

Mathematical Analysis of the Dynamic Modeling of Heart Rate

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INTRODUCTION & AIM

Most existing heart rate monitoring systems rely on fixed threshold values to trigger alerts, resulting in reactive responses that often fail to capture early physiological deterioration. Such approaches neglect the temporal structure of heart rate signals and their dynamic behavior.

We propose a predictive monitoring framework that analyzes not only the current HR value, but also how that value is changing: its speed, acceleration, and deviation from the individual's baseline. The framework models the heart rate signal as a time-dependent function and applies discrete approximations of its first and second derivatives to quantify rate of change and acceleration.

Traditional devices cannot distinguish between a 20-year-old athlete reaching 165 bpm during training - a physiologically normal condition - and an elderly patient reaching the same value, which may be life-threatening. The proposed framework adapts detection thresholds to each individual's physiological baseline, moving from reactive monitoring to a predictive model that identifies clinical deterioration early and with fewer false alarms.

METHODOLOGY

- HR SIGNAL ACQUISITION**
Heart rate $HR(t)$ is recorded continuously, building a time-dependent signal function whose temporal structure is the core input to all subsequent analysis.
- FIRST DERIVATIVE - RATE OF CHANGE**
A discrete approximation quantifies how fast HR is changing over each time step Δt , providing a measure of instantaneous variation.
- SECOND DERIVATIVE - ACCELERATION**
Measures whether the rate of change is itself increasing (acceleration) or decreasing (deceleration), revealing the concavity of the HR trajectory.
- ROBUST BASELINE - MEDIAN & MAD**
The individual baseline is estimated using the median, which is resistant to outliers, and the Median Absolute Deviation (MAD) scaled by 1.4826 to obtain a robust standard deviation consistent with normal variability.
- Z-SCORE ANOMALY DETECTION**
Applied to HR, f' , and f'' to quantify deviation from the personalized baseline in a standardized and interpretable way, enabling consistent risk stratification across different physiological profiles.
- TIME-TO-CRITICAL PREDICTION**
Linear extrapolation is used under steady-rate conditions; a quadratic formulation solved analytically is applied when positive acceleration is detected, providing a quantitative estimate of the time remaining until $Z = 3.5$.

RESULTS & DISCUSSION

The proposed framework identifies risk trajectories that remain undetected by static threshold methods, particularly in scenarios involving rapidly accelerating heart rate. By combining first- and second-order derivative analysis with a personalized Z-score baseline, the system captures the dynamics of physiological deterioration rather than reacting only to absolute values. The use of median and MAD instead of mean and standard deviation provides robustness against sensor noise and wearable device artifacts, significantly reducing false alarms. In a representative scenario ($Z_0 = 2.1$, $dZ/dt = 0.04 \text{ s}^{-1}$, acceleration = 0.003 s^{-2}), the quadratic model predicted time-to-critical ($Z = 3.5$) in 28.6 s, versus 35.0 s by linear extrapolation — an 18.3% earlier anticipation representing clinically relevant lead time for intervention.

MATHEMATICAL FRAMEWORK

FIRST DERIVATIVE (FINITE DIFFERENCE)

$$f'(t) = \Delta HR / \Delta t = [HR(t) - HR(t-1)] / \Delta t$$

Example: $(76 - 70) / 1.5 \text{ s} = +4 \text{ bpm/s}$

SECOND DERIVATIVE (ACCELERATION)

$$f''(t) = \Delta f' / \Delta t = [f'(t) - f'(t-1)] / \Delta t$$

$f'(t)$	$f''(t)$	Clinical Implication
+	+	Rate of ascent accelerating — Risk rising rapidly
-	-	Rate of descent accelerating — Rapid recovery
+	-	Rate of increase slowing — Trend improving
-	+	Rate of decrease slowing — Watch carefully

ROBUST STANDARD DEVIATION

$$MAD = \text{median}(|x_i - \text{median}(x)|)$$

$$\sigma \approx 1.4826 \times MAD$$

Z-SCORE (PERSONALIZED BASELINE)

$$Z = \frac{x - \mu}{\sigma}$$

Z-Score	Probability Above	System Status
≥ 2.0	$\approx 2.5\%$	Attention (Yellow Alert)
≥ 3.5	$\approx 0.02\%$	Critical (Red Alert)

TIME-TO-CRITICAL - LINEAR (STEADY REGIME)

