Envelopping algebras from stochastic automata of topological subshifts: integrable dynamical system of newly-weakened hyperbolicity IOCMA 2026 Conference MDPI

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Introduction and aim

The procedures are here proven, to write the weakened ismorphism from the toral homeomorphisms which is issued from the weak topology:

1. the index set of the automata subshifts is here newly defined from the directed acyclic graphs, whose trees are analyzed after substochastic probability matrices, after marginalization of the opportune variables after a new Tucker-like method, of the new suitable accumulation, time-dependent model;

2. the *-envelopping algebras corresponding to the functionals of stochastic automata from topological subshifts are newly spelled out as the restriction of all the functions of isometric isomorphism on the weak open set.

Results and discussion

- The original functional is smooth: the homeomorphism in the weak topology is a diffeomorphism in the strong topology-

- The newly-found dynamical system is an integrable dynamical system of newly- 'weakened' hyperbolicity.

- The Oseledez multipicative ergodig theorem is upgraded on the obtained torus with one marked point, where the trajectories have Ljapunov exponent equal to 0.

- This dynamical system is compared with that from the Ruelle diffeomorphism when the Sinai-Ruelle-Bowen measure is substituted with the new op- portune uniform-entropy measure.

- The new corresponding Miatello-Wallach family of meromorphic functions is written: the variances are indicated to be calculated after application of generic potential theory.

- The star-algebras are proven to be reduced to envelopping algebra, to which the Tomita-Takesaki paradigm is proven to be applied.

Methods

-The Fréchet operator is descended to the ordinary differentials in finite-dimensional spaces, after which the measure from the functional is written after the Radon-Nikodym derivative $d\mu/d\nu$: the transition probabilities are the derivatives of the functionals, which is a density function with respect to the measure ν.

- The Fréchet operator is written as a map, which is used to define isomorphisms between Fréchet spaces:
• after having constructed the isomorphism using the weak-*-topology on the dual space, this map is used to transfer the isomorphism to a weak-topology-norm space.

- The isometric isomorphism is a specification of the Rokhlin aperiodic endomorphism.

- The non-empty compact space, the distance on it, and the functional constitute a triple which is a Smale space.

Technicalities I

The dynamical system on a torus
The Oseledez Theorem is here upgraded as
Theorem a): The dynamical system is a hyperbolic manifold as it obeys the generalized Hadamard Lemma as

$$\lambda(e^{k+r}, e) = \lambda(e^k, a)\lambda(e^r, a) = 1.$$

Proof a): From the Generalized Hadamard Lemma,

$$\lambda(e^{k+r}, e) = \lambda(e^k, a)\lambda(e^r, a)det[A]$$

with $det[A] = 1$. □
Let d be the dimension of the dynamical system:
Theorem b): The dimension d d the dynamical system is $d = 1$ for $k \equiv r$ from the toral coordinates.
Proof b): As

$$\lambda(a_t \cdot e^k, a_t^{-1}) = \frac{1}{\lambda(e^k, a_t)}$$

□.

Technicalities II

From the work of Rokhlin,
Theorem c): The system is ergodic.
Proof c): Ergodicity is a spectral property. □

Remarks:
i) The Oseledez multiplicative ergodic theorem can therefore be applied.
ii) The space $L_2(\mathcal{M})$ can also be used to study the new Miatello-Wallach functions.
iii) The Kubo-Martin-Schwinger (KMS) conditions are fulfilled:
• the Tomita-Takesaki theory is applied.

References

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Future work

How to **map isomorphisms from a Lebesgue space to transformations on a weak topology norm space**:
- the map must be an isometric isomorphism ($T : X \to Y$);
- use its dual map to create an isomorphism between the dual spaces (Y^*) and (X^*):
• the new dual map ($T^* : Y^* \to X^*$) is defined as ($T^*(f) = f \circ T$) for any functional ($f \in Y^*$) isometric isomorphism;
- the dual map (T^*) is a homeomorphism between the weak topologies of (Y^*) and (X^*).
*The space of norms is defined by the weak * topology.*

How to **make a homeomorphism with respect to weak topologies correspond to a specific diffeomorphism** and how to **guarantee the existence of one in a stronger (e.g., smooth) topology**:
- define the Borel Sigma algebra of the topological space.

How to **transform an isomorphism from a Lebesgue space to the isomorphisms on a weak-topology-norm space** after the utilization of the dual space for the isomorphism to be transferred making use of the Fréchet operator:
- define the spaces and the dual space is a Lebesgue space (L^p), which is a Banach space.
The isomorphism is ($T : L^p \to W$) *where W is the weak-topology-norm space.*
How to **u**tilize a Fréchet operator: - after constructing the isomorphism using

the weak-* topology on the dual space:
• this map can be used to transfer the isomorphism to a weak-topology-norm space.

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