

Can Replacement - Level Fertility Reverse Population Decline? Evidence from Cohort – Component Projections

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INTRODUCTION & AIM

Countries with prolonged below replacement fertility result in contraction of cohort in the child bearing ages along with population aging and negative growth [1,2]. This contraction of reproductive cohort triggers a phenomenon known as negative population momentum, where the cohort size is not sufficient enough to generate births to offset deaths in comparatively larger older generations, even after fertility stabilizes at near replacement level, resulting in population ageing, shrinking labor force, high dependency ratio, slower economic growth in the long run [3].

South Korea represents the global extreme with below replacement fertility from 1984, dropped record low to 0.72 in 2023, showing little improvement at present. This study aims to investigate whether restoring fertility to the replacement level can reverse population decline or stabilize population size in a society shaped by such protracted low fertility [4].

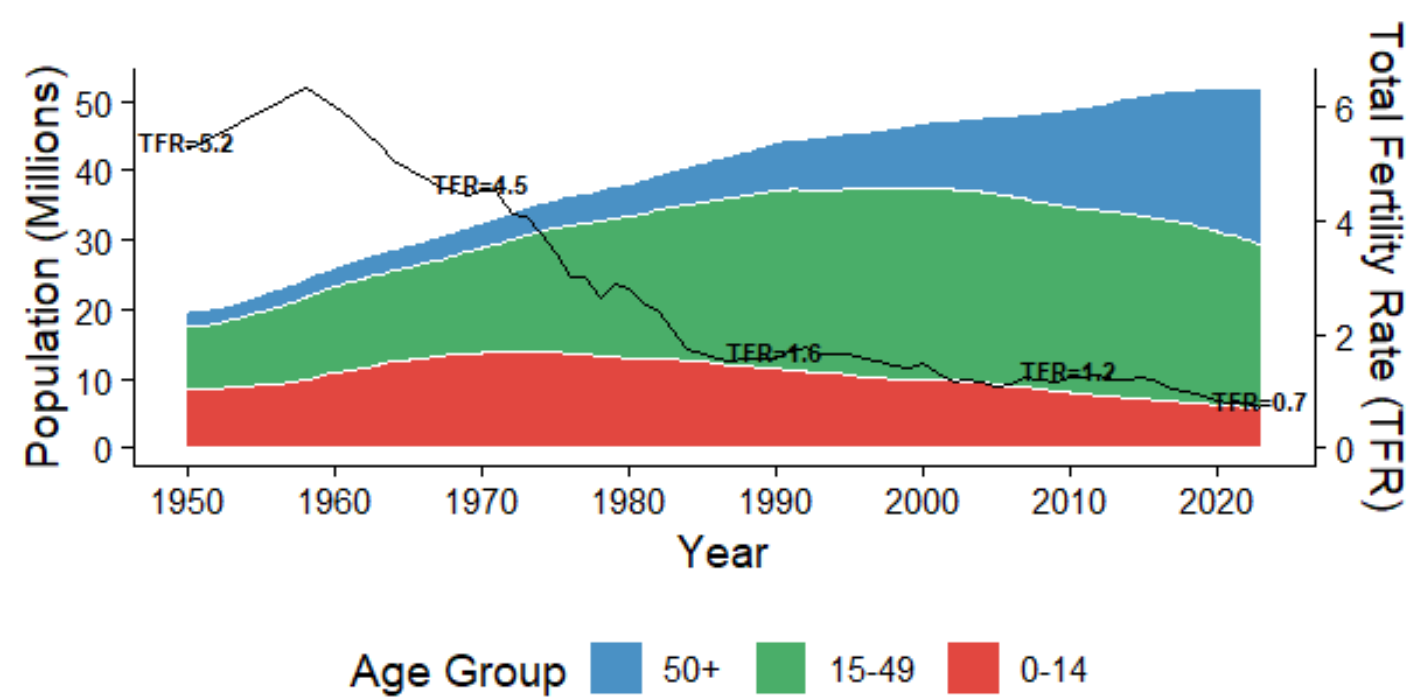


Figure 1: South Korea Demographic transition

METHOD

This study adopts theoretical cohort component projection approach by utilizing McKendrick-Leslie matrix framework [6], an age structured population model to project age specific population change over time by incorporating age-specific fertility and mortality rates. The continuous age structured partial differential equation developed by McKendrick–von Foerster is given by

