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Optimizing Emergency-Station Placement through Roman and Double Roman Domination in Zero-Divisor Graphs of Finite Rings

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INTRODUCTION & AIM

Graphs associated with algebraic structures have attracted considerable attention due to their ability to transform abstract algebraic concepts into analyzable combinatorial models.

Let R be a finite commutative ring. The **Zero-Divisor Graph** $\Gamma(R)$ is defined as the graph whose vertices are the non-zero zero divisors of R, where two distinct vertices x and y are adjacent whenever

$$xy = 0.$$

Domination concepts on graphs provide powerful mathematical tools for solving resource allocation and location problems.

Research Aim

This work investigates:

Roman Domination Numbers of zero-divisor graphs associated with finite rings.

Double Roman Domination Numbers of zero-divisor graphs associated with finite rings.

Applications of these domination models to optimal placement of emergency stations, hospitals, rescue units, and disaster-response facilities.

METHOD

Roman Domination

A function

$$f: V(G) \to \{0, 1, 2\}$$

is called a Roman dominating function if every vertex assigned 0 has a neighboring vertex assigned 2.

The Roman domination number is

$$\gamma_R(G) = \min \sum \quad f(v).$$

Double Roman Domination

A function

$$f: V(G) \to \{0, 1, 2, 3\}$$

is called a Double Roman Dominating Function (DRDF) if:

Every vertex assigned 0 is adjacent to either

- a vertex assigned 3, or
- at least two vertices assigned 2.

Every vertex assigned 1 is adjacent to a vertex assigned at least 2.

The Double Roman Domination Number is

$$\gamma_{dR}(G) = \min \sum \quad f(v).$$

Rings Investigated

Integer modulo rings $\mathbb{Z}_n$

Polynomial rings

Quotient polynomial rings

For each ring:

Construct the zero-divisor graph.

Determine structural properties.

Compute RD and DRD numbers.

Interpret optimal facility placement solutions.

RESULTS & DISCUSSION

Main Findings

✓ Roman domination numbers were computed for several classes of zero-divisor graphs arising from finite commutative rings.

✓ Double Roman domination values provide stronger protection and redundancy compared with standard Roman domination.

✓ DRD models guarantee enhanced coverage by allowing backup facilities for critical locations.

✓ Structural properties of the underlying ring significantly influence both domination parameters.

Facility Placement Interpretation

Graph-Theoretic Model	Practical Interpretation
Vertex	Location/Region
Edge	Direct service connection
Roman Dominating Vertex (2)	Fully equipped emergency station
Double Roman Value (3)	Major emergency hub
Dominated Vertex	Area receiving coverage

Application Areas

Emergency response systems

Hospital network planning

Disaster management

Fire station allocation

Security and surveillance networks

CONCLUSION

Zero-divisor graphs provide an effective bridge between algebraic structures and real-world optimization problems. Roman domination identifies minimum-cost emergency coverage strategies. Double Roman domination offers stronger reliability through backup coverage mechanisms. The obtained results demonstrate that domination theory can support optimal facility allocation and emergency planning in complex networks. Future developments may extend these models to fuzzy, weighted, and dynamic zero-divisor graphs.

FUTURE WORK / REFERENCES

Future Work

Triple Roman domination on zero-divisor graphs.

Fuzzy and weighted domination models.

Dynamic emergency-network optimization.

AI-assisted facility location using graph domination techniques.

References

I. Beck, Coloring of Commutative Rings.

D. F. Anderson and P. S. Livingston, The Zero-Divisor Graph of a Commutative Ring.

E. J. Cockayne et al., Roman Domination in Graphs.

Recent studies on Double Roman Domination and Network Optimization.