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A Beta-New XLindley Lifetime Model with Applications

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1. INTRODUCTION & AIM

Lifetime data analysis plays an essential role in engineering, medicine, finance, and reliability theory. Classical distributions often fail to model complex real-life data accurately.

To overcome this limitation, several generalized families have been proposed, such as beta-generated distributions including Beta-Exponential, Beta-Lindley, and Beta-Generalized Exponential models.

Aim of this work:

- Introduce a new flexible distribution called Beta-Polynomial Exponential Model (BPEM)
- Extend the Polynomial Exponential distribution using the beta-generated approach
- Study its statistical properties and real data performance

2. METHOD

The proposed model is constructed using the beta-generated family.

For a baseline CDF $F(x)$, the beta-generated distribution is defined as:

$$G(x) = \frac{1}{B(\alpha,\beta)} \int_0^{F(x)} t^{\alpha-1}(1-t)^{\beta-1} dt$$

with PDF:

$$g(x) = \frac{1}{B(\alpha,\beta)} f(x)[F(x)]^{\alpha-1}[1-F(x)]^{\beta-1}$$

Construction steps:

1. Start with Polynomial Exponential distribution
2. Apply beta transformation
3. Derive CDF and PDF
4. Obtain statistical properties

3. RESULTS & DISCUSSION

The baseline Polynomial Exponential distribution has CDF:

$$F_{\text{NPED}}(x,\theta) = 1 - \frac{L(n,\theta)\Gamma(n+1,x\theta)}{\Gamma(n+1)}$$

where:

$$L(n,\theta) = \frac{\sum_{n=0}^{N} \frac{a_{n,\theta}}{\theta^{n+1}}}{\sum_{n=0}^{N} a_{n,\theta}\frac{n!}{\theta^{n+1}}}$$

The corresponding PDF is:

$$f_{\text{NPED}}(x,\theta) = \frac{P(x,\theta)e^{-x\theta}}{\sum_{n=0}^{N} a_{n,\theta}\frac{n!}{\theta^{n+1}}}$$

where:

$$P(x,\theta) = \sum_{n=0}^{N} a_{n,\theta}x^n$$

4. BETA-POLYNOMIAL EXPONENTIAL MODEL

The cumulative distribution function (CDF) of the proposed model is given by:

$$G_{\text{BPE}}(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)} \sum_{k=0}^{\infty}\sum_{j=0}^{\infty} \frac{(-1)^{k+j}\Gamma(\alpha+k)}{k!\,j!\,\Gamma(\beta-k)} \times \frac{1}{\Gamma(\alpha+k+1-j)} (L(n,\theta))^j (\Gamma(n+1,x\theta))^j$$

Finally, the probability density function (PDF) is given by:

$$g_{\text{BPEM}}(x) = M_{\alpha,\beta,\theta,n}\exp(-x\theta)P(x,\theta) \times \sum_{j=0}^{\infty} \frac{(-1)^j}{j!\,\Gamma(\alpha-j)} (L(n,\theta))^{\beta+j-1} \times (\Gamma(n+1,x\theta))^{\beta+j-1}$$

where:

$$M_{\alpha,\beta,\theta,n} = \frac{\Gamma(\alpha+\beta)}{\Gamma(\beta)\sum_{n=0}^{N} a_{n,\theta}\frac{n!}{\theta^{n+1}}}$$

5. RELIABILITY AND HAZARD FUNCTIONS

Reliability function:

$$R(t) = 1 - G(t)$$

Hazard rate function:

$$h(t) = \frac{g(t)}{1-G(t)}$$

The model provides flexible shapes suitable for lifetime and reliability data analysis.

6. REAL DATA APPLICATION & COMPARISON

The model was applied to a dataset of 48 observations of luteinizing hormone levels.

Model	AIC	BIC	-2L	AICC
EX	182.05	183.92	180.05	182.13
L	166.24	168.11	164.24	166.33
NXL	170.27	172.14	168.27	170.36
BL	82.83	88.44	76.83	83.37
BE	82.82	88.44	76.82	83.37
BPE	**79.44**	**85.07**	**73.44**	**80.02**

7. CONCLUSION

- A new Beta-Polynomial Exponential Model was proposed.
- Several statistical properties were derived and tested.
- Real data shows superior performance over classical models.

8. REFERENCES

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