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On Uniform HX-Rings

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INTRODUCTION & AIM

The classical algebraic structures of groups and rings have been upgraded to the new structures of HX-groups and HX-rings, by a group of Chinese mathematicians [1,2,3].

An HX-group is a group defined on a nonempty subset $\mathcal{G}$ of P^*(G), where P^*(G) denotes the family of nonempty subsets of a group G, by extending the operation on G to an operation " $\circ$ " on P^*(G): $U \circ V = \{u \cdot v \mid u \in U, v \in V\}$, for all $U, V \in \mathcal{G}$. Similarly, consider a ring (R,+,·) and let P^*(R) be the family of all non-empty subsets of R. For any arbitrary subsets $A, B \in P^*$(R), define the HX-operations $A + B = \{a + b \mid a \in A, b \in B\}$ and $A.B = \{ab \mid a \in A, b \in B\}$. A nonempty subset $\mathcal{R}$ of P^*(R) is called an HX-ring on R if it forms a ring under the HX-operations defined above.

In this presentation we aim to recall the theory of HX-rings, focusing on the particular class of uniform HX-rings. Motivated by the connection established between HX-groups and hypergroups [5], we will try to construct hyperrings starting from uniform HX-rings.

DEFINITION AND EXAMPLES OF HX-RINGS

Let ($\mathcal{R}$,+,·) be an HX-ring on the ring (R,+,·). Denote by $\mathcal{Q}$ the zero element of $\mathcal{R}$ (it is not necessarily the set {0}, where 0 is the zero element of the ring R).

Definition [2] An HX-ring $\mathcal{R}$ on a ring R is called
(i) regular, if 0 ∈ $\mathcal{Q}$;
(ii) normal, if ($\mathcal{Q}$,+) is a monoid of (R,+);
(iii) uniform, if ($\mathcal{Q}$,+) is a subgroup of (R,+).

Example 1: Let R = Z_4 = $\{\bar{0}, \bar{1}, \bar{2}, \bar{3}\}$ be the ring of integers modulo 4. Consider the family $\mathcal{R}$ = $\{\{\bar{0},\bar{2}\}, \{\bar{1},\bar{3}\}\} \subseteq P^*$(R) ordered with the HX-operations represented by the following Cayley tables.

+	$\{\bar{0},\bar{2}\}$	$\{\bar{1},\bar{3}\}$
$\{\bar{0},\bar{2}\}$	$\{\bar{0},\bar{2}\}$	$\{\bar{1},\bar{3}\}$
$\{\bar{1},\bar{3}\}$	$\{\bar{1},\bar{3}\}$	$\{\bar{0},\bar{2}\}$

.	$\{\bar{0},\bar{2}\}$	$\{\bar{1},\bar{3}\}$
$\{\bar{0},\bar{2}\}$	$\{\bar{0},\bar{2}\}$	$\{\bar{0},\bar{2}\}$
$\{\bar{1},\bar{3}\}$	$\{\bar{0},\bar{2}\}$	$\{\bar{1},\bar{3}\}$

It is simple to check that ($\mathcal{R}$,+,·) is an HX-ring with the zero element $\mathcal{Q}$ = $\{\bar{0},\bar{2}\}$. Since ($\mathcal{Q}$,+) is a subgroup of R = $\mathbb{Z}_{\mathbf{4}}$, it follows that $\mathcal{R}$ is a uniform HX-ring.

Example 2: Let R = $\mathbb{Z} \times \mathbb{Z}$ be the ring defined as the direct product of $\mathbb{Z}$ with itself and consider $\mathcal{R}$ = {{(a,0)} | a∈ $\mathbb{Z}$ } ⊆ P^*($\mathbb{Z} \times$ {0}). We easily prove that both addition and multiplication are closed operations, but other axioms of a ring are not verified.

We notice that the set $\mathcal{Q}$ = {(0,0)} is a good candidate to be a zero element. If we take A = {(1,0),(2,0)} ∈ $\mathcal{R}$ and try to find its inverse with respect to the addition, we see that it should contain the set B = {(-1,0),(-2,0)} but then A + B = {(0,0),(-1,0),(1,0)} ≠ {(0,0)}. Thus, ($\mathcal{R}$,+) is not a group, so $\mathcal{R}$ is not an HX-ring on ring R.

CONSTRUCTIONS OF HX-RINGS

One method to construct HX-rings consists in defining them as quotient sets (that are not necessarily quotient rings).

Theorem 1: [4] Let H be a subring of a ring $\mathcal{R}$ and let I be a nonempty set of the ring $\mathcal{R}$ such that $I + I = I$. If an equality $(h_1 + I) \cdot (h_2 + I) = h_1 h_2 + I$ holds for any $h_1, h_2 \in H$, then $H/I = \{h + I \mid h \in H\}$ is an HX-ring on $\mathcal{R}$ with the zero element I.

Theorem 2 : [4] Let H be a subring of a ring $\mathcal{R}$ and I be a non-empty set of $\mathcal{R}$ such that $I + I = I.I = I.$ $If\ I.H = H.I = 0$, then $H/I = \{h + I \mid h \in H\}$ is an HX-ring on a ring $\mathcal{R}$.

DISCUSSION AND FUTURE WORK

Inspired by the previous results, we aim to find under which conditions (applied to the set I) the quotient H/I becomes a uniform HX-ring. Is this the only method to construct HX-rings or could we find a different one?

The other purpose of our future work is to define a hyperope-ration, i.e. a multivalued operation, starting from an HX-ring in order to get a hyperring structure. For doing this, we will start from the construction of Chinese hypergroupoids, given by Corsini as follows. Let $(\mathcal{G}, *)$ be an HX-group and G* = $\cup_{A \in \mathcal{G}} A$. Define on G* the hyperoperation:

$x \oplus y = \bigcup_{\substack{A,B \in \mathcal{G} \\ x \in A \\ y \in B}} A * B$, for any $x, y \in$ G*, where $x \in A$ and $y \in B$. If for any $A, B \in$ G* the implication $A \cap B \neq \emptyset \Rightarrow A = B$ holds, then (G*, $\oplus$) is a hypergroup.

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