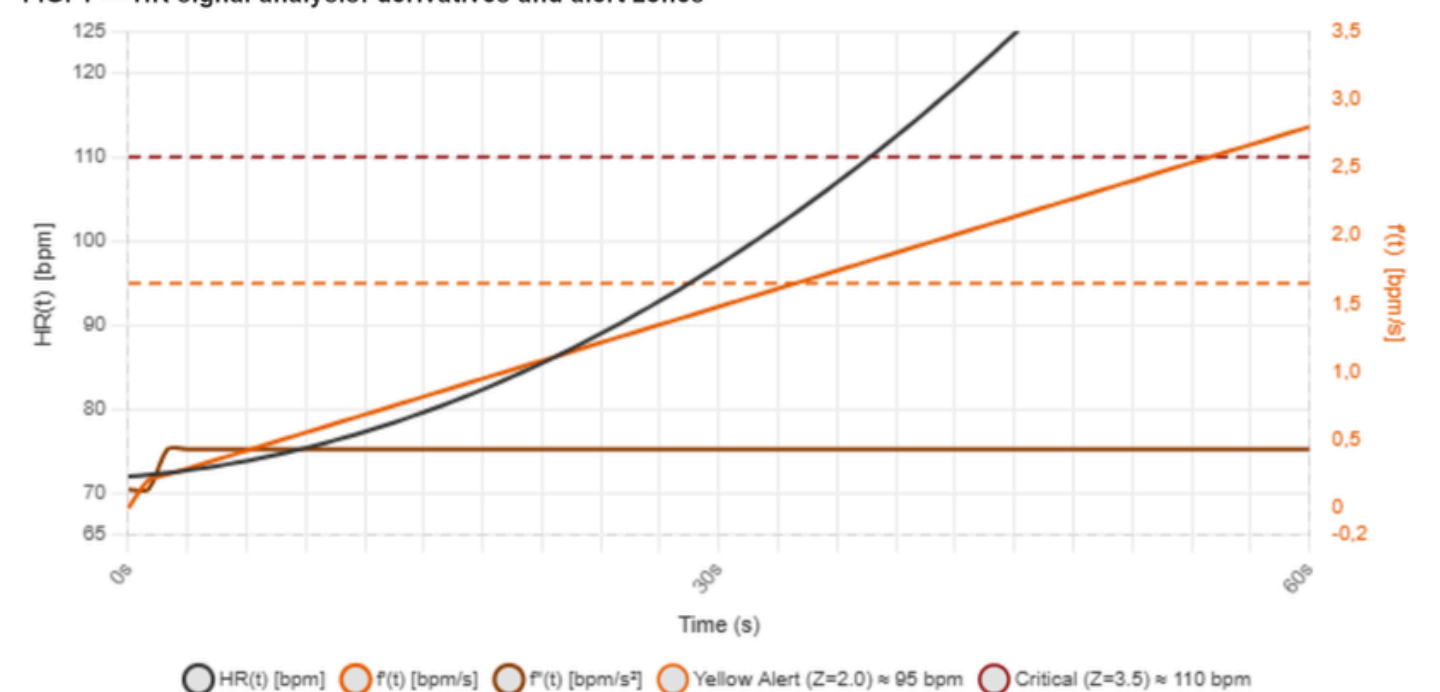
$$t_{\text{Min}} = \frac{Z_{\text{critical}} - Z_{\text{current}}}{\text{Rate of Change } Z}$$

TIME-TO-CRITICAL - QUADRATIC ($f'' > 0$, BHASKARA)

$$\frac{1}{2} \cdot a \cdot t^2 + v_0 \cdot t + (Z_0 - Z_{\text{critical}}) = 0$$

Solved analytically via the quadratic formula; only the positive root is retained.

FIG. 1 — HR signal analysis: derivatives and alert zones



CONCLUSION

The proposed framework integrates calculus-based temporal analysis with robust statistical modeling into a transparent and mathematically consistent predictive system for physiological monitoring. By combining discrete derivative analysis with individualized Z-score baselines, the system moves from reactive to anticipatory monitoring, providing healthcare professionals and individuals with quantitative lead time before critical cardiovascular events.

KEYWORDS

Heart Rate Prediction · Temporal Dynamics · Robust Statistics · Z-Score · Median Absolute Deviation · Finite Difference Methods · Predictive Monitoring · Calculus · Derivative Analysis

SESSION

Statistics and Operational Research

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