

An Improved Version of Exponential Rayleigh Distribution with Statistical Properties and Diverse Applications

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INTRODUCTION & AIM

Numerous classical probability distributions have shown little adaptability in recent decades when successfully representing the complexity of real-world data sets, including complex, symmetric, asymmetric, and skewed data sets. This has encouraged researchers to add more parameters to traditional models to create more flexible families of distributions. In this study, we introduced a novel extension of the exponential Rayleigh distribution utilizing a weighted general class of distributions (WG-ERD).

About its density and hazard functions, the suggested model is highly adaptable and provides a variety of behaviors. The statistical and reliability properties of the proposed distribution are thoroughly established, including the quantile function, moments, skewness, kurtosis, and order statistics. Parameter estimation employs both Bayesian and classical methods, including maximum likelihood, maximum product spacing, and Bayes estimators based on various loss functions. A simulation experiment illustrating the efficacy of the suggested distribution reveals that the Bayes estimator based on the square error loss function generates more precise parameter estimates than other approaches. Finally, two data sets are analyzed to investigate the new distribution's advantage and adaptability. When the suggested model is applied to the various probability distributions currently in use, we find that it provides the best fit compared to competing distributions based on specific evaluation criteria.

METHOD

The cumulative distribution function (CDF) and probability distribution function (PDF) of the proposed WG-ERD are given respectively as

$$G(w) = \log(1 + (1 - \exp(-aw^2 - cw))^b) / \log(2),$$

And

$$g(w) = h(\lambda aw + c) \exp(-aw^2 - cw) (1 - \exp(-aw^2 - cw))^{b-1} / \log(2) (1 + (1 - \exp(-aw^2 - cw))^b)$$

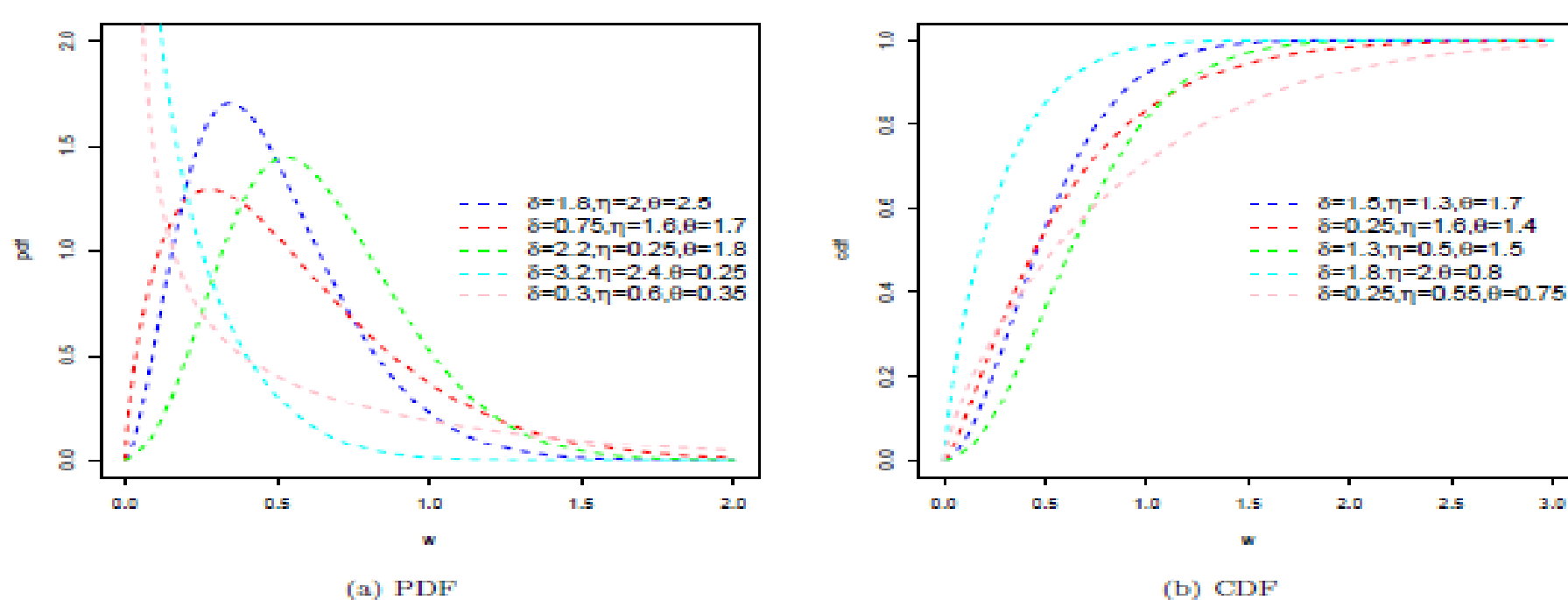


Figure (1) presents the PDF and CDF of the WG-ERD for various combinations of parameters a, b, and c, illustrating diverse shape patterns. The PDF plots show that the distribution is unimodal, right-skewed, and decreasing. These illustration patterns ensure the model's versatility in modeling several real data sets.