$$\frac{\partial n}{\partial t} + \frac{\partial n}{\partial a} = -\mu(a)n(a, t) \quad (1)$$

where, $n(a, t)$ is the number of people of age a and time t , $\mu(a)$ is the age specific mortality rate. The number of birth at time t is defined by the integral:

$$n(0, t) = \int_0^{\infty} b(a)n(a, t)da$$

where, $b(a)$ is the age specific fertility rate

A discrete numerical approximation of the continuous model is used to project population for 100 years in 5 year age cohort by Leslie matrix L , such that

$$\mathbf{n}(t + 1) = L\mathbf{n}(t) \quad (2)$$

where, \mathbf{n} is an age distribution vector, L is a linear operator, top row of it contains age specific fertility rates, survival probabilities at sub diagonal.

After projecting population in 5 yearly cohort, the old age dependency ratio is calculated as

$$DR = \frac{n_{(65+)}}{n_{(15-64)}} \times 100 \quad (3)$$

The analysis is structured around four distinct counterfactual scenarios to compare future age structure and see how old age dependency ratio vary at different circumstances with population and fertility data obtained from the United Nations World Population Prospects, while mortality data and survivorship proportions are derived from the Human Mortality Database.

1. Projection the female population for 100 years using constant baseline fertility and mortality of South Korea, to see how population age structure changes if no further change in vital rates happens.
2. Projection According to the United Nations World Population Prospects (Medium Variant) as an external benchmark representing a gradual, expert-assumed fertility recovery [5].
3. To investigate the effect of negative momentum [6], fertility is adjusted instantaneously such that net reproduction rate becomes 1, keeping mortality constant. The scaling factor k is solved such that $\sum L(k \cdot fert)pf = 1$, where pf is the female ratio
4. In the last scenario, adjusted replacement fertility is paired with shifting mortality where mortality is held constant for ages below 25, while the hazard curve for older ages shifts rightward by a postponement rate of 0.2 years per calendar year, to align with modern demographic reality which aims at increased survival at older ages [7].

RESULTS & DISCUSSION

Age structured female population projection is shown in three major groups: younger cohort, reproductive age and post reproductive age, also the burden of old age is marked with black line.

