IOCMA 2026 Conference

The 2nd International Online Conference on Mathematics and Applications

10–12 June 2026 | Online

MDPI

Kernel-Based Nonparametric Tests for Exponentiality Against Decreasing or Increasing Residual Entropy Alternatives

HASSINA BENAOUDIA

[1] Department of Probability and Statistics, Faculty of Mathematics,
University of Science and Technology Houari Boumediene (USTHB), Algeria
Laboratory of Research in Intelligent Informatics, Mathematics and Applications (RIIMA),
[2] Faculty of Economics, University of Tipaza, Algeria

benaoudia.hassina@univ-tipaza.dz

INTRODUCTION & AIM

Testing for exponentiality is essential in reliability theory and survival analysis because the exponential distribution is uniquely characterized by a constant hazard rate and memoryless property.

Information-theoretic measures, especially residual entropy, are effective to detect departures from exponentiality through decreasing or increasing residual life uncertainty (Ebrahimi, 1997; Benaoudia & Aissani, 2023). Under exponentiality, residual entropy is constant.

Aim: We propose a kernel-based nonparametric test for exponentiality based on Shannon entropy and residual entropy, with strong theoretical properties and high empirical power against monotonic alternatives (DURL/IURL).

METHOD

Entropy Measures

Shannon entropy and residual entropy quantify the uncertainty of the lifetime and the remaining lifetime after time t.

Hypotheses

H_0: The data follow an exponential distribution (constant residual entropy).
H_1: The data follow a distribution with decreasing or increasing residual entropy.

Test Statistic

A statistic S_n (and $-S_n$ for IURL) is built from kernel estimators of entropy, density and survival function based on Ebrahimi's derivative criterion.

Asymptotic Property

Under regularity conditions, S_n converges almost surely to its theoretical counterpart S under H_0.

Critical Values

Obtained via Monte Carlo simulations for different sample sizes and significance levels.

Performance Evaluation

Power studies under Weibull and Gamma alternatives and Pitman ARE comparisons with competing entropy-based tests.

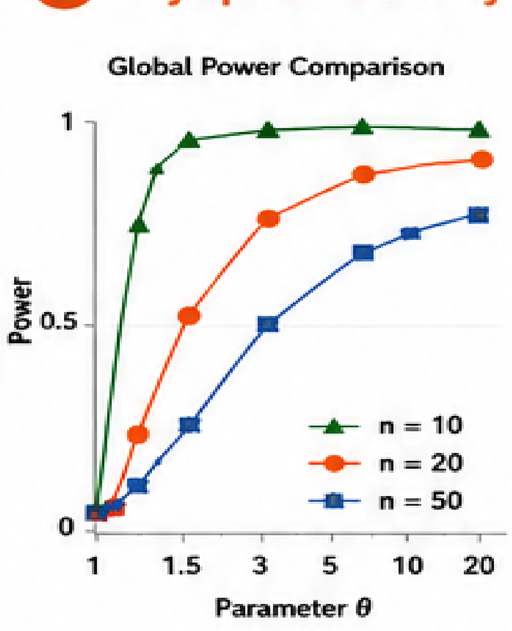
RESULTS & DISCUSSION

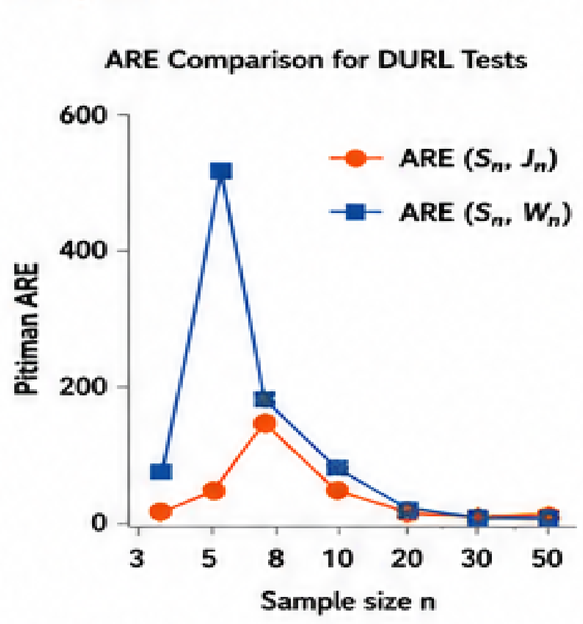
1 Critical Values (Monte Carlo Simulations)