RESULTS & DISCUSSION

Table 3: Simulation values of Ests and MSEs of the WG-ERD using Case 1.

n		θ					δ					η				
		MLE	MPS	Bayes			MLE	MPS	Bayes			MLE	MPS	Bayes		
				SE	LI	GE			SE	LI	GE			SE	LI	GE
25	Est	0.8877	0.7868	0.7402	0.7454	0.7335	0.5555	0.5495	0.4861	0.4921	0.4708	0.8899	0.7651	0.8109	0.8231	0.7980
	MSE	0.1401	0.1513	0.0239	0.0243	0.0240	0.1738	0.1929	0.0245	0.0248	0.0250	0.4130	0.5076	0.0520	0.0523	0.0527
50	Est	0.8200	0.8199	0.8413	0.8482	0.8356	0.5504	0.3909	0.4924	0.4955	0.4853	0.7945	0.8264	0.8226	0.8257	0.819
	MSE	0.0453	0.0681	0.0213	0.0216	0.0219	0.0816	0.0975	0.0124	0.0128	0.0130	0.2542	0.2717	0.0176	0.0179	0.0182
75	Est	0.8278	0.7534	0.8577	0.8592	0.8561	0.4931	0.4897	0.4486	0.4507	0.4440	0.8003	0.7963	0.8953	0.8972	0.8924
	MSE	0.0352	0.0526	0.0091	0.0093	0.0096	0.0687	0.0937	0.0108	0.0111	0.0113	0.2028	0.2584	0.0118	0.0121	0.0124
100	Est	0.8065	0.8113	0.8100	0.8106	0.8092	0.5409	0.5146	0.4993	0.5003	0.4980	0.7698	0.7398	0.7437	0.745	0.7419
	MSE	0.0327	0.0457	0.0028	0.0029	0.0031	0.0630	0.0722	0.0027	0.0031	0.0034	0.1701	0.2162	0.0053	0.0056	0.0059