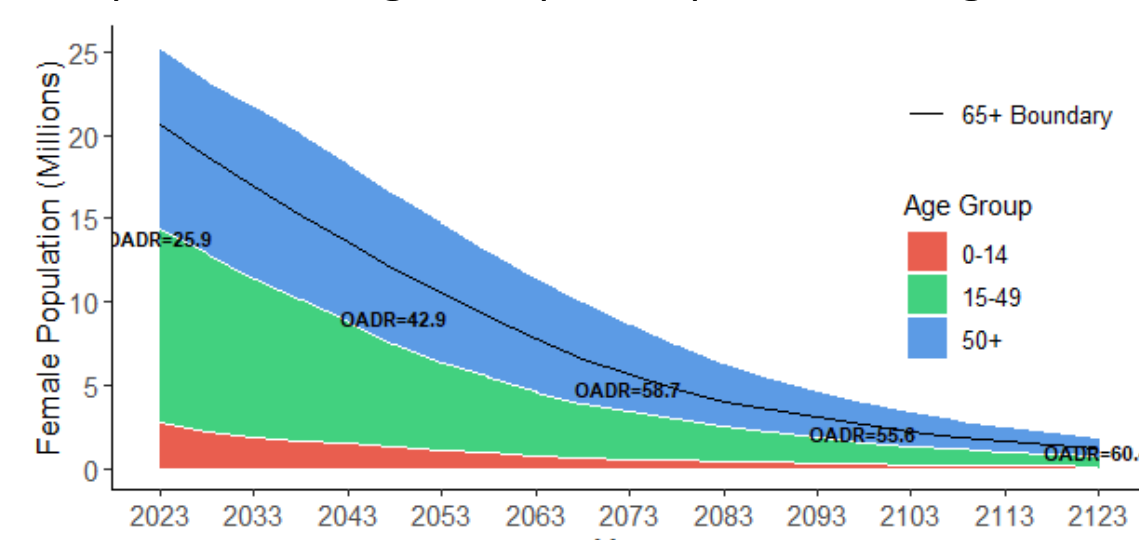


Figure 2: Population projection with constant Fertility and Mortality

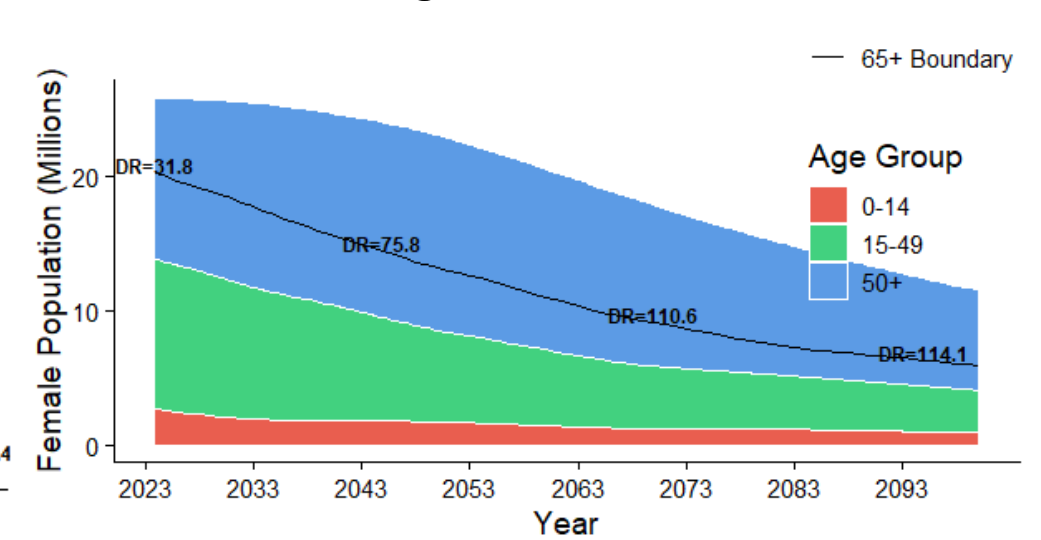


Figure 3: Projection according to UN

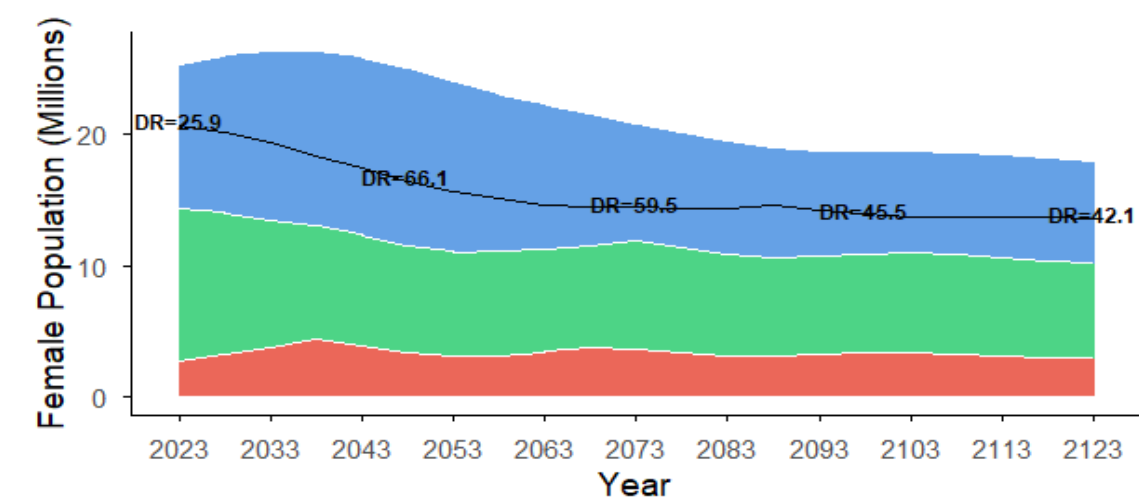


Figure 4: Population projection with replacement Fertility and constant Mortality

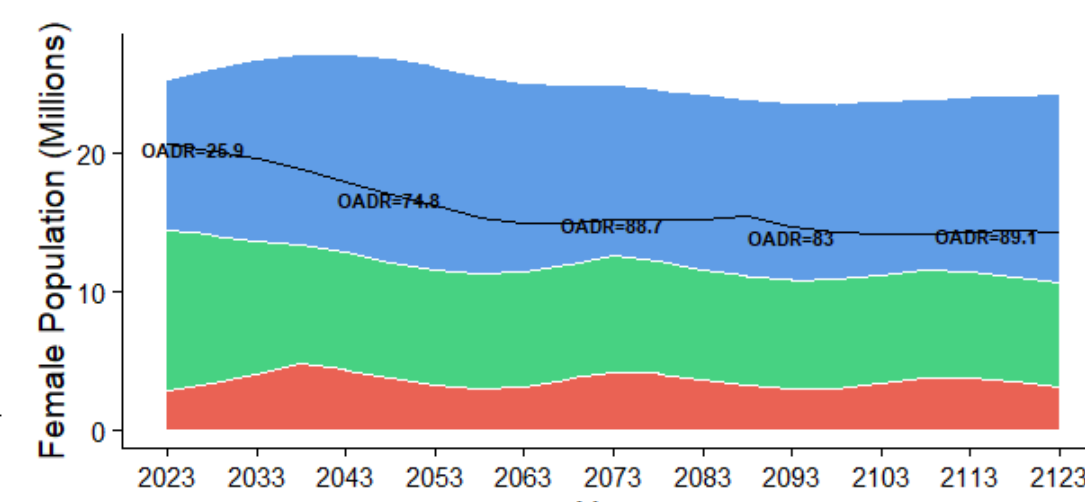


Figure 5: Population projection with replacement Fertility and Shifting Mortality

Figure1 reveals that the population faces rapid exponential collapse if no intervention done to change vital rates and dependency ratio climbs towards unsustainable levels. Figure 2 serves as realistic benchmark, also projects 50% population loss and dependency ratio expected to be raised to maximum among all scenarios. The effect of negative momentum is shown in Figure 3 by forcing an immediate jump to replacement fertility, keeping mortality constant, population continues to shrink for more than 60 years before stabilizing, also reaching dependency ratio at comparatively favourable level. Figure 4 reveals increased longevity cushions the population loss better compared to others, but the resulting stationary state is larger old age cohort and higher dependency ratio.

CONCLUSION / FUTURE WORK

The four scenarios collectively demonstrate that South Korea is currently a victim of its own demographic history, as past birth rates were so less, there are simply not enough potential mothers entering the reproductive cohort to generate surplus birth to offset death. The primary driver of future depopulation are not the vital rates but the contraction of cohorts in the childbearing ages. Even shifting mortality to right only expands aging layer and stressed dependency ratios. Thus, policy responses should extend beyond fertility promotion and incorporate long-term strategies addressing large old age cohort and sustained demographic contraction. Fur future work, this study can be extended by incorporating stochastic modeling to account for uncertainty in fertility recovery.

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