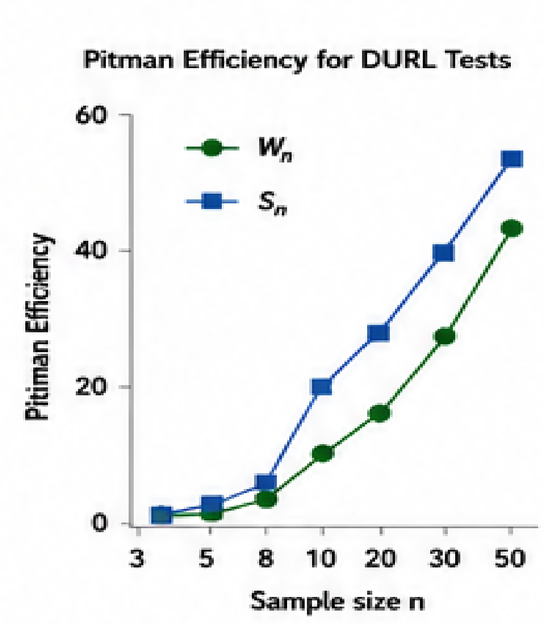
Sample size (n)	Significance level (α)	
	0.05	0.01
10	2.7039	3.2265
20	2.8424	3.1649
50	2.9213	3.1718

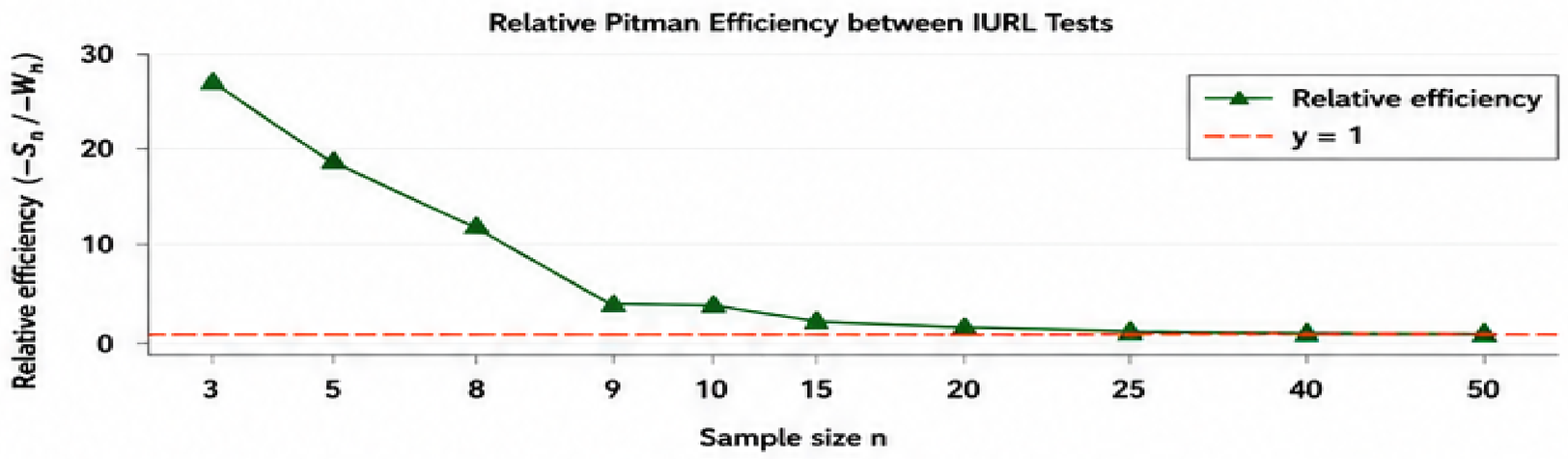
Critical values were obtained via Monte Carlo simulation.

2 Asymptotic Efficiency Comparisons









Relative Pitman efficiency of $-S_n$ with respect to $-W_n$ under IURL alternatives.
The horizontal line $y = 1$ represents equal efficiency.

The kernel-based statistic outperforms several entropy-based tests in both finite samples and asymptotically.

CONCLUSION

- A new kernel-based residual entropy test for exponentiality is proposed.
- The test has strong theoretical properties and converges almost surely under the null hypothesis.
- Simulation results show high power, especially against increasing residual life uncertainty alternatives.
- Pitman efficiency comparisons confirm the superiority of the proposed test over several existing tests.

FUTURE WORK / REFERENCES

Future Work

Extend the methodology to multivariate lifetime models and explore applications under dependent censoring and competing risks.

References

- Hassina Benaoudia and Amar Aissani. Nonparametric test for decreasing uncertainty of residual life distribution (DURL). *Mathematics*. MDPI. (2023).
- Nader Ebrahimi. Testing whether lifetime distribution is decreasing uncertainty. *Journal of Statistical Planning and Inference*. 1997.
- F. Belzunce, A. Güllamon, J. Navarro and M. Ruiz. Kernel estimation of residual entropy. *Communication Statistics – Theory and Methods*. 2001.