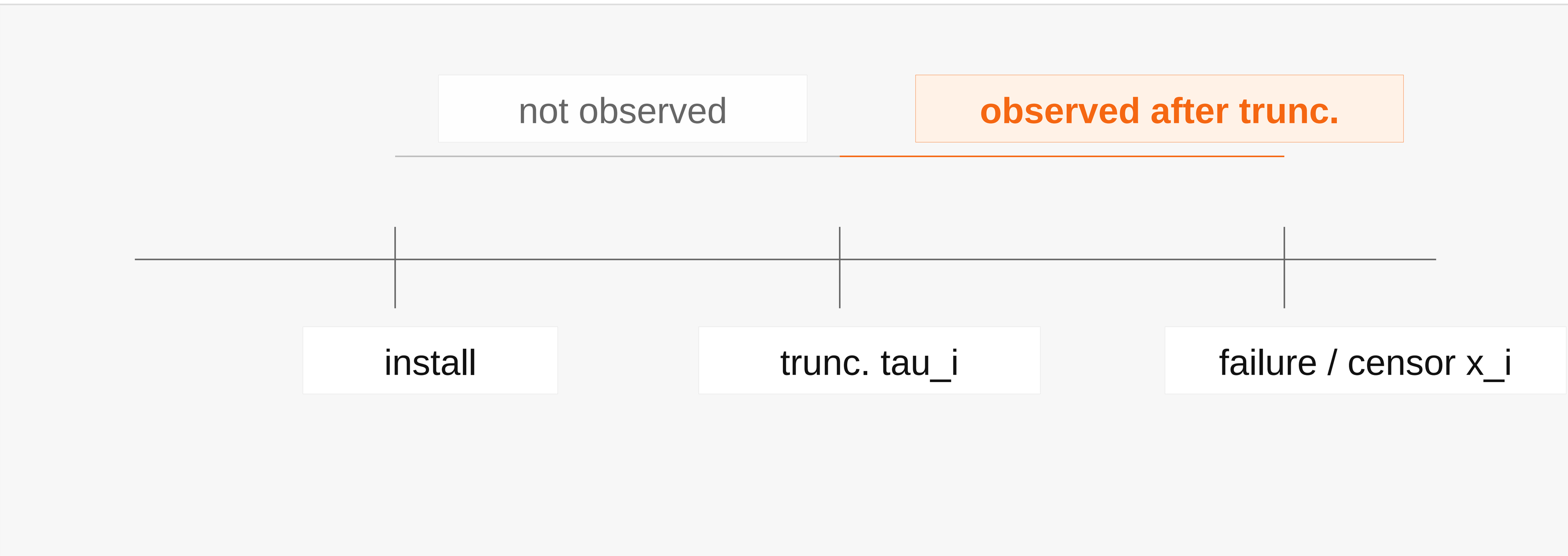
INTRODUCTION & AIM

Research objective

Accurate failure-time distribution modeling is fundamental in reliability engineering, directly influencing maintenance planning and risk assessment. Transformer data are left-truncated and right-censored; ignoring LTRC mechanisms can bias survival-tail estimates. Aim: compare five classical parametric models and CF1 Phase-Type (PH) models under a unified LTRC likelihood using AIC and EIC.

LTRC observation scheme



Observed times are conditional on survival beyond the truncation time. Failures and censored records therefore require different likelihood contributions.

METHOD

1. LTRC likelihood

For observation i, x_i is the observed failure/censoring time, tau_i is the left truncation time, and delta_i is the failure indicator.

$$D = \{(x_i, \tau_i, \delta_i)\}_{i=1}^{n}, \quad \delta_i \in \{0,1\}$$

$$\ell(\theta) = \sum_{i=1}^{n} \{\delta_i \log f(x_i \mid \theta) + (1-\delta_i) \log S(x_i \mid \theta) - \log S(\tau_i \mid \theta)\}$$

2. Phase-Type model / CF1

A PH distribution is the absorption time of a finite-state continuous-time Markov chain. CF1 uses ordered phases and has k = 2m - 1 free parameters.

Total lifetime = time to absorption

$$f(x) = \boldsymbol{\alpha} \exp(\boldsymbol{T}x)\boldsymbol{\xi}, \quad S(x) = \boldsymbol{\alpha} \exp(\boldsymbol{T}x)\mathbf{1}$$

3. Model selection

AIC uses a fixed 2k penalty. EIC replaces this with a bootstrap bias estimate and is useful when flexible PH models may be misspecified.

$$\mathrm{AIC} = -2\ell(\hat{\theta}) + 2k$$
$$\mathrm{EIC} = -2\{\ell(\hat{\theta}; D) - \hat{b}_{\mathrm{EIC}}\}$$

RESULTS & DISCUSSION

Main findings to communicate

- Transformer LTRC data: n = 286 records, 39 failures, and 247 right-censored units.
- Among classical models, the lognormal distribution has the lowest AIC and EIC.
- CF1 PH with m = 50 has the minimum EIC and outperforms the classical models under EIC.
- Tail behavior differs by model, affecting long-term reliability estimates.

Model	Role	Interpretation
Classical	baseline	interpretable but limited hazard/tail shapes
Lognormal	best classical	lowest AIC/EIC among classical fits
PH / CF1	flexible model	captures complex hazard and tail behavior
EIC	selection	bootstrap bias correction controls complexity

Distribution	k	AIC	EIC
Exponential	1	470.2810	470.3173
Weibull	2	471.7929	471.1914
Gamma	2	471.9665	471.2771
Lognormal	**2**	**469.3730**	**469.0939**
Inverse Gaussian	2	513.3378	521.9878

Table 1. Classical distribution comparison under the LTRC setting. Lognormal gives the smallest AIC and EIC among classical models.

m	k	AIC	EIC
2	3	471.5467	469.3489
5	9	480.1898	468.1962
10	19	497.4230	469.2416
20	39	534.5468	468.0078
50	**99**	**642.5906**	**463.1938**
70	139	716.0610	467.6242
100	199	830.1636	467.1209

Table 2. AIC/EIC comparison for CF1 PH distributions. EIC is minimized at m = 50.

CONCLUSION

- LTRC likelihood enables fair comparison of classical and CF1 PH lifetime models.
- Lognormal is best among the classical models; CF1 PH with m = 50 gives the lowest EIC overall.
- Distribution choice and tail behavior matter for long-term reliability and risk assessment.

FUTURE WORK / REFERENCES

Future work: validate on additional reliability datasets and quantify maintenance-policy sensitivity.

Conflict of Interest: none. **Acknowledgments:** JST BOOST JPMJBS2424.

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