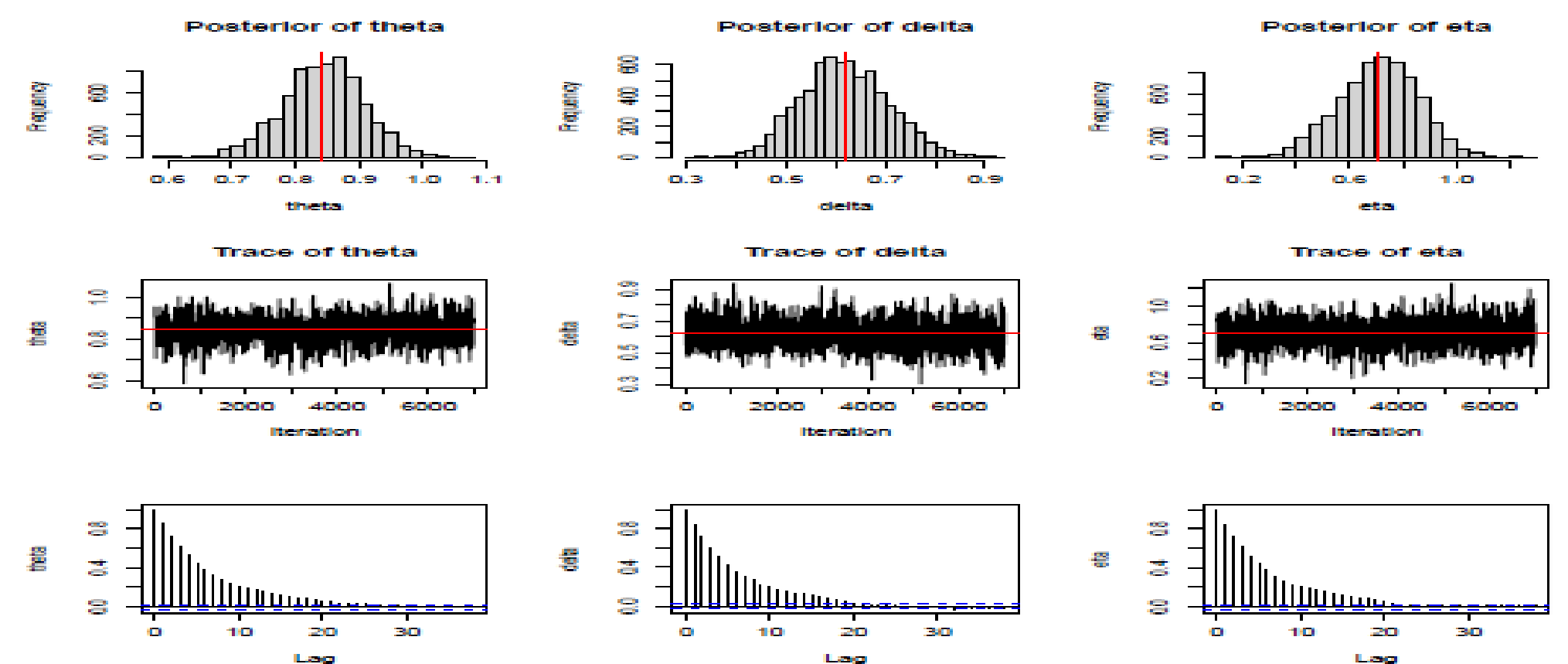
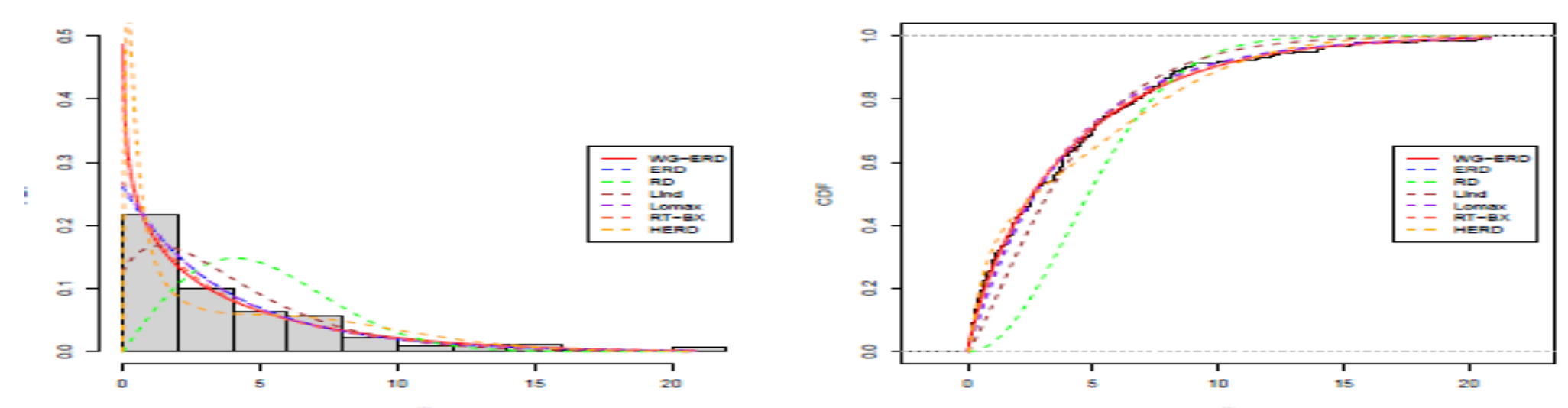


Table 7: Estimate and comparison criterion for the two applied data.

Data	Model	$\hat{\theta}$	$\hat{\delta}$	$\hat{\eta}$	\mathcal{R}_1	\mathcal{R}_2	\mathcal{R}_3	\mathcal{R}_4
1	WG-ERD	0.8298	0.0034	0.1550	982.713	992.697	0.0381	0.9246
	HERD	0.0056	0.0157	8.0646	1002.249	1012.233	0.0820	0.1246
	RT-BX	0.3122	0.6542	0.4269	984.101	994.085	0.0460	0.7742
	ERD		0.0012	0.2606	986.641	993.297	0.0727	0.22573
	Lomax		55.895	14.962	986.434	993.090	0.0689	0.2815
	RD			4.0964	1298.433	1301.761	0.3247	< 0.00001
	Lind			0.4255	1018.518	1021.846	0.1443	0.0003



CONCLUSION

This research aimed to create an efficient, adaptable extension of the traditional exponential Rayleigh distribution that could overcome its limitation in hazard-rate shaping and tail modeling. We introduced the WG-ERD and derived its complete set of analytical properties in closed form.

The practical applicability of the WG-ERD was demonstrated using two real-world datasets. The proposed WG-ERD demonstrated its effectiveness and versatility in analyzing heavy-tailed data, offering superior compatibility in both cases compared to competing models.

FUTURE WORK / REFERENCES

Bakouch, H., Chesneau, C., Enany, M. A weighted general family of distributions: Theory and practice, *Comp and Math Methods*, 3:e1135, 2021
 Jamal, F., Nasir, M. Generalized Burr X family of distributions, *International Journal of Mathematics and Statistics*, 19(1), 55-73, 2022.